Journal of Universal Computer Science, vol. 4, no. 1 (1998), 82-88
submitted: 25/9/97, accepted: 1/11/97, appeared: 28/1/98 © Springer Pub. Co.

Modelling the Measurement Uncertainty by Intervals

Fabiana Zamora Wilke
(Instituto de Informática - UFRGS - Brazil
zamora@inf.ufrgs.br)

Beatriz Regina Tavares Franciosi
(Instituto de Informática - UFRGS/PUCRS - Brazil
bea@inf.ufrgs.br)

Paulo Werlang Oliveira
(Instituto de Matemática - PUCRS - Brazil
pwerlang@euler.mat.pucrs.br)

Dalcidio Moraes Claudio
(Instituto de Matemática - PUCRS - Brazil
dalcidio@music.pucrs.br)

Abstract: This paper presents a proposal of measuring uncertain modelling referred to a geometrical model obtained from a *Coordinate Measuring Machine (CMM)*. The measurement of objects by *CMM* is achieved considering a particular region of measurement described through a set of finitely many points which defines the ideal reference mesh and it is exposed to, at least, three kinds of error: systematical, aleatory and rounding error. For each point of this mesh is associated the measure error bounds. Those ones have two components: error direction, and error numerical value. The measure error bound identification, as showed in this paper, makes possible to know about the best region to measure a specific object with smallest error bound.

Key Words: mathematical modelling, interval computing, measure uncertainty, coordinate measure machine.

Category: G.0

1 Introduction

Measures obtained from a *CMM (Coordinate Measuring Machine)* makes able the set of numerical features of the object. This instrument, the *CMM*, as described in [Alves 1996], makes possible the point-by-point metrological recognition by the grouping of the constituent surfaces of the object.

The measurement of objects by coordinates measuring is achieved considering a particular measurement region, that is defined by the displacement system of the mobile parts of the *CMM*. This measurement is submitted to, at least, three kinds of error: systematical, aleatory and rounding error.

The systematical error changes according to a determined rule. It can be determined by the *CMM* self-verification software. The aleatory error is caused by alterations (not perceptible) of the instrument, of the object to be measured, of the environment and of the users. These errors can not be determined separately but, nevertheless, can be caught quantitatively. On the other hand, the rounding error [see Kulisch 1981] is inherent to hardware and, by this, its monitoration depends on numerical methods and techniques of scientific computation. The problem at issue can be associated to, at least, two subproblems: control of the aleatory and control of the rounding error.

This paper presents a strategy to control the aleatory and rounding errors through the knowledge of the error bounds associated to different positions of the measurement region.

2 Basic Concepts

The study efforts presented here were concentrated on bidimensional measure region (2D region), i.e., on regions which can be described by a finite set of points (x_i, y_j), where x_i, y_j are real numbers, $i = 1, ..., m_x$, $j = 1, ..., m_y$. This set defines the mesh called the *reference mesh* of the measurement region, denoted by Set_{xy}.

$$Set_{xy} := Set_x \times Set_y$$

where $Set_x := \{x_i \,|\, i = 1, ..., m_x\}$, $Set_y := \{y_j \,|\, j = 1, ..., m_y\}$ and the integers m_x, m_y are the numbers of the x-coordinates and of the y-coordinates, respectively.

The error verification, associated to the different positions of the measurement region, is characterized by a correlation between the numerical value of the error, and its direction.

Then, each point of the *reference mesh* suffer a displacement which is associated to a numerical value (that expresses the amount of error associated to each point of this) and to a direction vector. On the following section these error measure components will be discussed.

3 The Components of the Measure Error

3.1 The Numerical Value

To obtain the numerical value of the error associated with the point (x_i, y_j) of the *reference mesh* the folowing set is considered:

$$Set_{\overline{xy}} := \{([x]^{(i,j)}, [y]^{(i,j)}) \,|\, [x]^{(i,j)} = \left[\min M_x^{(i,j)}; \max M_x^{(i,j)}\right],$$
$$[y]^{(i,j)} = \left[\min M_y^{(i,j)}; \max M_y^{(i,j)}\right]\},$$

where $M_x^{(i,j)} = \{\overline{x}_1^{(i,j)}, \overline{x}_2^{(i,j)}, \ldots, \overline{x}_n^{(i,j)}\}$ and $M_y^{(i,j)} = \{\overline{y}_1^{(i,j)}, \overline{y}_2^{(i,j)}, \ldots, \overline{y}_n^{(i,j)}\}$ contain the x- and y-coordinates, respectively, of the measurements for the reference mesh point (x_i, y_j) [see Tab. 1].

In fact, the $Set_{\overline{xy}}$ is formed by intervals vectors $([x]_i, [y]_j)$, the two components of wich are intervals of uncertainty of measurements. The use of such vectors make able the control of the rounding and aleatory error, because the measured values are framed inside an interval.

measurement	**measured value**	
k	$M_x^{(i,j)}$	$M_y^{(i,j)}$
1	$\overline{x}_1^{(i,j)}$	$\overline{y}_1^{(i,j)}$
2	$\overline{x}_2^{(i,j)}$	$\overline{y}_2^{(i,j)}$
3	$\overline{x}_3^{(i,j)}$	$\overline{y}_3^{(i,j)}$
...	...	...
n	$\overline{x}_n^{(i,j)}$	$\overline{y}_n^{(i,j)}$

Table 1: Measured values for the point (x_i, y_j) of the reference mesh

Then, the numerical value of the measurement error associated to the point (x_i, y_j) is denoted by $\varepsilon^{(i,j)}$ and it is calculated through the formula:
$\varepsilon^{(i,j)} = \frac{\delta_{x_i}^3 + \delta_{y_j}^3}{\delta_{x_i}^2 + \delta_{y_j}^2}$,
where
$\delta_{x_i} = \frac{w([x]_i)}{2}, \delta_{y_j} = \frac{w([y]_j)}{2}$, $w([x]_i) = maxM_x^{(i,j)} - minM_x^{(i,j)}$,
$w([y]_j) = maxM_y^{(i,j)} - minM_y^{(i,j)}$.

The proof is collected in the Appendix.

3.2 The Direction

The direction of the error is obtained by the identification of the *translation vector*, denoted by $\boldsymbol{T}_{x_i y_j}$, with origin (x_i, y_j) and extremity $(x_i + T_{x_i}, y_j + T_{y_j})$.
$T_{x_i} = m[x]_i - x_i, T_{y_j} = m[y]_j - y_j$,
$m[x]_i = \frac{maxM_x^{(i,j)} + minM_x^{(i,j)}}{2}$
$m[y]_j = \frac{maxM_y^{(i,j)} + minM_y^{(i,j)}}{2}$.

4 Example of Application

Given the sets Set_{xy} and $Set_{\overline{xy}}$, the numerical values of the measurement error $(\varepsilon^{(i,j)})$ and the translation vectors $(\boldsymbol{T}_{x_i y_j})$ associated to each point (x_i, y_j) (see Table 2).

Set_{xy}		**measured value**		$Set_{\overline{xy}}$		$\boldsymbol{T}_{x_iy_j}$		
x_i	y_j	$M_x^{(i,j)}$	$M_y^{(i,j)}$	$[x]_i$	$[y]_j$	T_{x_i}	T_{y_j}	$\varepsilon^{(i,j)}$
0	0	0.0001 0.0003	0.0000 0.0002	$[0.0001; 0.0003]$	$[0.0000; 0.0002]$	0.0002	0.0001	0.0002
1	0	−1.0002 −1.0001	0.0000 0.0001	$[-1.0002; -1.0001]$	$[0.0000; 0.0001]$	−2.0001	0.0000	−1.0001
2	0	2.0000 2.0000	0.0000 0.0000	$[2.0000; 2.0000]$	$[0.0000; 0.0000]$	0.0000	0.0000	2.0000
3	0	3.0000 3.0001	0.0001 0.0007	$[3.0000; 3.0001]$	$[0.0001; 0.0007]$	0.0000	0.0004	3.0001
4	0	4.0000 4.0001	0.0000 0.0001	$[4.0000; 4.0001]$	$[0.0000; 0.0001]$	0.0001	0.0000	4.0001
0	1	−0.0003 −0.0002	1.0000 1.0001	$[-0.0003; -0.0002]$	$[1.0000; 1.0001]$	−0.0003	0.0001	1.0000
1	1	1.0004 1.0005	1.0008 1.0009	$[1.0004; 1.0005]$	$[1.0008; 1.0009]$	0.0004	0.0009	1.0007
2	1	−2.0002 2.0004	1.0006 1.0008	$[-2.0002; 2.0004]$	$[1.0006; 1.0008]$	−1.9999	0.0007	1.0007
3	1	−3.0003 −3.0002	1.0008 1.0009	$[-3.0003; -3.0002]$	$[1.0008; 1.0009]$	−6.0002	0.0009	−2.5996
4	1	−4.0002 −4.0001	1.0011 1.0012	$[-4.0002; -4.0001]$	$[1.0011; 1.0012]$	−8.0001	0.0011	−3.7053
0	2	−0.0004 −0.0003	2.0000 2.0000	$[-0.0004; -0.0003]$	$[2.0000; 2.0000]$	−0.0003	0.0000	2.0000
1	2	−1.0002 −1.0002	2.0011 2.0012	$[-1.0002; -1.0002]$	$[2.0011; 2.0012]$	−2.0002	0.0012	1.4012
2	2	2.0000 2.0001	2.0008 2.0009	$[2.0000; 2.0001]$	$[2.0008; 2.0009]$	0.0000	0.0008	2.0005
3	2	−3.0004 −3.0002	2.0003 2.0006	$[-3.0004; -3.0002]$	$[2.0003; 2.0006]$	−6.0003	0.0005	−1.4613
4	2	−4.0003 −4.0002	2.0000 2.0001	$[-4.0003; -4.0002]$	$[2.0000; 2.0001]$	−8.0002	0.0000	−2.8003
0	3	0.0000 0.0000	3.0000 3.0000	$[0.0000; 0.0000]$	$[3.0000; 3.0000]$	0.0000	0.0000	3.0000
1	3	−1.0003 −1.0002	3.0008 3.0009	$[-1.0003; -1.0002]$	$[3.0008; 3.0009]$	−2.0003	0.0008	2.6008
2	3	2.0000 2.0001	3.0007 3.0008	$[2.0000; 2.0001]$	$[3.0007; 3.0008]$	0.0000	0.0007	2.6929
3	3	3.0002 3.0003	3.0009 3.0009	$[3.0002; 3.0003]$	$[3.0009; 3.0009]$	0.0002	0.0009	3.0006
4	3	4.0002 4.0002	3.0010 3.0011	$[4.0002; 4.0002]$	$[3.0010; 3.0011]$	0.0002	0.0010	3.6404
0	4	0.0000 0.0000	4.0000 4.0000	$[0.0000; 0.0000]$	$[4.0000; 4.0000]$	0.0000	0.0000	4.0000
1	4	1.0008 1.0009	4.0016 4.0017	$[1.0008; 1.0009]$	$[4.0016; 4.0017]$	0.0009	0.0016	3.8250
2	4	2.0000 2.0000	4.0009 4.0010	$[2.0000; 2.0000]$	$[4.0009; 4.0010]$	0.0000	0.0009	3.6009
3	4	3.0005 3.0006	4.0007 4.0009	$[3.0005; 3.0006]$	$[4.0007; 4.0009]$	0.0006	0.0008	3.6407
4	4	4.0000 4.0001	4.0001 4.0002	$[4.0000; 4.0001]$	$[4.0001; 4.0002]$	0.0001	0.0001	4.0001

Table 2: Set_{xy}, $Set_{\overline{xy}}$ and Estimates of Error $\varepsilon_{(i,j)}$ and $\boldsymbol{T}_{x_iy_j}$

For instance, if we want to put in this mesh a square, passing by four points and without points in the center, we must put this above and right-hand of the mesh because the error will be smaller (see Figure 1 and Figure 2).

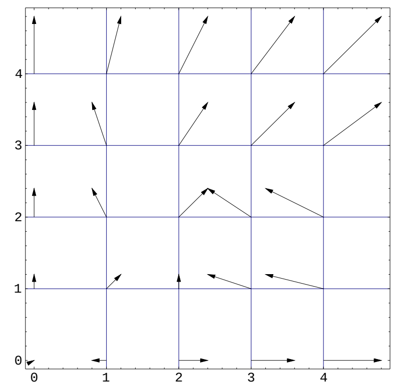


Figure 1: Vector map

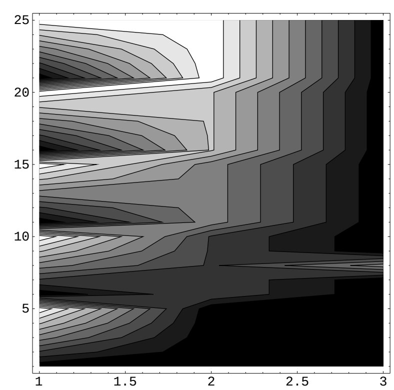


Figure 2: Contour map

Even so, there is a distortion of the square because there is a translation error. Besides, there is an uncertainty error of the measure associated to each point and it is also smaller in this region of the mesh.

5 Remarks

This paper look at the practical issues of using computers in the difficult process of re-build objects from its numerical features measured by a coordinate measuring machine.

The process works with ill-formed, ambiguous and vague ideas. This ambiguity and inaccuracy are carried over into the geometrical representation of such objects.

Despite these problems, the proposal of modelling presented here might be improved in a way. This way results of a strong interlink between scientific visualization and scientific computing. Informally, visualization is the transformation of data or information into pictures.

According to [Schroeder 1996] scientific visualization is the formal name given to the field of computer science that encompasses user interface, data representation and processing algorithms, visual representations, and other sensory presentations such as sound or touch.

In this sense, this paper presents the first part of a graphical environment design oriented to the scientific visualization of the *CMM's* error bound.

Nowadays, a mathematical modelling of measuring uncertainty referred of such machines was build. Because the real problem demands, the distortion error has the highest priority than the absolute error and this priority has a strong influence on the interpretation of such error. The graphic presentation of errors is like following: the distortion error will be show by a vector map and the absolute error by a contour map. Then the best region to measure a specific object is obtain by the analysis on graphic presentation.

6 References

[Alves 1996] Alves, A.S.: "Metrologia Geométrica"; Notas de Aula, Faculdade de Ciência e Tecnologia de Coimbra / Portugal (1996).

[Kulisch 1981] Kulisch, U.; Miranker, W.L.: "Computer Arithmetic in Theory and Practice"; Academic Press / New York (1981).

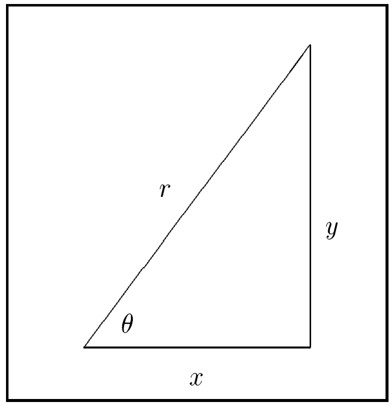
[Neumann 1991] Kulisch, H.J.: "Carl Zeiss Coordinate Measuring Machines Upgrade Quality in the Automotive Industry, Quality in Automobile Production"; June (1981).

[Schroeder 1996] Schroeder, W.; Martin, K.; Lorensen, B.: "The Visualization Toolkit: An Object-oriented Approach to 3D Graphics"; Prentice Hall, Upper Saddle River / New Jersey 07458 (1996).

Appendix

Proof of Formula.

Consider the figure below:



By Pitágoras, we have

$$x^2 + y^2 = r^2.$$

Therefore, $r = \sqrt{x^2 + y^2}$.

Doing $S = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$, onde $S = sen(\theta)$ e $C = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$, onde $C = cos(\theta)$, at that time, we have that $C^2 + S^2 = 1$. Thus, $S^2 = 1 - C^2$.

And doing $\alpha = C^2$, we weigh

$$V = \alpha \cdot x + (1-\alpha) \cdot y = C^2 \cdot x + S^2 \cdot y = \left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 \cdot x + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 \cdot y = \frac{x^3 + y^3}{x^2 + y^2}.$$

Therefore, in the case of the interval vectors, we have the semi-diameter weighed by

$$\varepsilon^{(i,j)} = \frac{\delta_{x_i}^3 + \delta_{y_j}^3}{\delta_{x_i}^2 + \delta_{y_j}^2}.$$