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Computational Complementarity and Shift Spaces[1]

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Abstract: Computational complementarity was introduced to mimic the physical complementarity in terms of finite automata (with outputs but no initial state). Most of the work has been focussed on "frames", i.e., on fixed, static, local descriptions of the system behaviour. The first paper aiming to study the asymptotical description of complementarity was restricted to certain types of sofic shifts. In this paper we continue this work and extend the results to all irreducible sofic shifts. We also study computational complementarity in terms of labelled graphs rather than automata.
Key Words: Complementarity principles, finite automata, sofic shifts, graphs.
Category: Category F.1.1 - Models of Computation

1 Motivation

Finite automata (with outputs but no initial states) have been extensively used as models of computational complementarity, a property which mimics the physical complementarity. All this work (e.g. Moore [12], Conway [7], Finkelstein and Finkelstein [8], Svozil [14], Calude, Calude, Svozil, Yu [3], Calude, Calude and Khoussainov [2], Calude and Lipponen [6]) was focussed on "frames", i.e., on fixed, static, local descriptions of the system behaviour.

In Calude and Lipponen [4] we took another view as we were mainly interested in the asymptotical description of complementarity. We studied the asymptotical behaviour of two complementarity principles — motivated by Moore's work [12] (see also Conway [7, p. 21] and Svozil [14]) and introduced by Calude, Calude, Svozil, Yu [3] — by associating to every incomplete deterministic automaton (with output, but no initial state) — studied in Calude and Lipponen [6] — certain sofic shifts.

In this paper we continue this research by extending the results to larger class of sofic shifts. We will prove that there is a strong relation between "local complementarity", as it is perceived at the level of "frames", and "asymptotical complementarity" as it is described by the irreducible sofic shift, as graphs having the same behaviour correspond to a unique sofic shift. To this aim we define the notion of complementarity properties to graphs but are also able to find a connection to finite automata.

2 Notations

If S is a finite set, then $|S|$ denotes the cardinality of S. A **partial** function $f : A \overset{\circ}{\to} B$ is a function defined for some elements from A. In case f is not

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defined on $a \in A$ we write $f(a) = \infty$. Let $D(f) = \{a \in A \mid f(a) \neq \infty\}$ denote the domain of f. If $D(f) = A$, we say that f is **total**. Two partial functions f and g are equal, when $D(f) = D(g)$ and $f(a) = g(a)$, for every $a \in D(f)$. For any two sets A and B, we denote their symmetrical difference by $\triangle$, $A \triangle B = (A \setminus B) \cup (B \setminus A)$. If Σ is a finite set, called alphabet, then Σ^* stands for the set of all finite words over Σ and the empty word, denoted by λ, whereas $\Sigma^{\mathbf{Z}}$ is the set of all bi-infinite words over Σ. An element of $\Sigma^{\mathbf{Z}}$ is a sequence $x = (x_n)_{n \in \mathbf{Z}} = \ldots x_{-1}x_0x_1 \ldots$

A **(labeled) graph** is a triple $G = (S_G, \mathcal{E}, \mathcal{L})$, where $\mathcal{E}$ is an edge set, S_G the vertex set, and the **labeling** $\mathcal{L} : \mathcal{E} \to \Sigma$ assigns to each **edge** e of G a label $\mathcal{L}(e)$ from the finite alphabet Σ.

A graph is **right-resolving** if for each vertex $p \in S_G$ the edges starting from p carry different labels. Since we are only dealing with right-resolving graphs in this article, instead of using the labeling function $\mathcal{L}$ we will borrow a notion of **transition function** from automata theory. So every (right-resolving) graph is a pair (S_G, Δ_G) where Δ_G is a partial function from the set of vertices S_G and the labels Σ to the set of vertices S_G. For instance, $\Delta_G(p, a) = q$ means that there is an edge labeled by a from the vertex p to the vertex q. And if $\Delta_G(p, b) = \infty$ then there is no edge with the label b starting from the vertex p. The transition diagram Δ_G can be naturally extended to a partial function, $\Delta_G : S_G \times \Sigma^* \overset{\circ}{\to} S_G$ as follows: for every $p \in S_G$, $w \in \Sigma^*$ and $\sigma \in \Sigma$, $\Delta_G(p, \lambda) = p$, and $\Delta_G(p, \sigma w) = \Delta_G(\Delta_G(p, \sigma), w)$ if $\Delta_G(p, \sigma) \neq \infty$. Hence right-resolving graphs have a "deterministic behaviour" in the sense that for every word $w \in \Sigma^*$ and every vertex p, there is at most one path starting from p which is labeled with w.

For all $p \in S_G$, the **follower set** $\mathcal{F}_G(p) = \{w \in \Sigma^* \mid \Delta(p, w) \neq \infty\}$ consists of all sequences of labels of paths from the vertex p to any other vertex $q \in S_G$.

Further on, we say that a graph $G = (S, \Delta)$ is **strongly connected** if for every pair of vertices $p, q \in S$ there is a word $w \in \mathcal{F}(p)$ such that $\Delta(p, w) = q$.

Follower sets naturally define how a graph G_1 can be thought to be simulated by another graph G_2 meaning that any vertex p of G_1 has a corresponding (not necessarily unique) vertex $q \in G_2$ such that the paths starting from p have exactly the same labels as the paths starting from q. Formally, a graph $G_1 = (S_1, \Delta_1)$ is $\mathcal{F}$**-simulated** by a graph $G_2 = (S_2, \Delta_2)$ if there is a mapping $h : S_1 \to S_2$ such that $\mathcal{F}_{G_1}(p) = \mathcal{F}_{G_2}(h(p))$ for all $p \in S_1$. If G_1 and G_2 are both $\mathcal{F}$-simulating each other, we say that G_1 and G_2 are $\mathcal{F}$**-equivalent**. If, moreover, the mapping $h : S_1 \to S_2$ is one-to-one and onto, then G_1 and G_2 are **isomorphic**. Notice that this isomorphism is only between vertices. The usual **labeled-graph isomorphism** is between edges (and labels) as well. But for our purposes in this article the first type is sufficient.

A graph G_1 is **minimal** if every graph G_2 which is $\mathcal{F}$-equivalent to G_1 has at least as many vertices as G_1, $|S_1| \leq |S_2|$.

3 Computational Complementarity

We say that two vertices $p, q \in S$ in a graph $G = (S_G, \Delta_G)$ are **indistinguishable** if they have the same follower sets, $\mathcal{F}_G(p) = \mathcal{F}_G(q)$. Hence, if there is a nonempty word w in the set $\mathcal{F}_G(p) \triangle \mathcal{F}_G(q)$ we say that p and q are **distinguishable** by w.

Using this definition, we extend the notions of the properties **A**, **B**, **C** which were introduced for automata (with outputs) in Calude, Calude, Svozil, Yu [3] to cover graphs. A graph G has property **A** if every pair of its distinct vertices are distinguishable. G has **B** if for every vertex p of G there exists a word which distinguishes p from all the other vertices. Finally, G has **C** if there exists a word which distinguishes between any two distinct vertices of G.

Further on, G satisfies a complementarity principle CI if it has **A** but not **B** and CII if it has **B** but not **C**.

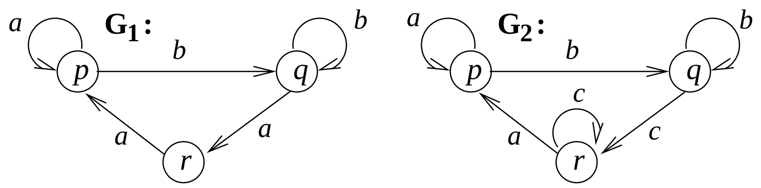

Figure 1: A graph G_1 satisfies principle CI and G_2 satisfies CII.

Example 1. The graph G_1 presented in Figure 1 has property **A** since the vertices p and r are distinguishable by $w = b$ and p and q by $w = ab$. But G_1 does not have **B** since there is no word which distinguishes the vertex p from the other vertices. Any word starting with a belongs to both $\mathcal{F}_G(p)$ and $\mathcal{F}_G(r)$ and any word starting with b belongs to $\mathcal{F}_G(p) \cap \mathcal{F}_G(r)$.

The graph G_2 has property **B** since the words a, b and c distinguish the vertices q, r and p, respectively, from the other vertices. On the other hand, any word starting with a cannot distinguish between p and r, with b cannot distinguish between p and q and with c cannot distinguish between q and r, respectively. Hence G_2 does not have **C**.

Compared to automata with outputs, graphs turn out to be more restricted in terms of complementarity properties.

Proposition 1. *Let $G = (S_G, \Delta_G)$ be a graph. If $|S_G| \geq 3$ then G cannot have property* **C**.

Proof. Let $p, q, r \in S$ be three distinct vertices. If there is a word w which can distinguish between all of them and $w \in \mathcal{F}_G(p) \backslash \mathcal{F}_G(q)$ then $w \in \mathcal{F}_G(p) \backslash \mathcal{F}_G(r)$, and hence, $w \notin \mathcal{F}_G(q) \cup \mathcal{F}_G(r)$, a contradiction. □

Lemma 2. *Isomorphism preserves properties* **A**, **B** *and* **C**.

Proof. Since isomorphism preserves the follower sets, also the properties **A**, **B** and **C** are invariant. □

The relation of indistinguishability $\equiv$ partitions the vertex set of a graph G into disjoint equivalence classes. Let $[s]$ denote the class of the vertex $s \in S_G$, $[s] = \{p \in S_G \mid p \equiv s\}$. The **merged graph** $M(G) = (S_M, \Delta_M)$ can be defined by setting $S_M = \{[s] \mid s \in S_G\}$ and $\Delta_M([s], a) = [r]$ if there are vertices $r, s \in S_G$ such that $\Delta_G(p, a) = q$.

Theorem 3. *Let G be a graph. Then*

1) *$M(G)$ has property* **A**.
2) *$M(G)$ and G are $\mathcal{F}$-equivalent.*
3) *$M(G)$ is minimal.*

Proof. 1) The merged graph $M(G)$ has property **A** by the definition.

2) Notice first that for each class $[s]$ we can fix a unique representative to be the vertex $s_{rep} \in S_G$ having, say, the minimum index between all the vertices in the same class. Thus the mapping $h : S_M \to S_G$, $h([s]) = s_{rep}$ is well-defined and for all $[s] \in S_M$, $\mathcal{F}([s]) = \mathcal{F}(s_{rep})$ by the definition. Hence $M(G)$ is $\mathcal{F}$-simulated by G via h; on the other hand, G is $\mathcal{F}$-simulated by $M(G)$ via the natural mapping $g : S_G \to S_M$, $g(s) = [s]$.

3) Let $B = (S_B, \Delta_B)$ be a graph which is $\mathcal{F}$-equivalent to G via the mappings $h_1 : S_G \to S_B$ and $h_2 : S_B \to S_G$. Consider the mapping $l : S_B \to S_M$ defined by $l(s) = [h_2(s)]$. Since $s \equiv h_2(h_1(s))$ for all $s \in S_G$, and $t \equiv h_1(h_2(t))$ for all $t \in S_B$, it follows that for any $[q] \in S_M$, there is a vertex $h_1(q) \in S_B$ such that $l(h_1(q)) = [h_2(h_1(q))] = [q]$. So l is an onto function from the finite set S_B to the finite set S_M, and hence, $|S_M| \leq |S_B|$. □

Theorem 4. *Two merged graphs G_1 and G_2 are $\mathcal{F}$-equivalent iff they are isomorphic.*

Proof. Assume first that G_1 and G_2 are $\mathcal{F}$-equivalent via the mappings $h_1 : S_1 \to S_2$ and $h_2 : S_2 \to S_1$. By Theorem 3, $|S_1| = |S_2|$. Indeed, since $G_1 = M(G_1)$ and $G_2 = M(G_2)$, we have $|S_1| \leq |S_2|$ and $|S_2| \leq |S_1|$. On the other hand, since all the states in G_1 and G_2 are distinguishable this implies that the mappings h_1 and h_2 are isomorphisms. The second part of the statement follows from the definition. □

Corollary 5. *Two graphs G_1 and G_2 are $\mathcal{F}$-equivalent iff their merged graphs $M(G_1)$ and $M(G_2)$ are isomorphic.*

4 Sofic shifts

A **sofic shift** X is a subset of $\Sigma^{\mathbf{Z}}$ consisting of all bi-infinite walks (sequences of labels) on some graph G. We say that G is a **presentation** of X, and we write $X = X_G$. By a well-known result (see Lind and Marcus [11]) every sofic shift X has a right-resolving presentation. The **language** of a shift X is the set $\mathcal{B}(X)$ of all subwords of sequences in X.

In this section we will consider mainly strongly connected graphs which correspond irreducible sofic shifts. Recall that a shift X is called **irreducible** if for all words $u, v \in \mathcal{B}(X)$ there is a word $w \in \mathcal{B}(X)$ such that $uwv \in \mathcal{B}(X)$. By Lind and Marcus [11], the shift X_G is irreducible iff its presentation G is strongly connected.

A **minimal (right-resolving) presentation** of a sofic shift X is a presentation which has the fewest number of vertices among all (right-resolving) presentations of X. A minimal presentation coincides with a minimal graph and is unique up to an isomorphism as the following results will show. Our aim in this section is to characterize all the right-resolving presentations corresponding to the same irreducible sofic shift.

The first result, see Lind and Marcus [11], shows that if G is a presentation of a shift space X then also its merged graph $M(G)$ is a presentation of X.

Proposition 6. *For a graph G, $X_G = X_{M(G)}$. Furthermore, if G is irreducible (resp. right-resolving), then so is $M(G)$.*

The following two results are due to Fischer [9, 10].

Proposition 7. *Any two minimal right-resolving presentations of an irreducible sofic shift are isomorphic as labeled graphs.*

Proposition 8. *Let X be an irreducible sofic shift and G its irreducible right-resolving presentation. Then the merged graph $M(G)$ is the minimal right-resolving presentation of X.*

We are now ready to prove our main result.

Theorem 9. *Two graphs G_1 and G_2 are $\mathcal{F}$-equivalent iff $X_{G_1} = X_{G_2}$.*

Proof. If G_1 and G_2 are $\mathcal{F}$-equivalent then $M(G_1)$ and $M(G_2)$ are isomorphic by Corollary 5. But this implies that $X_{M(G_1)} = X_{M(G_2)}$, and hence by Proposition 6, $X_{G_1} = X_{G_2}$.

On the other hand, if $X_{G_1} = X_{G_2}$ then by Propositions 7 and 8, $M(G_1)$ and $M(G_2)$ are isomorphic and again by Corollary 5, G_1 and G_2 are $\mathcal{F}$-equivalent. □

Theorem 9 can be used to express complementarity principles in terms of properties of sofic shifts.

Corollary 10. *Let G_1 and G_2 be two graphs. If G_1 satisfies CI and G_2 satisfies CII, then the sofic shifts X_{G_1} and X_{G_2} cannot be equal.*

Corollary 11. *If G_1 and G_2 are merged graphs presenting the same shift, $X_{G_1} = X_{G_2}$, then they satisfy the same principle CI/CII.*

Example 2. Notice that Theorem 9 is not valid for graphs which are not strongly connected. Indeed, the graphs G_1 and G_2 in Figure 2 are presentations of the same sofic shifts but are not $\mathcal{F}$-equivalent. Furthermore, G_1 has property **C** while G_2 satisfies principle CII.

5 Incomplete automata

In this section we will approach the sofic shifts from another point of view by studying the theory of incomplete automata introduced in Calude and Lipponen [6].

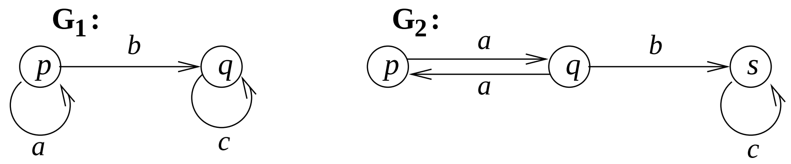

Figure 2: G_1 and G_2 generate the same shift but are not $\mathcal{F}$-equivalent.

An incomplete automaton can be seen as a labeled graph with an extension that every vertex produces an output. Formally, a **deterministic (finite) incomplete automaton** over the alphabets Σ (**input** symbols) and O (**output** symbols) is a triple $A = (S_A, \Delta_A, F_A)$, where the **set of states** S_A is finite and nonempty, the **transition table** Δ_A is a partial function from $S_A \times \Sigma$ to the set of states S_A, and the **output function** F_A is a (total) mapping from the set of states S_A into the output alphabet O. The graph $G_A = (S_A, \Delta_A)$ is said to be the **underlying graph** of the automaton A.

The counterpart of the follower set in a graph is the set of **applicable words** in an automaton. For all $p \in S_A$, the set $W_A(p) = \{w \in \Sigma^* \mid \Delta_A(p, w) \neq \infty\}$ consists of all words leading to complete computations on state p. Furthermore, an automaton $A = (S_A, \Delta_A, F_A)$ is **strongly connected** if the underlying graph G_A is strongly connected, that is, for every pair of states $p, q \in S_A$ there is a word $w \in W_A(p)$ such that $\Delta_A(p, w) = q$.

The **response** of an automaton $A = (S_A, \Delta_A, F_A)$ to an **input signal** is a partial function $R_A : S_A \times \Sigma^* \overset{\circ}{\to} O^*$ defined such that for every $s \in S_A$, $R_A(s, \lambda) = F_A(s)$, and

$$R_A(s, \sigma_1 \dots \sigma_n) = F_A(s) F_A(\Delta_A(s, \sigma_1)) F_A(\Delta_A(s, \sigma_1\sigma_2)) \dots F_A(\Delta_A(s, \sigma_1 \dots \sigma_n)),$$

if $\sigma_1 \dots \sigma_n \in W_A(s)$, $\sigma_i \in \Sigma$, $n \geq 1$ and $1 \leq i \leq n$.

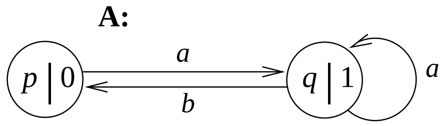

Figure 3: An automaton with two states.

Example 3. Let $\Sigma = \{a, b\}$, $O = \{0, 1\}$, and consider the strongly connected two-state automaton A presented in Figure 3. The state p emits an output 0, $F_A(p) = 0$, and the state q emits an output 1, $F_A(q) = 1$. The responses to an input ab are $R_A(p, ab) = 010$, $R_A(q, ab) = 110$, and to an input ba, $R_A(p, ba) = \infty$, $R_A(q, ba) = 101$.

In automata theory β-simulation comes from behavioral simulation, meaning that an automaton can perform all computations performed by another automaton and produces the same outputs. We say that an automaton

$A = (S_A, \Delta_A, F_A)$ is β**-simulated** by an automaton $B = (S_B, \Delta_B, F_B)$ if there is a mapping $h : S_A \to S_B$ such that for all $s \in S_A$, $W_A(s) = W_B(h(s))$ and $R_A(s, w) = R_B(h(s), w)$, for all $w \in W_A(s)$. Correspondingly, if A and B both β-simulate each other, we say that they are β**-equivalent**, and if the mapping $h : S_A \to S_B$ is one-to-one and onto, and for all $s \in S_A$ and $\sigma \in W_A(s) \cap \Sigma$, $h(\Delta_A(s, \sigma)) = \Delta_B(h(s), \sigma)$, then A and B are **isomorphic**. An automaton A is **minimal** if every automaton B which is β-equivalent to A has at least as many states as A, $|S_A| \le |S_B|$.

Two states $p, q \in S_A$ are **indistinguishable** iff $W_A(p) = W_A(q)$ and $R_A(p, w) = R_A(q, w)$ for all $w \in W_A(p)$. If p and q are not indistinguishable we say that they are **distinguishable** and every word from the set

$$\{w \mid w \in W_A(p) \triangle W_A(q) \text{ or } R_A(p, w) \ne R_A(q, w)\}$$

is said to **distinguish** between p and q. Using this definition, we define properties **A**, **B** and **C** as well as principles CI and CII in the same way as for the graphs in Section 3.

In the previous section we considered sofic shifts which are a special class of shift spaces. Formally, a subset X of $\Sigma^{\mathbf{Z}}$ is a **shift space** if it is topologically closed (with respect to the natural metric on $\Sigma^{\mathbf{Z}}$) and shift invariant, $\sigma(X) = X$, where $\sigma : \Sigma^{\mathbf{Z}} \to \Sigma^{\mathbf{Z}}$ is a **shift transformation** $\sigma(x)_i = x_{i+1}$. The set $\Sigma^{\mathbf{Z}}$ is called the **full shift**. Two shift spaces X and Y are **conjugate** if there is a one-to-one onto morphism $\phi : X \to Y$ which commutes with the shift transformation, $\phi \circ \sigma_X = \sigma_Y \circ \phi$. For more details, see Lind and Marcus [11].

Following Calude and Lipponen [4] we associate the **label-output shift** (of bi-infinite sequences) to each automaton $A = (S_A, \Delta_A, F_A)$:

$$\mathcal{S}_A^{\Sigma,O} = \{(a_i, x_i)_{i\in\mathbf{Z}} \mid q_i \in S_A, a_i \in \Sigma, x_i \in O, \Delta_A(q_i, a_i) = q_{i+1}, x_i = F_A(q_i)\}.$$

Other relations between finite automata (with initial states) and sofic shifts were explored by various authors; see Béal and Perrin [1], Perrin [13].

In Calude and Lipponen [4] we proved that every label-output shift is a sofic shift by introducing for each strongly connected automaton A a corresponding graph G_A such that $\mathcal{S}_A^{\Sigma,O} = X_{G_A}$. The following example shows the opposite situation, a way to find an automaton which corresponds to the given graph.

Figure 4: The label-output shift $\mathcal{S}_A^{\Sigma,O}$ and the sofic shift X_G correspond to each other.

Example 4. Consider the graph G and the automaton A presented in Figure 4. We notice that the sequences running through the first coordinates in the label-output shift $\mathcal{S}_A^{\Sigma,O}$ make the sofic shift X_G.

Formally, let $G = (S_G, \Delta_G)$ be a graph over the alphabet Σ. The automaton $A_G = (S_A, \Delta_A, F_A)$ has G as its underlying graph and we let every state $p \in S_A$ emit an output 0, $F(p) = 0$. It follows immediately that $a \in W_A(p)$ iff $a \in \mathcal{F}_G(p)$.

Hence we can find for any sofic shift X a corresponding label-output shift $\mathcal{S}_A^{\Sigma,O}$ where $O = \{0\}$ and $A = A_G$ for some right-resolving presentation G of X such that $X = h(\mathcal{S}_A^{\Sigma,O})$ where $h : (\Sigma \times O)^* \to \Sigma$ is defined by $h(a, 0) = a$ for all $a \in \Sigma$.

The automaton A_G is strongly connected and deterministic iff the graph G is strongly connected and right-resolving. In what follows, we consider only right-resolving graphs; however, we do not assume them to be strongly connected.

Theorem 12. *Two states $p, q \in S_{A_G}$ are indistinguishable iff $\mathcal{F}_G(p) = \mathcal{F}_G(q)$.*

Proof. If the states p, q in A_G are indistinguishable then $W_A(p) = W_A(q)$ and $R_A(p, w) = R_A(q, w)$ for all $w \in W_A(p)$. Since the output alphabet of A_G consists of only one letter, all the responses are just sequences of this letter. So two states in A_G are indistinguishable iff $W_A(p) = W_A(q)$. But this means that in the graph G, the follower sets of the vertices p and q are the same, $\mathcal{F}_G(p) = \mathcal{F}_G(q)$.

In the same way we prove the other implication. $\square$

For an incomplete automaton $A = (S_A, \Delta_A, F_A)$ the length of the shortest words to check whether two states $p, q \in S_A$ are distinguishable is $|S_A| - 1$ (see Calude and Lipponen [6]). With this in mind we are able to improve the bound presented in Lind and Marcus [11] for right-resolving graphs.

Theorem 13. *The shortest word needed to check whether two vertices in graph $G = (S, \Delta)$ are distinguishable is of length $|S| - 1$.*

Notice that Example 1 shows that this limit cannot be further improved. Indeed, the shortest word to distinguish between the vertices p and q in G_1 is ab and $|ab| = |S_1| - 1$.

Theorem 14. *The automaton A_G has property* **A** *(resp.* **B** *or* **C***) iff G has property* **A** *(resp.* **B** *or* **C***).*

Proof. Follows from the relation $w \in W_A(p)$ iff $w \in \mathcal{F}_G(p)$ for all $p \in S_G$. $\square$

In Calude and Lipponen [6] we constructed an algorithm which decides whether an automaton has properties **A**, **B** or **C**.

Corollary 15. *The properties* **A**, **B** *and* **C** *are decidable for graphs.*

References

[1] Marie-Pierre Béal and Dominique Perrin. Symbolic dynamics and finite automata, In G. Rozenberg and A. Salomaa, editors, *Handbook of Formal Languages, Vol. 2*, pages 463-506. Springer-Verlag, Heidelberg, 1997.

[2] C. Calude, E. Calude, and B. Khoussainov. Deterministic automata: simulation, universality and minimality. *Annals of Pure and Applied Logic*, 90, 1-3: 263–276, 1997.

[3] C. Calude, E. Calude, K. Svozil, and S. Yu. Physical versus computational complementarity I. *International Journal of Theoretical Physics*, 36:1495–1523, 1997.

[4] C. S. Calude and M. Lipponen. Computational complementarity and sofic shifts. In X. Lin, editor, *Computing Theory '98*, pages 277–290. Springer, Singapore, 1998.

[5] E. Calude and M. Lipponen. Minimal deterministic incomplete automata. *Journal of Universal Computer Science*, 3(11):131–149, 1997.

[6] E. Calude and M. Lipponen. Deterministic incomplete automata: simulation, universality and complementarity. In C. S. Calude, J. Casti, and M. J. Dinneen, editors, *Unconventional Models of Computation*, pages 131–149. Springer-Verlag, Singapore, 1998.

[7] J. H. Conway. *Regular Algebra and Finite Machines.* Chapman and Hall Ltd., London, 1971.

[8] D. Finkelstein and S. R. Finkelstein. Computational complementarity. *International Journal of Theoretical Physics*, 22(8):753–779, 1983.

[9] R. Fischer. Sofic systems and graphs. *Monatshefte für Mathematik*, 80(3):179–186, 1975.

[10] R. Fischer. Graphs and symbolic dynamics. In I. Csiszár and P. Elias, editors, *Topics in Information Theory 16*, pages 229-243. North-Holland, Budapest, 1975.

[11] D. Lind and B. Marcus. *Symbolic Dynamics and Coding*, Cambdridge University Press, 1995.

[12] E. F. Moore. Gedanken-experiments on sequential machines. In C. E. Shannon and J. McCarthy, editors, *Automata Studies*, pages 128–153. Princeton University Press, Princeton, 1956.

[13] D. Perrin. Symbolic dynamics and finite automata. In J. Wiedermann and P. Hájek, editors, *Proc. 20th International Symposium MFCS'95*, pages 94–104. Springer-Verlag, Heidelberg, 1995.

[14] Karl Svozil. *Randomness & Undecidability in Physics.* World Scientific, Singapore, 1993.