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Modeling Motion by the Integration of Topology and Time

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Abstract: A qualitative representational model and the corresponding reasoning process for integrating time and topological information is developed in this paper. In the calculus presented, topological information in function of the point of the time in which it is true is represented as an instance of the Constraint Satisfaction Problem. The resulting method can be applied to qualitative navigation of autonomous agents. The model presented in this paper will help us during the path planning task by describing the sequence of topological situations that the agent should find during its way to the target objective. A preliminary result of that application has been obtained by using qualitative representation of such spatial aspects for the autonomous simulated navigation of a Nomad-200 robot, on a structured environment of an easy corridor in a building.

Key Words: Qualitative Reasoning, Temporal Reasoning, Spatial Reasoning, Constraint Logic Programming, Autonomous mobile robot Navigation, Integration.
Categories: H.1, D.1.6.

1 Introduction

An autonomous mobile robot should be able to understand and reason with spatial aspects such as orientation, named distances, compared distances, cardinal directions, topology and time, in such a straightforward way as humans do. Spatial information that humans obtain through perception is coarse and imprecise, thus qualitative models which reason with distinguished characteristics rather than with exact measures seems to be more appropriated to deal with this kind of knowledge.

Several qualitative models have been developed for dealing with spatial concepts such a orientation [Guesguen 89], [Jungert 92], [Mukerje & Joe 90], [Freksa 92, Freksa & Zimmerman 92], [Hernández 94], named distances [Zimmermann 93], [Jong 94], [Clementini et al. 95], [Escrig & Toledo 00, 01], cardinal directions [Frank 92], an so on. A good state of the art of work in qualitative theories as a basis for commonsense can be found in [Cohn & Hazarika 01]. The concept of qualitative motion has been dealt in [Zimmermann and Freksa 93], [Musto, Stein et al. 00] and [Musto et al. 99]. In most of these approaches, motion has been modeled as a sequence of changes of positions, taking into account conceptual neighborhood, but without integrating the concept of time into the same model. The approach by [Escrig and Toledo 02] introduces an algebra of a qualitative model for representing and reasoning with velocity. Our aim is to formalize the intuitive notion of spatio-temporal continuity for a qualitative theory of motion. As motion can be seen as a form of spatio-temporal change, the paper presents a qualitative representation model for integrating qualitative time and topological information for modeling motion and reasoning about dynamic worlds in which spatial relations between regions may change with time.

The bases for the integration in the spatial reasoning field of different spatial aspects, have been inspired in the temporal reasoning field, where the integration of point algebra, interval algebra and metric information has been successfully accomplished [Meiri 91]. In order to accomplish the task of integrating different spatial aspects in the same model, the next three steps are defined [Escrig and Toledo 00]:

- the representation of each spatial aspect to be integrated
- the definition of the Basic Step of the Inference Process (BSIP). It is defined such as: given the spatial relationship between objects A and B, and the spatial relationship between objects B and C, the BSIP consists of obtaining the spatial relationship between A and C.
- the definition of the Complete Inference Process (CIP), that consists of repeating the BSIP as many times as possible, with the initial information and the information provided by previous steps of the BSIP, until no more information can be inferred.

The concepts of orientation, cardinal directions, and absolute and relative distances have been integrated into the same model thanks to consider the representation and the reasoning process of each aspect as an instance of the Constraint Satisfaction Problem [Escrig and Toledo 98, 00].

In this paper, topological together with time information will also be integrated into the previous mentioned model, following the same idea.

The paper is structured as follows, first section 2 provides a brief description of the bases for the integration of several temporal aspects which inspired the bases for the integration of several spatial aspects. In section 3, the bases for the integration of several spatial aspects are provided as well as the topology and time algebras which integrate the motion model. Section 4 explains one of the applications of the model presented to real mobile robot navigation. Finally section 5 concludes and explains the future work to be done.

2 Bases for the Integration of Several Temporal Aspects

The integration of several temporal aspects has been accomplished by considering them as instances of the Constraint Satisfaction Problem (CSP). A CSP for binary constraints can be formulated such that: given a set of variables $\{X_1, ..., X_n\}$, a discrete and finite domain for each variable $\{D_1, ..., D_n\}$, and a set of constraints $\{c_{ij}(X_i, X_j)\}$, which define the relationship between every couple of variables X_i, X_j, $(1 \leq i < j \leq n)$; the problem is to find an assignment of values $\langle v_1, ..., v_n \rangle$, $v_i \in D_i$ to variables such that all constraints are satisfied, i.e. $c_{ij}(X_i,X_j)$ is true for every i,j ($1 \leq i<j \leq n$). Every different assignment of values that satisfies all the constraints is called a solution.

A Constraint Satisfaction Problem is usually represented as a graph, called *Constraint Graph* where the nodes are the variables and the arcs are the binary constraints. Unary constraints can be disposed of by just redefining the domains to contain only the values that satisfy all the unary constraints. Higher order constraints are represented by hyperarcs. In the following we restrict our attention to the case of unary and binary constraints.

Generate and test and backtracking are algorithms which solve the CSP, although in a very inefficient way. These algorithms have an exponential cost. Research in the field tries to improve efficiency of the backtracking algorithm (a review of the state of the art can be found in [Meseguer 89] and [Kumar 92]). A set of these algorithms modify the search space before the search process starts, to make the search process easier. They are called algorithms which improve consistency. These algorithms are based on the idea of making explicit the implicit constraints by means of the constraint propagation process. Unfortunately the complete constraint propagation process is also hard, therefore the process is approximated by local constraint propagation, as path consistency. If the constraint graph is complete (that is, there is a pair of arcs, one in each direction, between every pair of nodes) it suffices to repeatedly compute paths of two steps in length at most. This means that for each group of three nodes (i,k,j) we repeatedly compute the following operation until a fix point is reached [Fruehwirth 94]:

$$c_{ij} := c_{ij} \oplus c_{ik} \otimes c_{kj} \qquad (1)$$

This operation computes the composition of constraints, denoted by c_{ij}, c_{ik} and c_{kj}, ($\otimes$) between nodes *ik* and *kj*, and the intersection ($\oplus$) of the result with constraints between nodes *ij*. The complexity of this algorithm is $O(n^3)$, where *n* is the number of nodes in the constraint graph (that is, the number of objects involved in the reasoning process) [Kumar 92; Mackworth and Freuder 85].

Constraint Handling Rules (CHRs) are a tool which helps to write the above algorithm. They are an extension of the Constraint Logic Programming (CLP) which facilitate the definition of constraint theories and algorithms which solve them. They facilitate the prototyping, extensions, specialization and combination of constraints [Fruehwirth 94]. There exist mainly two types of CHRs: propagation and simplification. *Propagation* CHRs add new constraints which are logically redundant but may cause further simplification. A *propagation* CHR is of the from:

$$H_1,\cdots,H_i \text{ ==> } G_1,\cdots,G_j \mid B_1,\cdots,B_k \qquad (i>0,\ j\geq 0,\ k\geq 0)$$

In the CHRs, the multi-head ($H_1,\ldots,H_i$) is a conjunction of user-defined constraints and the guard ($G_i,\ldots,G_j$) is a conjunction of literals. The propagation from user-defined constraints, H', means the addition of the set of constraints B to the initial set of constraints if H' matches the head (H) of a propagation rule and G is satisfied. This kind of rules are used to compute the part '$\otimes$' of formula (1).

Simplification CHRs replace constraints by simpler constraints preserving logical equivalence. A *simplification* CHRs is of the form:

$$H_1, \cdots, H_i \Leftarrow\Rightarrow G_1, \cdots, G_j \mid B_1, \cdots, B_k \qquad (i>0,\ j\geq 0,\ k\geq 0)$$

To *simplify* the user-defined constraints H' means to replace them by B if H' matches the head (H) of a simplification rule ant the guard G is satisfied. This kind of rules are used to compute the part '⊕' of formula (1).

3 Bases for the Integration of Several Spatial Aspects

Our aim is to formalize the intuitive notion of spatio-temporal continuity for a qualitative theory of motion. Therefore, we propose a constraint-based approach to integrate the topological calculus developed in [Isli, et al. 00] and temporal constraints.

3.1 Overview of the Topological Calculus

To make this paper self-contained we now explain the topological calculus selected to get the integration of topology and time. A fuller explanation can be found in [Isli, et al. 00].

The topological calculus selected is a constraint-based approach to the Calculus Based Method (CBM) developed by Clementini, Di Felice, and Oosterom ([Clementini & Di Felice 95]; [Clementini et al. 93]). The calculus is an algebra as the one of Allen (1983) presented for temporal intervals, of which the atomic relations will be the three relations resulting from the refinement of the *in* relation, together with the other four atomic relations of the CBM calculus. The calculus provides the result of applying the converse and the composition operations to the atomic relations: this is given as a converse table and composition tables. These tables in turn will play the central role in propagating knowledge expressed in the algebra using Allen's constraint propagation algorithm ([Allen 83]).

We have chosen this topological calculus because it allows us to reason about point-like, linear and areal entities and it is presented as an algebra alike to Allen's [Allen 83] temporal interval algebra. The fact of managing with point-like, linear and areal entities will allow us the use of different granularities of the same map. The calculus defines 9 topological relations, which are described bellow.

Before providing the formal definition of each topological relation we define a number of basic topological concepts needed to understand the definition of the topological relations, as it has been done in [Isli et al. 00]:

Definition 1. *The boundary of an entity h, called δh is defined as:*

- *We consider the boundary of a point-like entity (a point) to be always empty.*
- *The boundary of a linear entity (a line) is the empty set in the case of a circular line or the 2 distinct endpoints otherwise.*
- *The boundary of an area is the circular line consisting of all the accumulation points of the area.*

Definition 2. *The interior of an entity h, called h^o, is defined such as $h^o=h-\delta h$*

Definition 3. *The function **dim**, which returns the dimension of an entity of either the types we consider, or the dimension of the intersection of 2 or more of such entities, set denoted by S, is defined as follows (the symbol ∅ represents the empty set):*

If $S \neq \varnothing$ then
dim(S) = 0 if S contains at least a point and no lines and no areas.
dim(S)= 1 if S contains at least a line and no areas.
dim(S)= 2 if S contains at least an area.
Else dim(S) is undefined.

A topological relation r between two entities h1 and h2, denoted by (h_1,r,h_2), is defined on the right hand side of the equivalence sign in the form of a point-set expression. Definitions for each topological relation are given bellow.

Definition 4. *The touch (T) relation is defined such as:*

$(h_1, \textbf{touch}, h_2) \Leftrightarrow h_1^\circ \cap h_2^\circ = \varnothing \wedge h_1 \cap h_2 \neq \varnothing$

Definition 5. *The cross (C) relation is defined such as:*

$(h_1,\textbf{cross},h_2) \Leftrightarrow dim(h^\circ_1 \cap h^\circ_2) = max(dim(h^\circ_1),dim(h^\circ_2)) - 1 \;\wedge\; h_1 \cap h_2 \neq h_1 \wedge h_1 \cap h_2 \neq h_2$

Definition 6. *The overlap (O) relation is defined as follows:*

$(h_1, \textbf{overlap}, h_2) \Leftrightarrow dim(h^\circ_1) = dim(h^\circ_2) = dim(h^\circ_1 \cap h^\circ_2) \wedge h_1 \cap h_2 \neq h_1 \wedge h_1 \cap h_2 \neq h_2$

Definition 7. *The disjoint (D) relation is defined as follows:*

$(h_1, \textbf{disjoint}, h_2) \Leftrightarrow h_1 \cap h_2 = \varnothing$

Definition 8. *The equal (E) relation is defined such as:*

Given that $(h_1, in, h_2) \Leftrightarrow h_1 \cap h_2 = h_1 \;\wedge\; h^\circ_1 \cap h^\circ_2 \neq \varnothing$: *if* (h_2, in, h_1) *then* $(h_1, \textbf{equal}, h_2)$

Definition 9. The touching-from-inside (TFI) relation is defined as follows:

Given that (h_1, in, h_2) *and not* $(h_1, equal, h_2)$: *if* $h_1 \cap \delta h_2 \neq \varnothing$ *then* $(h_1,$ ***touching-from-inside***$, h_2)$.

Definition 10. The completely-inside (CI) is defined such as:

If (h_1, in, h_2), *not* $(h_1, equal, h_2)$ *and not* $(h_1, touching\text{-}from\text{-}inside, h_2)$ *then:* $(h_1,$ ***completely-inside***$, h_2)$.

Definition 11. The completely-inside$_i$ (CI_i) relation is defined as follows:

$(h_1, \textbf{completely-inside}_i, h_2) \Leftrightarrow (h_2, completely\text{-}inside, h_1)$

Definition 12. The touching-from-inside$_i$ (TFI_i) is defined such as:

$(h_1, \textbf{touching-from-inside}_i, h_2) \Leftrightarrow (h_2, touching\text{-}from\text{-}inside, h_1)$.

Table 1 shows some pictorial graphic examples of the different relations defined above.

The relations are mutually exclusive, that is, it cannot be the case that two different relations hold between two features. Moreover it can be proven that they form a full covering of all possible topological situations, that is, given two features, the relation between them must be one of the nine defined here. To prove these two characteristics we construct the *topological relation decision tree* depicted in [Fig. 1].

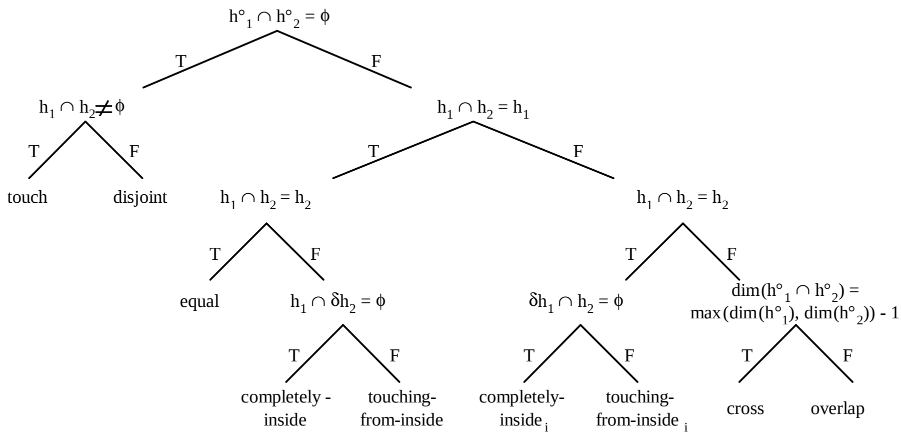


Figure 1: Topological relation decision tree.

Proof. Every internal node in this topological relation decision tree represents a Boolean predicate of a certain topological situation. If the predicate evaluates to true (T) then the left branch is followed, otherwise (the predicate evaluates to false (F)) the right branch is followed. This process is repeated until a leaf node is reached which will indicate which of the atomic topological relations this situation corresponds to. Two different relations cannot hold between two given features, because there is only one path to be taken in the topological relation decision tree to reach a particular topological relation. And there can be no cases outside the new calculus, because every internal node has two branches, so for every Boolean value of the predicate there is an appropriate path and every leaf node has a label that correspond to one of the atomic topological relations

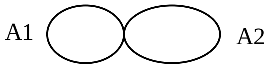
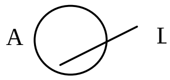
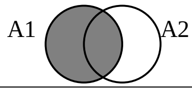
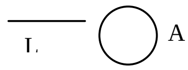
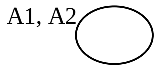
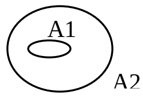
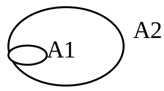
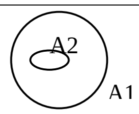
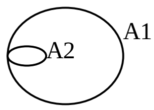
Relation	**Graphic Example**
touch	
cross	
overlap	
disjoint	
equal	
completely-inside	
touching-from-inside	
completely-inside$_i$	
touching-from-inside$_i$	

Table 1: Graphic Exmaples of the Topological relations defined.

Next tables [Tab. 2, Tab. 3-20] represent the converse and composition operations.

Given any three regions A,B and C such that (A, r1, B) and (B, r2, C), the composition tables should be able to provide the most specific implied relation R between the extreme regions, i.e. between A and C. If we consider all possibilities with A, B, and C being a point-like feature, a linear feature, or an areal feature, we would need 27 (3^3) tables. However, only 18 have been constructed, from which the other 9 can be obtained using the converse operation and the constructed tables. The 18 tables to be constructed split into 6 for B=point-like feature, 6 for B=linear feature and 6 for B=areal feature: when regions A is of type X, region B is of type Y, and region C is of type Z, with X, Y, and Z belonging to {P, L, A}, where P = point feature, L= line feature and A= area feature, the corresponding composition table will be referred to as the XYZ table.

R	**R$^{\cup}$**
Overlap	Overlap
Touch	Touch
Cross	Cross
Disjoint	Disjoint
Completely-inside	Completely-inside$_i$
Touching-from-inside	Touching-from-inside$_i$
Completely-inside$_i$	Completely-inside
Touching-from-inside$_i$	Touching-from-inside
Equal	Equal

Table 2: The converse table (R$\cup$ denotes the converse relation of R)

r_2 / r_1	**E**	**D**
E	E	D
D	D	{E, D}

Table 3: The PPP composition table

r_2 / r_1	**T**	**D**	**CI**
E	T	D	CI
D	{T, D, CI}	{T, D, CI}	{T, D, CI}

Table 4: The PPL composition table

r_2 / r_1	**T**	**D**	**CI**
E	T	D	CI
D	{T, D, CI}	{T, D, CI}	{T, D ,CI}

Table 5: The PPA composition table

r_2 / r_1	**T**	**D**	**CI**
T	{T, C, TFI}	{T, C, D}	{C, TFI, CI}
D	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}
CI$_i$	{T, C}	{T, C, D}	{C, CI, TFI}

Table 6: The LPA composition table

r_2 / r_1	**T**	**D**	**CI**
T	{T, O, C, E, TFI, TFI_i}	{T, D, O, C , TFI_i, CI_i}	{T, O, C, TFI, CI}
D	{T, D, O, C, CI, TFI}	{T, D, O, C,E,TFI,CI,TFI_i, CI_i}	{T, D, O, C, TFI, CI}
CI_i	{T, O, C, TFI_i, CI_i }	{T, D, O, C, TFI_i, CI_i}	{O,C,E,TFI,CI,TFI_i, CI_i}

Table 7: The LPL composition table

r_2 / r_1	**T**	**D**	**CI**
T	{T, O, E, TFI, TFI_i}	{T, O, D, CI_i, TFI_i}	{O, TFI, CI}
D	{T, O, D, TFI, CI}	{T, O, D, E, TFI, CI, TFI_i, CI_i}	{T, O, D, CI, TFI}
CI_i	{O, TFI_i, CI_i}	{T, O, D, CI_i, TFI_i}	{O, E, TFI, CI, TFI_i, CI_i}

Table 8: The APA composition table

r_2 / r_1	**T**	**D**	**CI_i**
T	{E, D}	D	D
D	D	{E, D}	D
CI	D	D	{E, D}

Table 9: The PLP composition table

r_2 / r_1	**T**	**C**	**D**	**CI**	**TFI**
T	{T, D}	{T, D, CI}	D	CI	{T, CI}
D	{T, D, CI}	{T, D, CI}	{T, D, CI}	{T, D, CI}	{T, D, CI}
CI	{T, D}	{T, D, CI}	D	CI	{T, CI}

Table 10: The PLA composition table

r_2 / r_1	**T**	**C**	**O**	**D**	**E**	**CI**	**TFI**	**CI_i**	**TFI_i**
T	{T, D, CI}	{T, D, CI}	{T, D, CI}	D	T	CI	{T, D}	D	{T, D}
D	{T, D, CI}	{T, D, CI}	{T, D, CI}	{T, D, CI}	D	{T, D, CI}	{T, D, CI}	D	D
CI	{T, D}	{T, D, CI}	{T, D, CI}	D	CI	CI	CI	{T, D, CI}	{T, D, CI}

Table 11: The PLL composition table

r_2 / r_1	**T**	**C**	**D**	**TFI**	**CI**
T	{T, C, D, TFI}	{T, C, D, TFI, CI}	{T, C, D}	{T, C, TFI, CI}	{C, TFI, CI}
D	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}
O	{T, C, D}	{T, C, D, TFI, CI}	{T, C, D}	{C, TFI, CI}	{C, TFI, CI}
C	{T, C, D}	{T, C, D, TFI, CI}	{T, C, D}	{C, TFI, CI}	{C, TFI, CI}
E	T	C	D	TFI	CI
TFI	{T, D}	{T, C, D, TFI, CI}	D	{TFI, CI}	CI
CI	{T, D}	{T, C, D, TFI, CI}	D	{TFI, CI}	CI
TFI_i	{T, C}	C	{T, C, D}	{TFI, CI}	{C, TFI, CI}
CI_i	{T, C}	C	{T, C, D}	{TFI, CI}	{C, TFI, CI}

Table 12: The LLA composition table

r2 / r1	**T**	**C**	**O**	**D**	**E**	**CI**	**TFI**	**CI_i**	**TFI_i**
T	{T, D, O, C, E, TFI, CI, TFI_i,}	{T, C, O, D, TFI, CI}	{T, C, O, D, TFI, CI}	{T, C, O, D, TFI_i, CI_i}	T	{T, C, O, TFI, CI}	{T, C, O, TFI, CI}	D	{T, D}
C	{T, C, O, D, TFI_i, CI_i}	{T, D, O, C, E, TFI, CI, TFI_i,CI_i}	{T,D, O, C, TFI, CI}	{T, D, O, C, TFI_i,, CI_i}	C	{C, O, TFI, CI}	{C, O, TFI, CI}	{T, C, D}	{T, C, D}
O	{T, C, O, D, TFI_i, CI_i}	{T, D, O, C, TFI_i,, CI_i}	{T, D, O, C, E, TFI, CI, TFI_i,, CI_i}	{T, D, O, C, TFI_i,, CI_i}	O	{O, TFI, CI}	{O, TFI, CI}	{T, O, D, TFI_i, CI_i}	{T, O, D, TFI_i , CI_i}
D	{T, C, O, D, TFI, CI}	{T, D, O, C, TFI, CI}	{T, D, O, C, TFI, CI}	{T, D, O, C, E, TFI, CI, TFI_i,, CI_i}	D	{T, D, O, C, TFI, CI}	{T, D, O, C, TFI, CI}	D	D
E	T	C	O	D	E	CI	TFI	CI_i	TFI_i
CI	D	{T, C, D}	{T, O, D, TFI, CI}	D	CI	CI	CI	{T,D ,O, E, TFI, CI, TFI_i, ,CI_i}	{T, O, D, TFI, CI}
TFI	{T, D}	{T, C, D}	{T, O, D, TFI, CI}	D	TFI	{T, D, O, TFI, CI, TFI_i,, CI_i}	{TFI, CI}	{T, O, D, TFI_i, CI_i}	{T, D,O , E, TFI, TFI_i
CI_i	{T, C, O, TFI_i, CI_i}	{C, O, TFI_I, CI_i}	{O, TFI_i, CI_i}	{T, D, O, C, TFI_i,, CI_i}	CI_i	{O, E, TFI, CI, TFI_i,, CI_i}	{O, TFI_i, CI_i}	CI_i	CI_i
TFI_i	{T, C, O, TFI_i, CI_i}	{C, O, TFI_i, CI_i}	{O, TFI_i, CI_i}	{T, D, O, C, TFI_i,, CI_i}	TFI_i	{O, TFI, CI}	{O, E, TFI, TFI_i}	CI_i	{CI , CI_i}

Table 14: The LLL composition table

r_2 / r_1	**T**	**C**	**D**	**CI**	**TFI**
T	{T, O, D, E, TFI, TFI_i}	{T, O, D, TFI, TFI_i}	{T, O, D, TFI_i, CI_i}	{O, TFI, CI}	{T, O, TFI, CI}
C	{T, O, D, TFI, TFI_i}	{T, O, D, E, TFI, CI, TFI_i, CI_i}	{T, O, D, TFI_i, CI_i}	{O, TFI, CI}	{O, TFI, CI}
D	{T, O, D, TFI, CI}	{T, O, D, TFI, CI}	{T, O, D, E, TFI, CI, TFI_i, CI_i}	{T, O, D, TFI, CI}	{T, O, D, TFI, CI}
CI_i	{O, TFI_i. CI_i}	{O, TFI_i, CI_i}	{T, O, D, TFI_i, CI_i}	{O, E, TFI, CI, TFI_i, CI_i}	{O,TFI_i, CI_i}
TFI_i	{T, O, TFI_i, CI_i}	{O, TFI_i, CI_i}	{T, O, D, TFI_i, CI_i}	{O, TFI, CI}	{O, E, TFI, TFI_i}

Table 13: The ALA composition table

r_2 / r_1	**T**	**D**	**CI_i**
T	{E, D}	D	D
D	D	{E, D}	D
CI	D	D	{E, D}

Table 15: The PAP composition table

r_2 / r_1	**T**	**C**	**D**	**CI_i**	**TFI_i**
T	{T, D, CI}	{T, D, CI}	D	D	{T, D}
D	{T, D, CI}	{T, D, CI}	{T, D, CI}	D	D
CI	D	{T, D, CI}	D	{T, D, CI}	{T, D, CI}

Table 16: The PAL composition table

r_2 / r_1	**T**	**O**	**D**	**E**	**CI**	**TFI**	**CI_i**	**TFI_i**
T	{T, D}	{T, D, CI}	D	T	CI	{T, CI}	D	{T, D}
D	{T, D, CI}	{T, D, CI}	{T, D, CI}	D	{T, D, CI}	{T, D, CI}	D	D
CI	D	{T, D, CI}	D	CI	CI	CI	{T, D, CI}	{T, D, CI}

Table 17: The PAA composition table

r_2 / r_1	**T**	**C**	**D**	**CI_i**	**TFI_i**
T	{T, D, O, C, E, TFI, CI, TFI_i, CI_i}	{T, D, C, O, TFI, CI}	{T, D, C, O, TFI_i, CI_i}	D	{T, D, O}
C	{T, D, C, O, TFI_i, CI_i}	{T, D, O, C, E, TFI, CI, TFI_i, CI_i}	{T, D, C, O, TFI_i, CI_i}	{T, D, C, O, TFI_i, CI_i}	{T, D, C, O, TFI_i, CI_i}
D	{T, D, C, O, TFI, CI}	{T, D, C, O, TFI, CI}	{T, D, O, C, E, TFI, CI, TFI_i, CI_i}	D	D
CI	D	{T, D, C, O, TFI, CI}	D	{T, D, O, C, E, TFI, CI, TFI_i, CI_i}	{T, D, C, O, TFI, CI}
TFI	{T, D, O}	{T, D, C, O, TFI, CI}	D	{T, D, C, O, TFI_i, CI_i}	{T, D, C, O, E, TFI, TFI_i}

Table 18: The LAL composition table

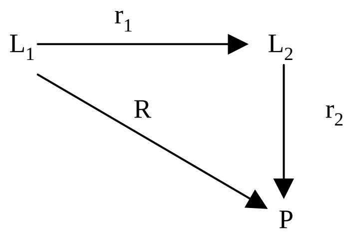
r_2 / **r_1**	**T**	**O**	**D**	**E**	**CI**	**TFI**	**CI_i**	**TFI_i**
T	{T, D, C, TFI}	{T, C, D, TFI, CI}	{T, D, C}	T	{C, CI, TFI}	{T, C, CI, TFI}	D	{T, D}
C	{T, D, C}	{T, C, D, TFI, CI}	{T, D, C}	C	{C, CI, TFI}	{C, CI, TFI}	{T, C, D}	{T, C, D}
D	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	D	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}	D	D
CI	D	{T, C, D, TFI, CI}	D	CI	CI	CI	{T, C, D, TFI, CI}	{T, C, D, TFI, CI}
TFI	{T, D}	{T, C, D, TFI, CI}	D	TFI	CI	{CI, TFI}	{T, C, D}	{C, T, D, TFI}

Table 19: The LAA composition table

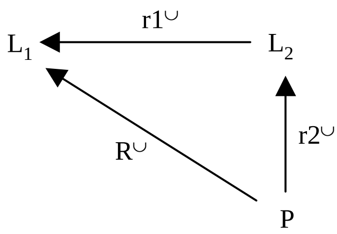
r_2 r_1	**T**	**O**	**D**	**E**	**CI**	**TFI**	**CI_i**	**TFI_i**
T	{T, D, O, E, TFI, TFI_i}	{T, D, O, TFI, CI}	{T, D, O, TFI_i, CI_i}	T	{O, CI, TFI}	{O, T, CI, TFI}	D	{D, T}
O	{T, D, O, TFI_i, CI_i}	{T, O, D, E, TFI, CI, TFI_i, CI_i}	{T, D, O, TFI_i, CI_i}	O	{O, CI, TFI}	{O, TFI, CI}	{T, D, O, TFI_i, CI_i}	{T, D, O, TFI_i, CI_i}
D	{T, D, O, TFI, CI}	{T, D, O, TFI, CI}	{T, O, D, E, TFI, CI, TFI_i, CI_i}	D	{T, D, O, TFI, CI}	{T, D, O, TFI, CI}	D	D
E	T	O	D	E	CI	TFI	CI_i	TFI_i
CI	D	{T, D, O, TFI, CI}	D	CI	CI	CI	{T, O, D, E, TFI, CI, TFI_i, CI_i}	{T, D, O, CI, TFI}
TFI	{T, D}	{T, D, O, CI, TFI}	D	TFI	CI	{CI, TFI}	{T, D, O, TFI_i, CI_i}	{T, D, O, E, TFI_i, TFI}
CI_I	{O, CI_i, TFI_i}	{O, CI_i, TFI_i}	{T, D, O, CI_i, TFI_i}	CI_i	{E, O, CI, TFI, CI_i, TFI_i}	{O, TFI_i, CI_i}	CI_i	CI_i
TFI_i	{T, O, CI_i, TFI_i}	{O, CI_i, TFI_I}	{T, D, O, CI_i, TFI_i}	TFI_i	{O, CI, TFI}	{O, E, TFI, TFI_i}	CI_i	{TFI_i, CI_i}

Table 20: The AAA composition table

To illustrate the process to obtain the composition of one of the 9 tables not explicitly constructed, let us consider the case h_2=linear entity. The six tables constructed for this case are the PLP, PLL, PLA, LLL, LLA, and ALA tables. From these six tables, we can get the other three, namely the LLP, ALP, and ALL tables. We illustrate this by showing how to get the $r_1 \otimes r_2$ entry of the LLP table from the PLL table. This means that we have to find the most specific relation R such that for any two linear spatial regions L_1 and L_2, and any point-like spatial region P, if (L_1, r_1, L_2) and (L_2, r_2, P) then (L_1, R, P). We can represent this as:



From the converse table we can get the converses $r1^{\cup}$ and $r2^{\cup}$ of r_1 and r_2, respectively. The converse $R^{\cup}$ of R is clearly the composition $r2^{\cup} \otimes r1^{\cup}$ of $r2^{\cup}$ and $r1^{\cup}$, which can be obtained from the PLL table:



Therefore, R is the converse of $R^{\cup}$: $R = (R^{\cup})^{\cup}$.

3.2 Defining Motion: The Integration of Topology and Time

In order to define motion as an integration of space and time, a topological calculus and a time algebra has to be selected and developed for their integration. Once we have decided the topological calculus, next step is the definition of the most suitable time algebra for modeling motion. Next section describe the time algebra developed.

3.2.1 The Time Algebra

We are going to define a temporal algebra, in which variables represent time points and there are five primitive constraints: ==*, next, prev,* >>, <<, which are defined as follows:

Definition 13. Given two time points, t and t’, t **==** t’ iff has not occurred a change between t and t’ (or between t’ and t) on any relation.

Definition 14. Given two time points, t an t’, t’ **next** t iff t’ > t and some relation or relations have changed to a neighbor relation between t and t’.

Definition 15. Given two time points, t and t’, t’ **prev** t iff t’ < t and some relation or relations have changed to a neighbor relation between t and t’.

Definition 16. Given two time points, t and t’, t’ **>>** t iff t’ > t and a relation has changed strictly more than once to a neighbor relation.

Definition 17. Given two time points, t and t', t' >> t iff t' < t and a relation has changed strictly more than once to a neighbor relation.

According to these definitions, time is represented by disjunctive binary constraints of the form $X\{r_1, ..., r_n\}Y$,

where each r_i is a relation that is applicable to X and Y. $X\{r_1, ..., r_n\}Y$ is a disjunction of the way $(X\ r_1\ Y) \vee .... \vee (X\ r_n\ Y)$ and r_i is also called primitive constraints.

We have used this time algebra because we are only interested in the point of the time in which one region is transformed into its topological neighborhood. The topological neighborhood of a region is that region to which the original region can be transformed to by a process of gradual, continuous change which does not involve the passage through any third region.

To reason about these temporal constraints we need to define the converse and composition operations and construct the converse and composition tables.

First of all we need to define what we understand as a general relation of the calculus because we are going to define the converse and composition operation in terms of general relations.

Definition 18. *A* ***general relation*** *R of the calculus is any subset of the set of all atomic relations.*

Definition 19. *The* ***converse*** *of a general relation R, called $R^{\cup}$ is defined as:*

$$\forall(X,Y)\ ((X,R,Y) \Leftrightarrow (Y,R^{\cup},X)) \quad (2)$$

Definition 20. *The* ***composition*** *R1 ⊗ R2 of two general relations R1 and R2 is the most specific relation R such that:*

$$\forall (h_1, h_2, h_3)\ ((h_1, R1, h_2) \wedge (h_2, R2, h_3) \Rightarrow (h_1, R, h_3) \quad (3)$$

The last three definitions are suitable for the temporal constraints chosen and the topological calculus defined in [Isli, Museros et alters 00].

[Tab. 21 and 22] shows the converse and composition operations respectively for the Time Algebra.

r	**$r^{\cup}$**
==	==
<<	>>
>>	>>
next	*prev*
prev	*next*

Table 21: The converse table for the Time Algebra.

r_1 / r_2	**<<**	**Prev**	**==**	**next**	**>>**
<<	{<<}	{<<}	{<<}	{prev,<<}	{<<,prev, ==, next, >>}
prev	{<<}	{<<,prev}	{prev}	{==,prev,next}	{next,>>}
==	{<<}	{prev}	{==}	{next}	{>>}
next	{<<,prev}	{prev,==,next}	{next}	{>>,next}	{>>}
>>	{<<,prev, ==, next, >>}	{>>,next}	{>>}	{>>}	{>>}

Table 22: The composition table for the Time Algebra

3.2.2 The representational model for Motion

The first step to define the framework to reason with motion is to create the representational model for topology and qualitative time points. The representational model follows the formalism used by Allen for temporal interval algebra [Allen 83]. The Allen style formalism will provide to our approach the possibility of reasoning with topology in dynamic worlds by applying the Allen's constraint propagations algorithm.

The binary relations between two objects, which can be points, lines or areas, h_1 and h_2 of the algebra in a point of time t are defined as tertiary constraints or propositions where the topological relation r between h_1 and h_2 in the point of time t is denoted by $(h_1,r,h_2)_t$. From this definition we specify a **general relation R** of the algebra during time t as:

$$\forall(h_1,h_2)\ ((h_1,R,h_2)_t \Leftrightarrow U_{r\in R}\ (h_1,r,h_2)_t) \quad (4)$$

Definition 21. *The* ***converse*** *of a general topological relation R in time t, denoted as* $R^{\cup}$, *is defined as follows:*

$$\forall(h_1,h_2)\ ((h_1,R,h_2)_t \Leftrightarrow (h_2,R^{\cup},h_1)_t) \quad (5)$$

From this definition we observe that the converse of the algebra defined integrating topology and time is the same as the converse defined only for topological relations because the converse is calculated in the same point of time, therefore time does not affect to the converse operation. In [Tab. 23] we can find the converse table constructed from this definition.

R	$R^{\cup}$
touch	*touch*
cross	*cross*
overlap	*overlap*
disjoint	*disjoint*
equal	*equal*
completely-inside	*completely-inside$_i$*
touching-from-inside	*touching-from-inside$_i$*
completely-inside$_i$	*completely-inside*
touching-from-inside$_i$	*touching-from-inside*

Table 23: The converse table for the spatio-temporal representation model

3.3 The Reasoning Process

3.3.1 The Basic Step of the Inference Process

The BSIP for topological information integrated with time (motion) consists of: *"given three objects A,B, C, if the topological relationship in time x between A and B and B and C are known, it is possible to obtain the topological relationship in timex between objects A and C".* To infer such topological relationship in the point of time called x we are going to define the *composition* operation for two general relations R1 and R2.

The composition for the model including topology and time has to be defined to include all the possibilities in four different ways as follows:

Definition 22. *The resulting general relation R obtained from the* ***composition*** *($\otimes$) operation could be calculated as:*

a) $(A,R1,B)t_0 \otimes (B,R2,C)t_0 \Rightarrow (A,R,C)t_0$

b) $(A,R1,B)t_0 \otimes (t_0, Reltime, t_1) \Rightarrow (A,R,B)t_1$

c) $(A,R1,B)t_0 \otimes (B,R2,C)t_1 / (t_0, Reltime, t_1) \Rightarrow ((A,R1,B)t_0 \otimes (t_0, Reltime, t_1)) \otimes (B,R2,C)t_1 \Rightarrow (A,R',B)t_1 \otimes (B,R2,C)t_1 \Rightarrow (A,R,C)t_1$

In this definition R1, R2 and R represent a general topological relation between the spatial regions A, B and C which can be points, lines or areas. Reltime represents a general time relation between time points t_0 and t_1. The first type of composition (*Definition22.a*) is the composition of the topological relations between three regions A, B and C, in the same point of time, where A, B, C belong to {point, line, area}. Then it is the usual topological composition, the time does not affect. To calculate this composition we will use the 18 composition tables and the converse table defined in [Isli, Museros et alters 00] and described in [Section 3.1 Overview of the Topological Calculus].

The second type of composition (*Definition22.b*) is the composition which implements the Freksa's conceptual neighborhood notion ([Freksa 91] and [Freksa 92]). It looks for the possible topological relations which will appear between two regions as time changes. To reason about this type of composition we need to construct 6 composition tables that will be referred as XY_t–table where the regions X and Y belongs to {point (P), line (L), area (A)} and t represents the dimension of time of the algebra. We would need 9 composition tables (3^2) if we consider all possibilities with X and Y being a point-like, a linear or an areal entity. However, we construct only 6 tables from which the other 3 tables can be obtained using the converse operation. We construct the AA_t-table, LA_t-table, PA_t-table, LL_t-table, PL_t-table and the PP_t-table, which are depicted in [Tab. 24 to 29] respectively. We have depicted in a common column the case for "next" and "prev" and a common column for the case of "<<" and ">>" because their entries are the same. The notation Reltop and Reltime are used to denote the topological relations and time relations respectively.

From the tables we can also infer that the == time relation represents the identity.

Reltime / RelTop	next or prev	<< or >>
T	{D,O,T}	{T, E, TFI, CI, TFI, CIi, TFIi}
O	{T,TFI,O}	{O, D, E, CI, TFIi, CIi}
D	{T,D}	{D, O, E, TFI, CI, TFIi, CIi, TFIi}
E	{O,E,}	{E, T, D, TFI, CI, TFIi, CIi}
TFI	{O,CI,TFI}	{TFI, T, D, E, CI, TFIi, CIi}
CI	{TFI,CI}	{CI, T, O, D, E, TFIi, CIi}
TFIi	{O,CIi,TFi}	{TFIi, T, D, E, CI, TFI}
CIi	{TFIi,CIi}	{CIi, T ,O, D, E, TFI, CI}

Table 24: AAt-table

Reltime / Reltop	next or prev	<< or >>	==
T	{C,D,T}	{T,TFI,CI}	{T}
C	{D,TFI,C}	{C,T,CI}	{C}
D	{T,D}	{D,C,TFI,CI}	{D}
TFI	{C,CI,TFI}	{TFI,T,D}	{TFI}
CI	{TFI,CI}	{CI,T,C,D}	{CI}

Table 25: LAt-table

Reltime / Reltop	next or prev	<< or >>	==
T	{D,CI,T}	{T}	{T}
D	{T,D}	{D,CI}	{D}
CI	{T,CI}	{CI,D}	{CI}

Table 26: PAt-table

Reltime / **Reltop**	**next or prev**	**<< or >>**	**==**
T	{D,O,C,T}	{T,E,TFI,CI, TFi,CIi}	{T}
D	{T,C,D}	{D,O,E,TFI,CI, TFIi,CIi}	{D}
O	{T,C,O}	{O,D,E,TFI,CI, TFIi, CIi}	{O}
C	{T,D,C}	{C,O,E,TFI,CI, TFIi, CIi}	{C}
E	{T,O,E}	{E,D,C,TFI,CI, TFIi,CIi}	{E}
TFI	{C,CI,T,TFI}	{TFI,D,O,E, TFIi,CIi}	{TFI}
CI	{TFI,C,CI}	{CI,T,D,O,E, TFIi,CIi}	{CI}
TFIi	{T,C,CIi,TFi}	{TFIi,D,O,E, TFI,CI}	{TFIi}
CIi	{C,TFIi,CIi}	{CIi,T,D,O,E, TFI,CI}	{CIi}

Table 27: LLt-table

Reltime / **Reltop**	**next or prev**	**<< or >>**	**==**
T	{D,CI,T}	{T}	{T}
D	{T,CI,D}	{D}	{D}
CI	{T,D,CI}	{CI}	{CI}

Table 28: PLt-table

Reltime / **Reltop**	**next or prev**	**<< or >>**	**==**
E	{D,E}	{E}	{E}
D	{E,D}	{D}	{D}

Table 29: PPt-table

As a relation t prev t’ corresponds to a change of some topological relation to a neighbour relation, the tables always keep the possibility that a relation has not changed between time t and t’, this situation model the fact that the time changes from t to t’ because other topological relationship has changed and the relationship between X and Y (RelTop) has not changed.

The three tables not constructed can be obtained by applying the converse operation. For example, the AL_t-table is not constructed but we can get any of its entries using the LA_t-table. This means that we have to find the most specific relation R such that, being X and Y an areal and a linear entity respectively:

$$(X, \text{Reltop}, Y)_{t0} \otimes (t_0, \text{Reltime}, t_1) \Rightarrow (X, R, Y)_{t1} \qquad (6)$$

From the LA_t-table and using the converse operation we will get such relation R as follows:

$$(Y, \text{Reltop}^{\cup}, X)_{t0} \otimes (t_1, \text{Reltime}^{\cup}, t_0) \Rightarrow (Y, R', X) \qquad (7)$$

Then the relation R that we are looking for is **R=(R')**$^{\cup}$.

For the third case of composition (*Definition22.c*) we want to infer the composition R in time t_1 between 3 regions, X, Y and Z having the topological relation in time t_0 between X and Y, the topological relation in time t1 between Y and Z and the qualitative time relation between times t_0 and t_1. To get the composition relation R, first we have to obtain the topological relations that can appear between X and Y in time t_1 using the composition tables defined for the case of *Definition22.b* above described. Then we have the general relation R' which appear between X and Y during t_1, this together with the general relation R2 between Y and Z in t_1 is a case suitable to apply the usual composition tables as explained for the case of *Definition22.a* and we will get the general composition relation R.

3.3.2 The Full Inference Process

For computing the Full Inference Process (FIP) of topological and time information we consider that:

1) each topological relationship between two objects in time t is seen as a constraint;
2) the set of topological relationships in time forms a constraint graph, where the nodes are spatial objects (points, lines and areas) and the arcs are the binary constraints between objects. This constraint graph is not complete at the beginning, that is, all the nodes are not bi-directional connected, because there is no initial topological relationship in time between all the objects in the space;
3) the fact of propagating the constraints for making explicit the topological relationships between all the nodes in the graph is seen as an instance of the CSP.

The formula (1), which approximated the solution for temporal objects, is rewritten for topological relations between spatial objects in a point of time in three formulas for each of the definition of composition given for the BSIP, as follows:

Case 1: $c_{a,c,t} := c_{a,c,t} \oplus c_{a,b,t} \otimes c_{b,c,t}$ (8)

Case 2: $c_{a,b,t1} := c_{a,b,t1} \oplus c_{a,b,t0} \otimes c_{t0,t1}$ (9)

Case 3: $c_{a,b,t1} := c_{a,c,t1} \oplus c_{a,b,t0} \otimes c_{b,c,t1}$ (10)

In our approach, the constraint $c_{a,b,t}$ (which represents the topological relationship holding between objects a and b in time t) is represented by the PROLOG predicate ctr_comp_top(TB,TA,A,B,Rel,t), where A and B are the spatial objects which holds the set of atomic topological relationships included in the set named *Rel* in the point of time t, TB and TA represents the types of the objects A and B, which can be point (p), line (l) or area (a). And the constraint $c_{to,t1}$ represents the time constraint between points of time t_0 and t_1, (t_0, Rtime,t_1), and is represented by the PROLOG predicated ctr_comp_time(t0,t1,Rtime), where Rtime represents the set of time relationships that can hold between t0 and t1.

The intersection ('⊕') and composition ('⊗') parts of formulas (8,9,10) are implemented with simplification and propagation CHRs, respectively.

The part of the intersection ($c_{a,b,t}\oplus$....) is implemented by the following simplification CHR:

```
ctr_comp_top(TB,TA,B,A,R1,t),ctr_comp_top(TB,TA,B,A,R2,t) <=>
  intersection(R1,R2,R3)|ctr_comp_top(TB,TA,B,A,R3,t).
```

where intersection/3 calculates the intersection between the set of relation R1 and R2, and the resulting set is stored in R3.

For supplying the lack of completeness of the constraint graph (because there is not a topological relation between every object in the graph), two CHRs more are defined, by applying the converse operation to the first and second constraints, respectively.

The part of the basic operation (8) related to the composition ($c_{a,b,t} \otimes c_{b,c,t}$) corresponds to the BSIP defined in the previous section. It is implemented by propagation CHRs in the next ways:

```
Case 1:
ctr_comp_top(TB,TA,B,A,R1,t1), ctr_comp_top(TC,TB,C,B,R2,t1), ==>
  composition(R1,R2,R3) | ctr_comp_top(TC,TA,C,A,R3,t1).

Case 2:
ctr_comp_top(TB,TA,B,A,R1,t0), ctr_comp_time(t0,t1,Rtime), ==>
  composition_motion(R1,Rtime,R3) | ctr_comp_top(TB,TA,B,A,R3,t1).
Case 3:
ctr_comp_top(TB,TA,B,A,R1,t0), ctr_comp_top(TC,TB,C,B,R2,t1),
  ctr_comp_time(t0,t1,Rtime) ==> composition_motion(R1,Rtime,R3),
  composition(R3,R2,R4) | ctr_comp_top(TC,TA,C,A,R4,t1).
```

where composition/3 refers to the set of facts of the PROLOG database which defines the composition operation for topological relations, in which time does not appears (*Definition22.a)*, and composition_motion/3 refers to the set of facts of the PROLOG database which defines the composition operation for the topological neighbourhood concept (motion), defined in *Definition22.b*.

As before, for the case in which the constraint graph is not complete, six other CHRs are defined by applying the converse operation to the first and second constraints of the head of each rule, respectively in each case.

Termination is guaranteed because the simplification rules replace R1 and R2 by the result R3 or R4 of intersecting R1 and R2 (and R3, R4 are the same as R1 or R2 or smaller), and because propagation CHRs are never repeated for the same constraint goals more than twice.

In the algorithm which implements the FIP no queue of modified constraints is needed because the new constraint goal itself will trigger new applications of the propagation CHRs.

4 Experiments

We have applied the model described in the previous sections (which integrates topology and time with orientation and distance) to mobile robot navigation. The robot used is a Nomad-200 robot, which has sixteen infrared sensors, sixteen ultrasonic sensors (numbered in the way shown in [Fig. 2]), and two ring of bumpers. The front of the robot corresponds to sensor 0. The application consists of controlling a robot which autonomously learns any structured environment and moves along a corridor. In the first step towards the solution, the structure of the corridor has been simplified to a rectangle where the offices can only be found to a side of the corridor, and with columns in the same side of the corridor where the offices are [Fig. 3].

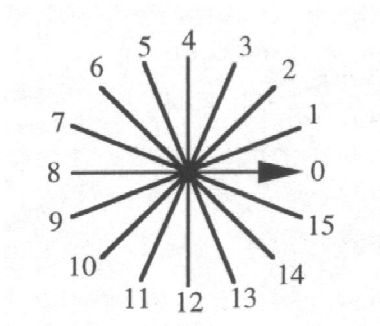


Figure 2: The robot has 16 infrared sensors and 16 ultrasonic sensors.

In a learning phase, the robot will capture the structure of the corridor. When the learning phase has finished, a guiding phase starts, in which the robot will guide any user to a goal office in the corridor. When the robot receives a request of guiding someone to an office, it will immediately answer:

1) the orientation of the goal office in qualitative terms. For instance: *"The office that you are looking for is behind me, to my right" or "The office that you are looking for is in front of me, to my left"*.
2) the qualitative distance of the goal office with respect the current position of the robot. For instance: *"The office that you are looking for is very close from here"*. The absolute distance reference system is an entry to the system.
3) Topological information about the relation between the column closer to the goal office. For instance: *"The office that you are looking for is touching the third column in our way until the office"*.
4) Spatio-temporal information describing the sequence of topological situations between the robot, seen as a mobile region, and the regions of the map of the corridor that we will find. For instance, if the origin region is region 1 and the target region is the region 2 the robot will answer: *"First we will be completely_inside the region 1 and disjoint the target region, which contains the office that you are looking for. Secondly we will be touching from inside region 1 and touching the target region. Then, we will be overlapping region 1 and target region. Afterwards we will be touching from inside the target region and touching the region 1. And finally we will be completely inside the target region, the James' Office Region and disjoint the starting region"*.

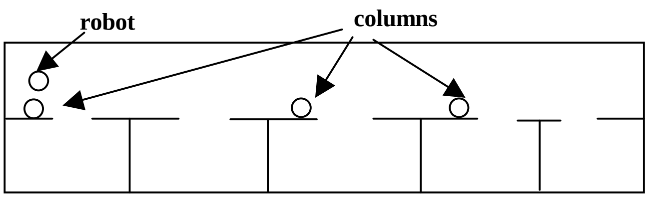


Figure 3: Structure of the corridor.

These are the main problems detected in our application:

- It is possible to know the current position of the robot related to its initial position by using the dead-reckoning information. However, this position contains a lot of imprecision caused by the slide of the wheels of the robot in its movement, which accumulates errors. The robot deals with this lake of precision of sensorial information.
- It is necessary that the robot detects and avoids obstacles without loosing the direction of movement. It also has to identify doors, columns and the ends of the corridor.
- When the robot learns the environment of the corridor all the doors has to be opened. However, in the guiding phase, the robot has to identify closed doors.

The three problems described are solved in the application we have implemented. In order to manage the lake of precision of the position of the robot, the space is divided into qualitative regions stored in a topological map.

To solve the second problem, the robot avoids obstacles interpreting the sensorial information. To help the robot does not loose the direction of the movement it should be centered and aligned in the corridor.

Finally to allow the robot to detect closed doors, the quantitative imprecise distance between offices is computed.

5 Conclusions and Future Work

The major contribution of the work presented in this paper is the definition of an approach for integrating topological aspects and time.

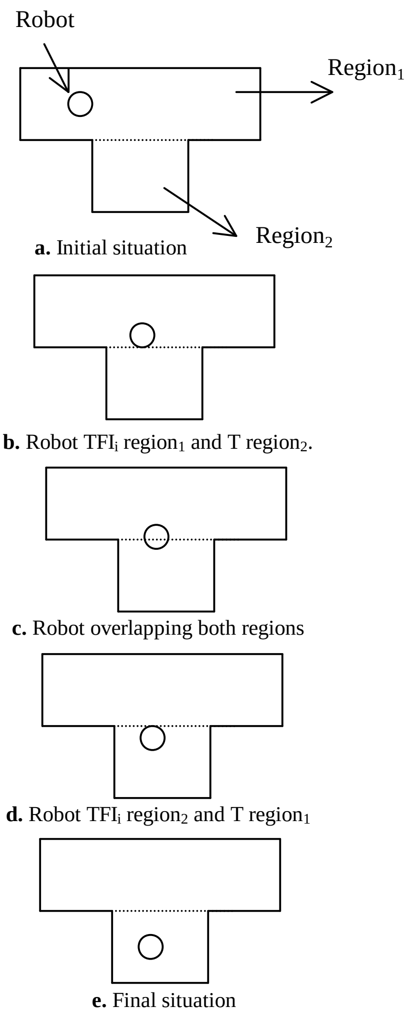
In order to make possible the integration it is necessary to define (1) its representation; (2) the basic step of the inference process; and (3) the full inference process. It is achieved by using constraint logic programming extended with constraint handling rules (CLP+CHR) as tool. The paradigm CLP+CHR is used to implement a constraint solver which solves in a straightforward way the complete inference process for each aspect of the space to be integrated. Therefore CLP+CHR provides a suitable tool for the integration.

Although only the topological together with time model has been described in this paper, qualitative orientation, cardinal direction and named and compared distances have also been integrated into the same model following the same steps described here with the topological and time information ([Escrig & Toledo 00]). Therefore a second contribution of this paper has been the integration of motion with other spatial aspects, such as orientation, distances and cardinal directions.

The application explained in the [Section 4 Experiments] has been solved using qualitative spatial representation techniques, including orientation, distance, topological relations and spatio-temporal information. Due to the simplicity of the corridor, we have not used qualitative reasoning techniques to infer new information. However, for dealing with a structure of the corridor with higher complexity (i.e. doors to both sides of the corridor, corridors with different shapes, more than one floor, etc.) it will be necessary a qualitative reasoning process for the orientation, distance and topological aspects. This is our current work.

For our future work, during the reasoning process, the spatio-temporal representational model described in this paper will help to reason about the sequence of topological situations that an autonomous robot should find during its way from a starting region to a target objective. It can also help to detect situations in which the robot is loosing its direction of movement. For instance, if we have a situation as the one depicted in [Fig. 4a]. in time t_0, and we want that the robot goes from region$_1$ to region$_2$, we know that the sequence of topological relations between the robot (interpreted as a mobile region) and the origin region, called region$_1$, and the target region, called region$_2$, is the next one:

(Robot,CI$_i$,Region$_1$)$_{t0}$ and (Robot,D,Region$_2$)$_{t0}$,
(Robot,TFI$_i$,Region$_1$)$_{t1}$ and (Robot,T,Region$_2$)$_{t1}$,
(Robot,O,Region$_1$)$_{t2}$ and (Robot,O,Region$_2$)$_{t2}$,
(Robot,T,Region$_1$)$_{t3}$ and (Robot,TFI$_i$,Region$_2$)$_{t3}$,
(Robot,D,Region$_1$)$_{t4}$ and (Robot,CI$_i$,Region$_2$)$_{t4}$
where t0 prev t1 prev t2 prev t3 prev t4.



a. Initial situation

b. Robot TFI$_i$ region$_1$ and T region$_2$.

c. Robot overlapping both regions

d. Robot TFI$_i$ region$_2$ and T region$_1$

e. Final situation

Figure 4: Graphical example of the initial situation

If during the robot's way until the target objective we find a situation which does not follow the sequence, for instance we find $(Robot,TFI_i,Region_1)_{t1}$ and $(Robot,D,Region_2)_{t1}$, the robot is losing its direction of movement. Therefore the robot should rectify its direction of movement. Then, we want to use this knowledge to the navigation of an autonomous robot integrating this knowledge to other qualitative spatial information such as orientation, distance and cardinal directions.

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