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Ordered Fuzzy Numbers Applied in Bee Swarm Optimization Systems

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Abstract: The paper presents an innovative OFNBee optimization method based on combining the swarm intelligence with the use of directed fuzzy numers OFN. In the introduction, the issues related to the subject of the study, including bee algorithms and OFN numbers, were reviewed. The innovative OFNBee algorithm was presented and verified against a set of known benchmarks functions such as Sphere, Rastrigin, Griewank, Rosenbrock, Schwefel and Ackley. These functions have been applied due to their reliability in the literature. In the further part of the study, the configuration of the algorithm parameters is carried out, including the launch of each mathematical function several dozen times for different data, such as different population sizes. The key part of the research and analysis was to compare OFNBee with six standard ABC, MBO, IMBO, TLBO, HBMO, BBMO bee algorithms. The article ends with a summary and an indication of the possible future works.

Key Words: OFNBee, OFN, Fuzzy logic, metaheuristic, ABC, MBO, IMBO, TLBO, HBMO, BBMO
Category: H.2, H.3.7, H.5.4

1 Introduction

Since 1992, when M. Dorigo published his doctoral thesis on the application of Ant Colony to solve TSP problems, a dynamic development of Swarm Intelligence methods within Artificial Intelligence has been observed. There have been many concepts of Multi-Agent System [Gil 2008] heuristics based on equal assumptions, among which Particle Swarm Optimization [Eberhart and Shi 2000] takes pride of place. According to the authors of this article, the most interesting inspirations come directly from nature. In recent decades, several hundred new optimization methods have appeared in this category, inspired by plankton beds, termite mounds, ant nests, swarms of locusts or bees [Slowik 2020] [Vasuki 2020] [Mirjalili and Dong 2020]. There are also interesting hybrid approaches combining Swarm Intelligence with Rough Set Theory or with Fuzzy Logic [Liu et al. 2007]. The authors presented in this article the concept of a new swarm optimization method which is a hybrid of bee swarm intelligence with generalization of calculations on fuzzy numbers, the so-called Ordered Fuzzy Numbers proposed by W. Kosinski [Prokopowicz, et al. 2017]. It is in this context of metaheuristic solutions that the method described in this article falls.

The new OFNBee swarm optimization method was proposed by Dawid Ewald in his doctoral thesis [Mirsadeghi 2015], [Czerniak et al. 2021] and in the paper [Ewald et al. 2018] [Nakrani 2004]. The inspiration for the method was a summer study on a paper by Martin Lindauer, who is a well-known scientist among entomologists. He noticed a characteristic "dance" performed by those bees that return to the swarm with food. On further observations, Lindauer found that the decision to change the food source is made with a relatively small percentage of the bees involved. Active bees by "dancing" gradually reduce the amount of proposed food sources, while most bees are inactive and wait for a decision to choose a single food source. It is therefore seemingly a kind of natural consensus. When there is only one food source left, most of the bees start fly towards the location indicated by the "dancing" bees. The theory of the meaning of the bee dance was later proved by subsequent entymologists, which was already mentioned at the beginning of this paper. They have incontrovertibly proven that with the use of dance and pheromone, a scout bee returning to the hive provides the swarm with information about the kind of food it has encountered. Bees fly during the day only because they navigate in accordance to the position of the sun (Fig. 1). One of the first information confirmed was that the angle which marks the position of the bee's body in relation to the sun is the direction of the route to the food source. Another conclusion was the discovery that the amplitude of the scout bee's vibrations carries information about the abundance of the food source, where higher amplitude means more food. Whereas the performed movement over a distance of a certain length towards the food informs the swarm about the amount of flight time necessary to reach the food source. It is difficult to represent this information using a single number, and nobody has done it so far. It was possible by means of the OFN notation and described in the following paragraph. It is also worth noting that to represent the distance to the food source, bees use three types of dance:

- round dance - it means that the food source is close to the hive,
- lourish dance - the distance to the hive is greater than 100 meters,

– crescent dance - the distance is less than 100 meters but not very close.

Figure 1 shows a dancing bee and illustrates the importance of the message submitted. One can also conclude, on the basis of entomologists' studies, that the method of selecting a new food location to visit is carried out during the dance of bees, this process can be seen as a kind of voting. However, this hypothesis had some imperfections, such as the observation that sometimes despite the lack of an unanimous decision by the dancing bee scouts, a swarm of waiting bees began to fly. It was also not detected how the waiting bees vote for the active dancing bees. As a result, two new hypotheses were formulated and attempted to be proved. The first is the quorum hypothesis, where it is sufficient that the majority of bees vote for a certain food source, and the second is the consensus hypothesis, seen here as a common concord of a sufficiently large subgroup of dancing scout bees. In the subsequent experimental studies, the quorum hypothesis was confirmed, while the consensus hypothesis was excluded. As it was observed in the case of reaching quorum, the dancing scouts returned to their swarms, emitting a peculiar sound that stimulated the remaining waiting bees to warm up their muscles. The duration of this warm-up usually lasted about an hour and was immediately followed by the flight of the bees to the new food location.

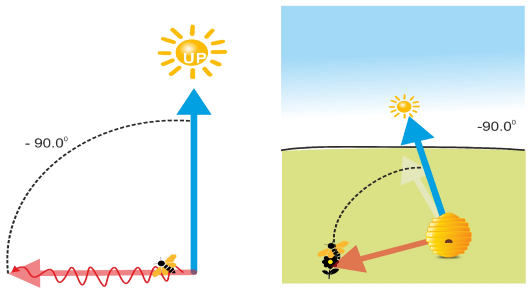

Figure 1: The dance angle indicates the navigation angle relative to the Sun

The way bees choose a new food source is an effective method and can undoubtedly be used in other areas. Many authors addressed the problems of consensus and compromise in decision making [Sadollah et. al. 3013][Zadeh 1965]. Scientists studying the issue of bee behavior have proposed the following three factors thanks to which the bee's decision should be correct.

– Organization is a key feature of an efficient decision making process. Due to the variety of objects and enabling them to fully share knowledge it is possible to make the right and unanimous decision.

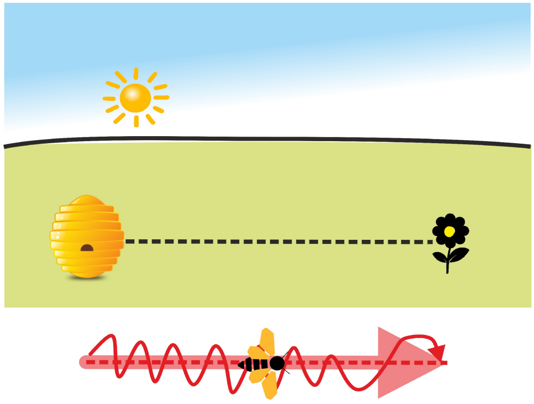

Figure 2: The time of dancing is proportional to the length of the flight

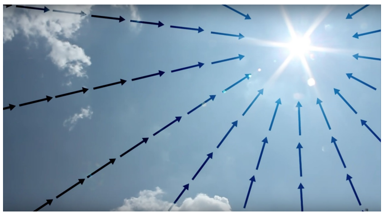

Figure 3: Bee navigation by the position of the sun.

- Competition is a feature that increases the quality of decisions made, because each of the objects continuously improves, and the more good decisions represented by the objects, the faster and easier the final decision.
- Balance is a feature that, on the other hand, affects the stabilization of group decisions and prevents stopping at local extremes which can be wrong decisions represented by individual objects. With that mechanism, certain elements of democracy strengthen the decisions represented by larger clusters of objects.

2 New OFNBee swarm optimization method

A well-suited way to describe the bee optimization and behavioral mechanisms observed in the hive is to use the OFN notation. Each bee provides input in the form of the following information:

- the direction towards which the food is located,
- the angle of navigation in relation to the position of the Sun,

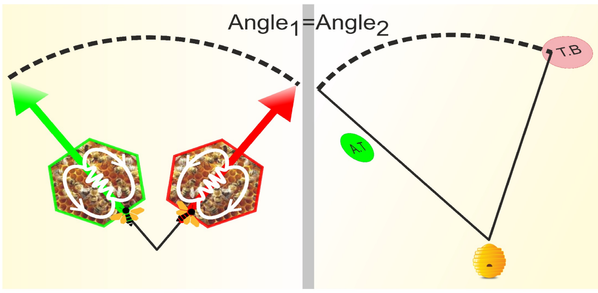

Figure 4: Two different flight directions indicated by dance

- the flight distance,
- affluence of the food source.

Figure 6 shows the OFN number that describes the information provided by a single scout bee:

- aF - the angle between the sun and the food source,
- sF - quantity of food,
- dF - distance to the food.

Part of the Figure 6 depicts the food source to which the bee should fly at an angle aF relative to the position of the sun. This Figure includes specified information including angle aF. The dF arm, in turn, determines the distance to the food source. On the other hand, sF is equivalent to the amplitude of sinusoidal vibrations of the bee and determines the amount of food. The ordered fuzzy number A is determined as shown below. In the first step, one can define the base of the trapezoid named support(A). In the next step, the angle aF is used to plot a rising slope of f(x). The intersection of the functions f(x) and y = 1 makes it possible to determine the second base of the trapezoid. The last side is a function of the falling edge g(x) which can be created by joining the two ends of the trapezoidal bases.

Below pseudocode of the new OFNBee algorithm proposed by the authors. Require: random determination of the initial food sources for the employed bees involved until required fulfillment of $N <> 0$ do

1. sending the employed bees involved to places near food sources stored in the memory and determining the amount of nectar contained therein
2. the process of fuzzyfication and selection of the places by onlooker bees

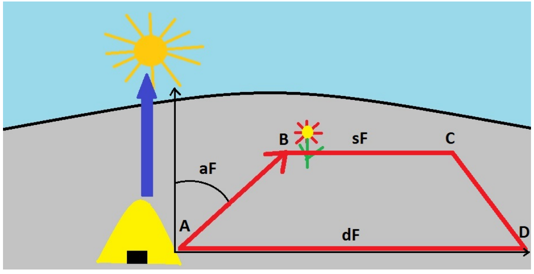

Figure 5: Graphical interpretation of the OFN number in OFNBee

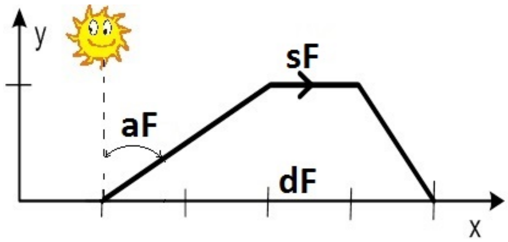

Figure 6: OFN number describes the information received from a scout bee

  - fuzzyfication of fit
  - fuzzyfication of sumyfit
  - fuzzyfication of unfeasible values
  - fuzzyfication of the sum of unfeasible values
  - calculation of the probability using the roulette wheel function - prob
  - defuzzyfication of prob[]
  - assignment of values and sending the onlooker bees

3. sending onlooker bees to locations near the selected food sources and determining the amount of nectar contained therein
4. cessation of the exploitation process for a source abandoned by the bees
5. sending scouts in order to randomly discover new food sources
6. sending scouts in order to randomly discover new food sources

end while

3 Application of the new OFNBee method

This chapter describes the tests performed and compares the results with standard algorithms.

3.1 Organization and course of research

Each new method should be tested against a set of known test functions. These functions were selected due to their frequent occurrence in the literature during the verification of the operation of optimization algorithms. The research was divided into two stages. The first one was to verify the correct operation of the new method by repeatedly finding the optimum of selected mathematical functions. The second stage of the verification of correctness was the solution of selected design problems, which are examples of multi-criteria optimization. For the needs of the first stage of the research, the most frequent functions were selected. Six commonly known mathematical functions were selected – [Abbass 2003] [Apiecionek et al. 2017] [Ewald et al. 2018] [Zarzycki et al. 2020] [Eberhart and Shi 2000] Sphere, Rosenbrock, Rastrigin, Griewank, Schwefel, Ackley – a more detailed description of them is provided below in chapter 3.2. As a result of the experiment, parameters of the OFNBee algorithm were selected, i.e. the blur and sharpen operator. Each series of trials included 30 separate runs of the algorithm for different operators. A detailed description can be found in subchapter 3.3. The next step was the experimental selection of the population size for the selected best operators. The last step is to compare the OFNBee results with the results for ABC and MBO, IMBO (Improved Marriage in Honey-Bees) [Gomez et al. 2008] TLBO (Teaching-Learning-Based Optimization) [Zarzycki et al. 2021], [Shimpi et al. 2015], HBMO (Honey Bees Mating Optimization Algorithm) and BBMO (Bumble Bees Mating Optimization) [Mernik et al. 2015].

3.2 Selected mathematical testing functions

Sphere function The Sphere function is described by the equation below [Surjanovic 2019]

$$f(x) = \sum_{i=1}^{n} x_i^2 \tag{1}$$

The graphical interpretation of the function is shown in the figure below.

- Recommended values of variables: : $-5.12 \leq x_i \leq 5.12$ $i = 1, 2, ..., n$
- Global minimum: $x = (0, ..., 0), f(x) = 0$

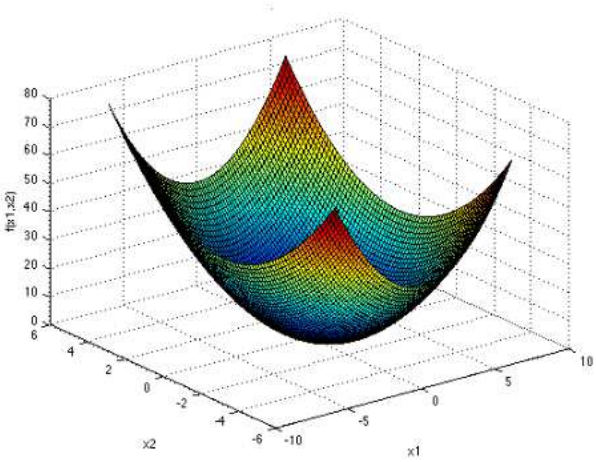

Figure 7: Sphere function presented in three-dimensional graphics

Rosenbrock function The Rosenbrock function is described by the equation below.

$$f(X) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \tag{2}$$

The graphical interpretation of the function is shown in Figure 8. Recommended variable values:

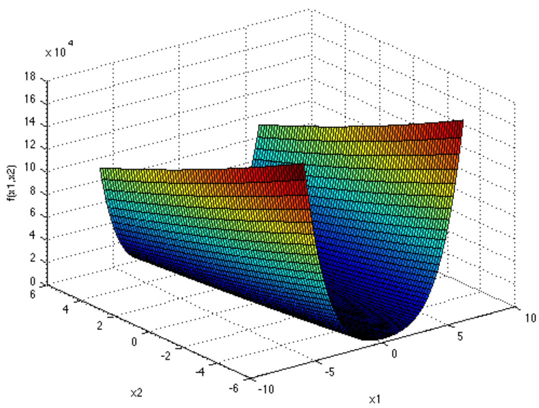

Figure 8: Graphical interpretation of Rosenbrock functionk [Surjanovic 2019]

- Recommended values of variables: $-2,048 \leq x_i \leq 2,048$ $i = 1, 2, ..., n$
- Global minimum: $x = (1, ..., 1), f(x) = 0$

Rastrigin function The Rastrigin function is described by the equation below.

$$f(X) = An + \sum_{i=1}^{n} [x_i^2 - Acos(2\pi x_i)] \tag{3}$$

The graphical interpretation of the function is shown in Figure 9.

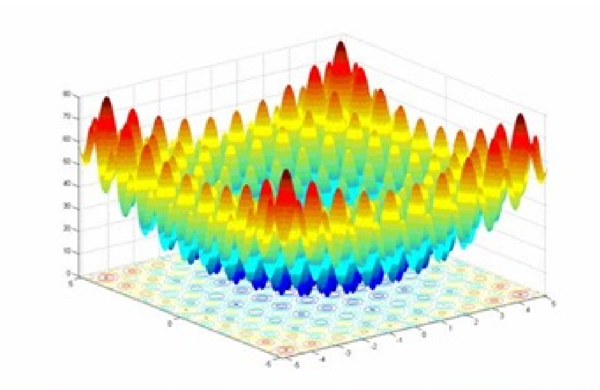

Figure 9: Graphical interpretation of the Rastrigin function [Surjanovic 2019]

- Recommended values of variables:: $-5,12 \leq x_i \leq 5,12$ $i = 1, 2, ..., n$
- Global minimum: $x = (0, ..., 0), f(x) = 0$

Griewank function The Griewank function is described by the following formula

$$f_n(x_1, ..., x_n) = 1 + \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}}) \tag{4}$$

The graphical interpretation of the function is shown in Figure 10. 10.

- Recommended values of variables: $-600 \leq x_i \leq 600$ $i = 1, 2, ..., n$
- Global minimum: $x = (0, ..., 0), f(x) = 0$

Schwefel function The Schwefel function is described by the following formula.

$$f(x) = \sum_{i=1}^{n} \Big[- x_i \sin(\sqrt{|x_i|}) \Big] \tag{5}$$

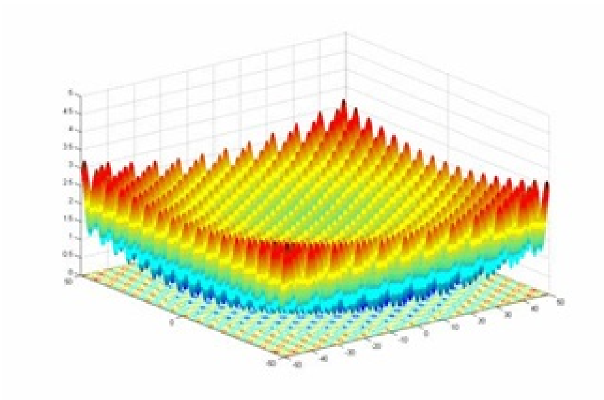

Figure 10: Graphical interpretation of the Griewank [Surjanovic 2019]

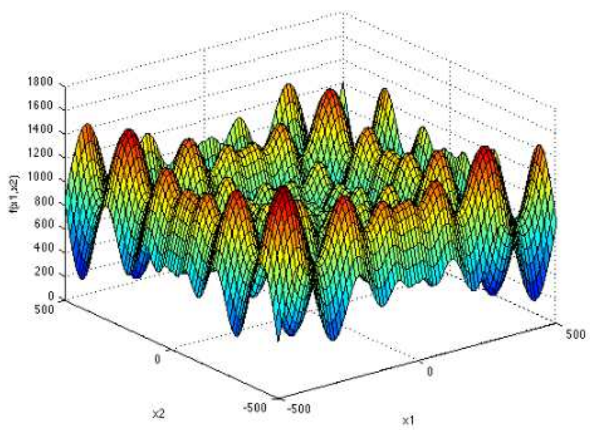

Figure 11: Overview graphic of the Schwefel function [Surjanovic 2019]

The test area is usually limited to a hypercube $-500 \leq x_i \leq 500,\ i = 1, ..., d$. Global minimum $f(x) = -n \cdot 418.9829; \quad x_i = 420.9687,\ i = 1, ..., n$.

- Recommended variable values: $-500 \leq x_i \leq 500\ i = 1, 2, ..., n$
- Global minimum: $x = (420.9687, ..., 420.9687), f(x) = 0$

Ackley function The Ackley function is described by the following formula

$$f(x) = -aexp(-b\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}) \tag{6}$$

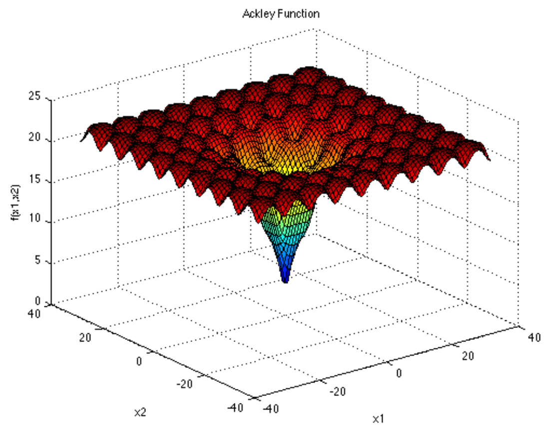

Figure 12: Graphical interpretation of the Ackley function [Surjanovic 2019]

- Recommended values of variables: $a = 20$, $b = 0.2$, $c = 2\pi$.
- Global minimum: $x = (0, ..., 0), f(x) = 0$

3.3 Configuration of OFNBee algorithm parameters

The new OFNBee algorithm is implemented in R language. The OFNBee algorithm includes several configuration options. For proper operation, when running the algorithm, you must specify the blur and sharpen operator and the size of the population. In the previous work of the authors, some of the experiments have already been carried out [Kumar et al. 2014], this concerns blur and sharpening operators.

The algorithm was run 30 times for each mathematical function from the test set and using the GR blur and sharpening operators based on [Kumar et al. 2014]. Each of the population sizes = 10, 50, 100, 300, 500, 800 and 1000 was checked. Tables 1, 2 and 3 show the mean results for all functions. When interpreting the results, it should be remembered that the lower the time value and, at the same time, the closer the result to the expected result, the better the algorithm works.

Analyzing the obtained results presented in Tables 1, 2 and 3, it can be noticed that only in the case of the NP - 50 population size configuration, exactly the minimum values of the function F (x) = 0 for four functions (Sphere, Rastrigin, Griewank, Ackley) are obtained. With each larger NP population - 100, 300, 500, 800, 1000, the algorithm's runtime increases, and at the same time the accuracy decreases - the results oscillate very close to the value of 0, but do not reach exactly this point

Table 1: Average results for 30 separate runs of the OFNBee algorithm. SD - standard deviation, AV - mean [10, 50]

Functions	OFNBee		
		10	50
Sphere	SD	0	0
	AV	0	0
	SD Time	0,0158295519	0,0187082869
	AV Time	0,0733333333	0,325
Rosenbrock	SD	0,0111787799	0,0031858532
	AV	0,0050520758	0,002336541
	SD Time	0,0074278135	0,0187696257
	AV Time	0,08	0,3683333333
Rastrigin	SD	0	0
	AV	0	0
	SD Time	0,0098552746	0,018695995
	AV Time	0,0783333333	0,3423333333
Griewank	SD	0	0
	AV	0	0
	SD Time	0,0169651434	0,0206336406
	AV Time	0,0786666667	0,3546666667
Schwefel	SD	50,2750418227	0,0590362232
	AV	30,3723629905	0,0126121427
	SD Time	0,0104166092	0,0299808368
	AV Time	0,1053333333	0,4366666667
Ackley	SD	0	0
	AV	4,44089209850063E-016	4,44089209850063E-016
	SD Time	0,0084486277	0,0209432731
	AV Time	0,081	0,376

3.4 The course of the experiment and the results

Using the known optimal configuration of the new method, its effectiveness in confrontation with known solutions should be checked. The introduction describes the most popular bee algorithms. The ABC algorithm is the most exploited solution. However, the BCO algorithm ceased to be developed. Recently, the MBO method, which already has numerous variations, has been very popular. In the literature, you can find many prepared results of the comparisons of MBO with its new hybrids. Therefore, in the further part of the study, the results of optimization of OFNBee, ABC mathematical functions and the MBO family algorithms will be compared. In the first part of the experiment, six bee

Table 2: Average results for 30 separate runs of the OFNBee algorithm. SD - standard deviation, AV - mean [100, 300]

Functions	OFNBee		
		100	300
Sphere	SD	2,02655857737991E-018	6,31025957557412E-019
	AV	2,34885444151442E-018	8,24894320059105E-019
	SD Time	0,0460496983	0,0911516357
	AV Time	0,8603333333	2,675
Rosenbrock	SD	0,001252226	0,0003925343
	AV	0,001516892	0,00040443
	SD Time	0,029048117	0,0593460531
	AV Time	0,761	2,3456666667
Rastrigin	SD	4,99448031707291E-014	0
	AV	9,11863177558795E-015	0
	SD Time	0,0651752721	0,0881880175
	AV Time	0,9406666667	2,8823333333
Griewank	SD	0,000028885	1,27870673323729E-008
	AV	1,0903397054011E-005	5,15877077722493E-009
	SD Time	0,0598551892	0,147276033
	AV Time	0,8863333333	2,8216666667
Schwefel	SD	0,0006950621	9,81530980475022E-008
	AV	0,0002142501	2,54877155498434E-005
	SD Time	0,0565522848	0,1313781941
	AV Time	0,8746666667	2,8346666667
Ackley	SD	0	0
	AV	4,44089209850063E-016	4,44089209850063E-016
	SD Time	0,1623958552	0,1420413571
	AV Time	1,21	3,6163333333

algorithms were used to compare their effectiveness with the new OFNBee algorithm:

- One entry in the list
- Another entry in the list

ABC, MBO, IMBO, TLBO, HBMO, BBMO these algorithms come from the group of bee algorithms based on the MBO idea, such as the HBPI (Honey Bees Policy Iteration) proposed by Chang in 2006 or by Afshar et al. in 2007, HBMO (Honey Bee Mating Optimization) [Chang 2006]. Another installment of the bee-based algorithm is proposed by Y. Marinakis, M. Marinaki, N.A.

Table 3: Average results for 30 separate runs of the OFNBee algorithm. SD - standard deviation, AV - mean [500, 800, 1000]

Functions	OFNBee			
		500	800	1000
Sphere	SD	4,51942100450434E-019	4,37374237462085E-019	1,67143365658109E-019
	AV	5,07797700363873E-019	3,758917040288E-019	2,17327638389E-019
	SD Time	0,1410571743	0,2637919766	0,4182845372
	AV Time	4,6783333333	8,23	11,0103333333
Rosenbrock	SD	0,0002794633	2,81409478453561E-005	9,86472388911669E-005
	AV	0,0002222803	6,00491221293E-005	8,88684794341611E-005
	SD Time	0,089324412	0,1015562808	0,1510579549
	AV Time	4,2026666667	7,1463333333	9,5656666667
Rastrigin	SD	0	0	0
	AV	0	0	0
	SD Time	0,1556406855	0,2766883387	0,4041741971
	AV Time	5,0403333333	8,7576666667	11,5446666667
Griewank	SD	8,11524068969743E-009	1,08234281894554E-008	7,8642539747665E-010
	AV	2,36041567032904E-009	2,70556494985E-009	3,86326233966135E-010
	SD Time	0,2076327992	0,4275814099	0,4765297207
	AV Time	5,043	8,555	11,4443333333
Schwefel	SD	1,98837903331214E-007	1,94918895665666E-008	0,00000006
	AV	0,000025513	2,54603135052397E-005	2,54714554330349E-005
	SD Time	0,2395359498	0,3530693006	1,8083592932
	AV Time	5,0213333333	8,802	13,1743333333
Ackley	SD	0	0	0
	AV	4,44089209850063E-016	4,44089209850063E-016	4,44089209850063E-016
	SD Time	0,7527122605	0,3751413527	2,8237452915
	AV Time	6,5703333333	11,23	17,1306666667

Matsatsinis in 2010 BBMO (Bumble Bees Mating Optimization) or its variant HBMO (Honey Bees Mating Optimization Algorithm) [Mernik et al. 2015]. Another well-known development of MBO is the IMBO (Improved Marriage in Honey Bees Optimization) algorithm, proposed in 2013 by Yuksel Celik and Erkan Ulker [Chang 2006]. In 2013, V. Patel and Rao R. Venkata proposed the TLBO (Teaching-LearningBased Optimization) method [Shimpi et al. 2015]. These methods were selected for comparison with the new OFNBee hybrid bee optimization method. Due to the lack of source codes in the R language, the presented results come from the literature. The use of these algorithms written in other languages could have a negative impact on time or exactness, which would degrade the reliability of the results obtained in this way. Currently, the most popular bee algorithm is ABC, which will also be compared in the experiment. For the ABC algorithm, the source code of which is available in the R language, the results were generated.

In Table 4, you can see that the new OFNBee method achieved the expected 0 results for the Sphere, Rosenbrock, Rastrigin, and Griewank functions, as did the

ABC and MBO algorithms. However, for the Schwefel and Ackley functions, none of the algorithms reached the expected value of 0. However, the new OFNBee algorithm achieved the closest result. Tables 4 and 5 compare the results for the selected functions. Table 4 shows the results for six selected mathematical functions generated by three main algorithms: OFNBee, ABC and MBO. As can be seen in Table 5, the results for the same functions for the algorithms from the MBO family are presented. These algorithms will serve as a benchmark for OFNBee as well as MBO and ABC.

Table 4: Comparison of results for 6 functions. SD - standard deviation, AV - mean

		OFNBee	ABC	MBO
Sphere	SD	0	5.21E - 18	3.36E + 01
	AV	0	4.88E - 17	0
Rosenbrock	SD	0,0031858532	0,008658828	8.67E - 03
	AV	0,002336541	0,013107593	0
Rastrigin	SD	0	0	0
	AV	0	0	0
Griewank	SD	0	0	0,32563139
	AV	0	0	0
Schwefel	SD	0,0590362232	9.09E - 13	1.94E + 16
	AV	0,0126121427	-4189.828873	-2,96E+211
Ackley	SD	0	3.57E - 17	0,0178353
	AV	4,44089209850063E-016	1.71E - 16	8.23045E - 15

Table 5 presents the results of the algorithms' operation. For the HBMO and BBMO algorithm and the Schwefel and Ackley functions, no results from the literature were presented. This is due to the fact that other studies were carried out using a different configuration of algorithms than the others, which would have an impact on the result, making it impossible to compare these algorithms

4 Analysis of the results

Tables 6 and 7 show a comparison of OFNBee with selected algorithms. As can be seen (x- with a better result, $\approx$ - with an equal result), the new algorithm obtained better results in all cases. It should be noted that when the algorithms obtained the expected results, the "x" sign was placed with both compared algorithms. This is because in such cases both algorithms are considered to be of

Table 5: Comparison of the results for the 6 functions. SD - standard deviation, AV - mean, NP - 50

		IMBO	TLBO	HBMO	BBMO
Sphere	SD	0	0		
	AV	0	0	0,67	0
Rosenbrock	SD	0,142502861	3.56E - 01		
	AV	0	47,0162	46,07	24,37
Rastrigin	SD	3.18323E - 14	3.56E - 01		
	AV	4,55E-15	2,03E-12	4,03	1.59E - 08
Griewank	SD	0	0		
	AV	0	0	1.44E - 02	0
Schwefel	SD	3.52272E + 30	1.48E + 02		
	AV	-6.17561E + 29	-20437.84		
Ackley	SD	2.30E - 15	8.32E - 31		
	AV	1.57E - 14	3.55E - 15		

similar effectiveness. The conducted experiment confirms that for selected mathematical functions the new hybrid optimization method obtains better results than the known algorithms for bee optimization. Table 6 presents a summary of the comparison of the operation of three algorithms - OFNBee with ABC and OFNBee with MBO. In the case of OFNBee's confrontation with ABC, the new method turned out to be better, and ABC received the same result as OFNBee in two cases. However, in the case of MBO, the new method also proved to be better, but MBO achieved the same result as OFNBee in four cases.

Table 6: Comparison of OFNBee with ABC and MBO

	OFNBee - ABC		OFNBee - MBO OFNBee	
	OFNBee	ABC	OFNBee	MBO
Sphere	x		x	x
Rosenbrock	x			x
Rastrigin	x		x	x
Griewank	x		x	x
Schwefel	x		x	
Ackley		x	x	
Sum	5	1	5	4

The results for the remaining algorithms are presented in Table 7. In all cases, the new method turned out to be better. IMBO got a score similar to OFNBee three times. In the case of TLBO, the algorithm scored OFNBee only twice. The HBMO algorithm always performed worse than OFNBee, and the BBMO achieved the expected result only twice.

Table 7: Comparison of OFNBee with IMBO, TLBO, and HBMO and BBMO

	OFNBee - IMBO -		OFNBee - TLBO -		OFNBee - HBMO		OFNBee - BBMO	
	OFNBee	IMBO	OFNBee	TLBO	OFNBee	HBMO	OFNBee	BBMO
Sphere	≈	≈	≈	≈	x		≈	≈
Rosenbrock	≈	≈	x		x		x	
Rastrigin	x		x		x		x	
Griewank	≈	≈	≈	≈	x		≈	≈
Schwefel		x	x					
Ackley	x		x					
Sum	2	1	4	0	4	0	2	0

The work presents a model of the OFNBee optimization algorithm, which achieves better results than the commonly known optimization algorithms reflecting the behavior of bees. The bee population sizes used by the algorithms differed in favor of OFNBee, i.e. the new method uses fewer resources, which in the case of very complex problems can significantly increase its efficiency. The operation of the new algorithm can be controlled by a number of parameters that allow it to influence its effectiveness and speed.

5 Conclusions and Future Work

In order to reliably compare and demonstrate the effectiveness of the new method of bee optimization using the arithmetic of Ordered Fuzzy Numbers for the experiment, the functions that can be found in the literature were selected. The choice of functions was dictated by the fact that they are used to test different bee algorithms and many other algorithms, which allowed to fairly compare the results. Due to the fact that the new method uses the arithmetic of Ordered Fuzzy Numbers, and this arithmetic is itself a complex topic, some elements have been described in a simplified manner. The fuzzyfication and defuzzyfication functions that are inherent in OFN and their impact on the operation of the new method are well documented in the literature cited and also will be part of a separate article.

Several new fuzzyfication operators were created in the course of research into a new optimization method. However, it seems reasonable to say that this could be the beginning of a new research topic. Since the fuzzyfication and defuzzyfication operators undoubtedly affect the optimization results of the new method, it may turn out that more research is needed on this issue. The direction of further research will therefore be the search for new fuzzyfication operators adapted to the problem of bee optimization, which could positively affect the operation of OFNBee. This approach would create a comprehensive OFNBee optimization environment. Investigating whether the development of new fuzzyfication and defuzzyfication operators can reduce the size of the bee population used by the algorithm seems to be an interesting field for further exploration. The next stage of research is an attempt to use the created algorithm in industry as a tool helping to obtain the most effective solutions. Simple configuration of the algorithm and low resource consumption makes it possible to use systems such as Arduino or Raspberry Pi to use OFNBee, help in the optimization of production or management processes. The simplicity and flexibility of the proposed testing environment created during the dissertation allows you to easily add new optimization problems. Thanks to this, it becomes possible to search for new potential places where the hybrid method could be used. The conducted research also revealed a problem with a uniform testing environment. The OFNBee algorithm was created in the R language, where you can get implemented mathematical functions and more and more new optimization methods, such as the ABC, PSO, GSO algorithm. Unfortunately, there are no older solutions, such as MBO or BCO. Therefore, while writing the dissertation, an environment was created, implemented in the R language, in which users will be able to test their algorithms under identical conditions and using the database of testing functions [Liu et al. 2007] [Yanxia 2017].

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Appendix

Table 8: Table of acronyms

Acronym	Full form
ABC	Artificial Bee Colony
BBMO	Bumble Bees Mating Optimization
BCO	Bee Colony Optimization
HBMO	Honey Bee Mating Optimization
HBPI	Honey Bees Policy Iteration
IMBO	Improved Marriage in Honey Bees Optimization
MBO	Marriage in Honey Bees Optimization
OFN	Ordered Fuzzy Numbers
OFNBee	Ordered Fuzzy Numbers Bee algorithm
TLBO	Teaching-LearningBased Optimization