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Integration and Selection of Linear SVM Classifiers in Geometric Space

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Abstract: Integration or fusion of the base classifiers is the final stage of creating multiple classifiers system. Known methods in this step use base classifier outputs, which are class labels or values of the confidence (predicted probabilities) for each class label. In this paper we propose an integration process which takes place in the geometric space. It means that the fusion of base classifiers is done using their decision boundaries. In order to obtain one decision boundary from boundaries defined by base classifiers the median or weighted average method will be used. In addition, the proposed algorithm uses the division of the entire feature space into disjoint regions of competence as well as the process of selection of base classifiers is carried out. The aim of the experiments was to compare the proposed algorithms with the majority voting method and assessment which of the analyzed approaches to integration of the base classifiers creates a more effective ensemble.

Key Words: classifier integration, ensemble of classifiers, svm
Category: Topic I.5.2 - Design Methodology

1 Introduction

In a typical supervised learning approach one classifier is used to designate the class label. The single classifiers have been known for many years, and one of the classifiers, which is derived from statistical methods of data analysis is Fisher's linear discriminant classifier. This classifier defines the decision boundary, which is a linear function (in a two-dimensional space) and defines the boundary in the feature space between two class labels. Another classifier, which defines a linear boundary between the two class labels is linear SVM classifier. The idea of single classifier is still used in recognition tasks, however, for over twenty years, also multiple classifiers systems have been used [Drucker et al. 1994, Xu et al. 1992]. The system consisting of more than one, so called, base classifier is defined as ensemble of classifiers (EoC) or multiple classifiers systems (MCSs) [Cyganek 2012, Giacinto and Roli 2001, Przybyła-Kasperek 2019, Woźniak et al. 2014]. The reasons for usage of a classifier ensemble include, for example, the fact that single classifiers are often unstable (small changes in input data may result in creation of very different decision boundaries) or they are overfitting.

The task of constructing MCSs can be generally divided into three steps: generation, selection and integration [Britto et al. 2014]. In the first step a set of base classifiers is defined. Classifiers can be generated using: different training sets, different models and different parameters of these models, manipulating the training objects or using different feature subspaces. The classifiers, which are called heterogeneous, belong to different machine learning algorithms, but they are trained on the same data set. In this paper, we will focus on homogeneous classifiers which are obtained by applying the same classification algorithm to different learning sets.

The second phase of building EoC is related to the choice of a set of classifiers or one classifier from the whole available pool of base classifiers [Aksela and Laaksonen 2006, Britto et al. 2014, Burduk and Walkowiak 2015]. This article will use the selection of classifiers, which is done independently in each region of competence. The selection uses the accuracy of the classification measured using validation set and heuristic division of the whole feature space into disjoint subspaces.

The integration or fusion process is widely discussed in the pattern recognition literature [Ponti Jr. 2011, Tulyakov et al. 2008]. This process uses the outputs of the base classifiers selected in the previous step. Generally, the output of a base classifier can be divided into two types: an abstract or measurement space [Kuncheva 2004]. With respect to the spaces presented here, in the literature there are many proposals of methods for combining the outputs of base classifiers in order to obtain a single class label.

If a set of labels is available (abstract space), various methods for combining the responses of the base classifiers can be used to obtain the final decision [Lam and Suen 1997, Przybyła-Kasperek et al. 2017, Ruta and Gabrys 2005]. One of simpler methods for combining the responses of classifiers is majority voting. In this method, each component classifiers in EoC casts an equal vote and the object is classified to the class, for which most base classifiers cast their votes. The advantage of this method is its simplicity and a lack of any calculations other than counting the votes cast by individual classifiers. One of disadvantages of using this approach for combining the responses is a frequent draw situation, which means that the same number of classifiers indicates more than one class. In two-class recognition tasks, this problem can be solved by using an odd number of base classifiers.

Another way to combine classifiers is weighted voting. In this approach, each classifier is assigned with a weight that is taken into account when determining the final decision of the ensemble. Typically, weights depend on the quality of the base classifiers assigned to them and are usually normalized to unity. Another way to calculate the weights is the approach in which each classifier is assigned with the number of weights equal to the number of the predefined classes in the recognition task. The resulting weights are also normalized to unity in each class separately. Another method based on counting the votes cast by individual classifiers is the Behavior-Knowledge Space method [Huang and Suen 1993]. In this method, an object is classified into the class, for which the conformity of responses from individual classifiers exceeds the predetermined threshold value.

Several methods for determining responses of EoC in the measurement level has been proposed in the literature [Fumera and Roli 2005, Kuncheva et al. 2001]. There can be distinguished the sum method, the product method or the methods based on position statistics. In the latter group, minimum, maximum and median methods are distinguished. The final decision of EoC is made according to the $\arg\max$ rule. The application of a combined classifier using position statistics, according to the minimum rule, gives the most pessimistic result. In this case, if only one of the base classifiers returns the zero value of a posteriori probability of the membership of the object in the selected class, the response of the entire ensemble for this class will be "no". The situation is quite opposite when the maximum method is used, where a posteriori probability equal to unity obtained for the selected class by one base classifier is associated with the selection of this class by the entire ensemble. A compromise between these methods is the use of the median method. In the methods discussed above, the support functions obtained from individual base classifiers have an equal share in the integration of responses of these classifiers. Weighted versions of these methods can also be easily created – as in the case of integration in the abstract space.

In this paper we propose the concept of the classifier integration process which takes place in the geometric space. It means that we use the decision boundary in the integration process instead of the base classifier output represented by the abstract or measurement space. In other words the integration of base classifiers is done using their decision boundaries instead of class labels or predicted probabilities.

The author's earlier work [Burduk 2017] presents results of the integration base classifier in the geometric space in which the base classifiers use Fisher's classification rule, while the process of the base classifier selection is performed in the regions of competence defined by the intersection points of decision boundary functions. The paper [Burduk 2018] presents results of the integration base classifier in the geometric space in which the harmonic mean is used.

In this paper we use a different linear classifier', the competence region will be defined by the heuristic division of the feature space and in addition, the median or weighed mean is used to determine the decision boundary of EoC.

The remainder of this paper is organized as follows. Section 2 presents the basic concept of the supervised classification and building EoC. Section 3 describes the proposed method for the integration of base classifiers in the geometric space in which the median or weighted mean is used after selection to determine the final decision boundary of EoC. The experimental evaluation is presented in Section 4. The discussion and conclusions from the experiments are presented in

Section 5.

2 Basic concept

Let us consider the binary classification task. It means that we have two class labels $\Omega = \{0, 1\}$. Each pattern is characterized by the feature vector x. The recognition algorithm Ψ maps the feature space x to the set of class labels Ω according to the general formula:

$$\Psi(x) \in \Omega. \tag{1}$$

Let us assume that K $(k \in \{1, 2, \ldots, K\})$ different classifiers $\Psi_1, \Psi_2, \ldots, \Psi_K$ are available to solve the classification task. In MCSs these classifiers are called base classifiers. In the binary classification task, K is assumed to be an odd number. As a result of all the classifiers' actions, their K responses are obtained. Usually all K base classifiers are applied to make the final decision of MCSs although some methods select just one base classifier from the ensemble. The output of only this base classifier is then used in the class label prediction. Another option is to select a subset of the base classifiers. Then, the combining method is needed to make the final decision of EoC.

The majority vote is a combining method that works at the abstract level. This voting method allows counting base classifiers outputs as a vote for a class and assigns the input pattern to the class with the greatest count of votes. The majority voting algorithm is defined as follows:

$$\Psi_{MV}(x) = \arg\max_{\omega} \sum_{k=1}^{K} I(\Psi_k(x), \omega), \tag{2}$$

where $I(\cdot)$ is the indicator function with the value 1 in the case of the correct classification of the object described by the feature vector x, i.e. when $\Psi_k(x) = \omega$. In the majority vote method each of the individual classifiers takes an equal part in building EoC.

3 Proposed method

Conventional fusion methods fuse the class labels or confidence values produced by the base classifiers to produce the class labels of EoC. Assuming that the decision boundary for each base classifier is known we present the integration process that is performed in the geometric space. Therefore in the proposed method we don't use in fusion the class labels or confidence values. The geometric approach discussed in [Li et al. 2012] is applied to find characteristic points in the geometric space. These points are then used to determine the decision boundaries.

Thus, the results presented in [Pujol and Masip 2009] do not concern the process of integration of base classifiers, but a method for creating decision boundaries.

The proposed algorithm to find the final decision boundary of EoC in geometry space is defined as:

Algorithm 1: Algorithm for finding decision boundary of combining classifier in geometry space Ψ_{GS}

Input : K base classifiers $\Psi_1, \ldots, \Psi_K$, M number of regions of competence

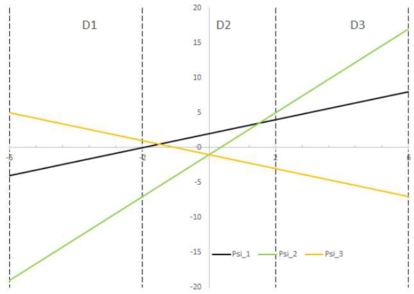
Output: Decision boundary of combining classifier Ψ_{GS}

1 Train each of the base classifiers $\Psi_1, \Psi_2, \ldots, \Psi_K$.
2 Divide feature space into non-overlapping M subspaces.
3 Evaluate competence of each base classifier on every subspace using the validation data set.
4 Having quality measures select l best classifiers, where $1 < l < K$ is calculated in each region of competence.
5 Define the decision boundary of the proposed EoC classifier Ψ_{GS} as median Ψ_{MdGS} of the decision boundaries of base classifiers in each region of competence separately or as weighted mean Ψ_{wMeGS}.

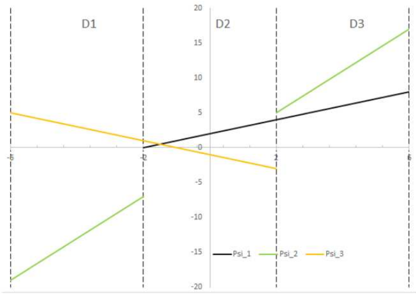
The graphical interpretation of the proposed method for the two-dimensional feature space, three regions of competence and three base classifiers is shown in Fig. 1. Decision boundaries defined by 3 linear classifiers are presented in Fig. 1(a). In addition, there are also marked three regions of competence labeled $D1, D2, D3$. In each region of competence one base classifier is rejected which is shown in Fig. 1(b). For example, in the region labeled $D1$ the classifier Ψ_1 is rejected i.e. this classifier has the lowest quality of classification on the validation set. Fig. 1(c) shows the decision boundary defined by the proposed method Ψ_{MdGS} (the blue piecewise linear function), which is the median of the values of decision boundaries after the selection process. Fig. 1(d) shows, on the other hand, the decision boundary defined by the majority vote rule Ψ_{MV} (the red piecewise linear function). Therefore, Fig. 1(c)-1(d) show the visual difference between the decision boundary of EoC proposed in the paper and Ψ_{MV}.

4 Experimental Setup

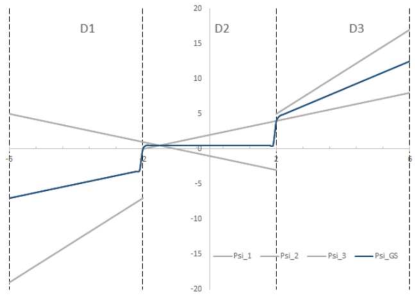
In the experiment we used the linear SVM method as a base classifier and we created a pool of classifiers consisting of nine base classifiers. SVM classifier implementation from python library scikit-learn [Pedregosa et al. 2011] was



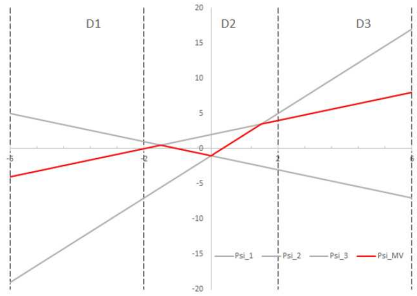
(a) Decision boundaries of the base classifiers Ψ_1, Ψ_2, Ψ_3 and regions of competence $D1, D2, D3$



(b) Decision boundaries of the base classifiers after selection one base classifier in each region of competence



(c) Decision boundary of proposed classifier – blue



(d) Decision boundary of majority vote classifier – red

Figure 1: An example of the proposed method with three base classifiers and three regions of competence

utilised. The integration algorithm was implemented in python using numpy [Travis E. 2006], scikit-learn and scipy [Jones et al. 2018]. The latter was also used to conduct statistical tests.

In selection steps we removed basic classifiers from the pool of all base classifiers, for example Ψ^5_{MdGS} means that EoC after the selection consisted of five base classifiers i.e. four base classifier has been removed in each region of competence. All base classifiers are taken into consideration in the majority voting (MV) rule Ψ_{MV}, that is why the number of classifiers is omitted. The experiments were conducted for space division into $3, 5, 7$ and 9 regions of competence. The division into regions of competence is heuristic. These regions are created by the orthogonal division in relation to one of the features.

The experiments were conducted using open-source data sets available on platforms UCI Machine Learning Repository [Dheeru and Karra 2017] and KEEL Data Set Repository [Alcalá-Fdez et al. 2011]. For clarity following abbreviations for data sets names are used: bio – Biodeg, bup – Bupa, cry – Cryotherapy, dat – Data banknote authentication, hab – Haberman, ion – Ionosphere, met – Meter a, pop – Pop failures, sei – Seismic bumps, two – Twonorm, wis – Wisconsin. For all data sets the feature selection process [Guyon and Elisseeff 2003, Rejer 2015] was performed to indicate two most informative features.

It was asserted, that after data set division, all base classifiers were oriented in the same way. That means, that whenever there was an unordered model among all base classifiers (it classified objects under decision boundary with a certain label, which was restricted for objects over boundary in other models), the data set was rejected.

5 Results and Discussion

The main aim of the experiments was to compare the quality of classification [Trawiński et al. 2012] of the proposed method of integration base classifiers in the geometric space Ψ_{wMeGS} and Ψ_{MdGS} with MV rule Ψ_{MV}. Additionally statistical tests were performed to compare the improvement achieved by using median and weighted mean in composing decision boundary to determine the best setup possible for the used set of benchmarking datasets. In order to compare the quality of the classification, we used two classification measures: accuracy (ACC) and Matthews correlation coefficient (MCC). Tab. 1– 4 show the results of ACC and Tab. 5– 8 show the results of MCC. Along with quality measures, average ranks obtained in nonparametric Friedman tests are written in the last column. This test was carried out for each division of feature space (feature space was divided in 3, 5, 7 and 9 regions of competence) and method used for integration of decision boundaries separately, which means that 16 (8 tests for each quality measure) tests were performed. The difference in the quality of the classification in some cases was observed, because Bonferroni – Dunn test requires difference in Friedman ranks to be at least 2.45 (7 algorithms are compared against the reference method, 12 data sets are used) to reject the null hypothesis at the significance level of $\alpha = 0.1$.

Bonferroni-Holm post-hoc tests were conducted in order to determine whether the proposed integrated classifiers Ψ_{MdGS} or Ψ_{wMdeGS} are better or worse than the reference method Ψ_{MV}. Tab. 9 shows p-values of Bonferroni–Holm test gathered for classification measures ACC, MCC and number regions of competence based on non-parametric Friedman tests presented in previous tables. This test pointed out that there is a statistical difference in the classification results for the three examined cases at the significance level of $\alpha = 0.1$. The p-values for Holm test and the selected cases are shown in Tab. 10.

Mean ranks presented in Tab. 1- 8 and p-values of Bonferroni–Holm test from Tab. 10 indicate that the median method of integrating base classifiers in

Table 1: ACC and mean rank for space division into 3 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.724	0.580	0.827	0.888	0.774	0.886	0.832	0.927	0.931	0.747	0.935	0.946	4.25
Ψ^3_{wMeGS}	0.715	0.575	0.855	0.888	0.774	0.889	0.835	0.928	0.931	0.747	0.935	0.948	4.25
Ψ^4_{wMeGS}	0.708	0.571	0.862	0.887	0.764	0.889	0.849	0.927	0.935	0.748	0.936	0.948	4.67
Ψ^5_{wMeGS}	0.721	0.574	0.869	0.886	0.760	0.884	0.851	0.924	0.908	0.748	0.939	0.948	4.17
Ψ^6_{wMeGS}	0.724	0.580	0.860	0.886	0.758	0.882	0.846	0.924	0.926	0.748	0.940	0.949	**4.12**
Ψ^7_{wMeGS}	0.722	0.583	0.867	0.886	0.762	0.879	0.846	0.923	0.937	0.748	0.939	0.948	4.79
Ψ^8_{wMeGS}	0.720	0.584	0.867	0.887	0.761	0.878	0.842	0.924	0.933	0.748	0.937	0.947	4.33
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**5.42**
Ψ^2_{MdGS}	0.724	0.577	0.825	0.888	0.775	0.886	0.810	0.927	0.931	0.747	0.933	0.947	**3.75**
Ψ^3_{MdGS}	0.725	0.582	0.880	0.888	0.778	0.889	0.805	0.926	0.937	0.747	0.933	0.948	5.17
Ψ^4_{MdGS}	0.724	0.576	0.859	0.886	0.775	0.893	0.807	0.926	0.911	0.748	0.935	0.950	4.33
Ψ^5_{MdGS}	0.725	0.583	0.889	0.887	0.775	0.891	0.799	0.925	0.937	0.748	0.936	0.948	5.00
Ψ^6_{MdGS}	0.724	0.581	0.872	0.887	0.773	0.896	0.791	0.924	0.922	0.748	0.939	0.948	4.08
Ψ^7_{MdGS}	0.725	0.582	0.889	0.891	0.772	0.889	0.765	0.923	0.938	0.748	0.938	0.947	5.00
Ψ^8_{MdGS}	0.724	0.578	0.877	0.889	0.771	0.893	0.755	0.924	0.925	0.748	0.941	0.946	4.00
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**4.67**

Table 2: ACC and mean rank for space division into 5 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.727	0.574	0.834	0.898	0.761	0.891	0.777	0.926	0.931	0.748	0.932	0.947	4.42
Ψ^3_{wMeGS}	0.726	0.582	0.847	0.895	0.766	0.894	0.787	0.927	0.931	0.748	0.935	0.950	5.42
Ψ^4_{wMeGS}	0.713	0.576	0.831	0.893	0.760	0.893	0.807	0.926	0.935	0.748	0.935	0.950	4.08
Ψ^5_{wMeGS}	0.721	0.580	0.847	0.895	0.755	0.887	0.824	0.923	0.909	0.748	0.937	0.949	**3.50**
Ψ^6_{wMeGS}	0.724	0.581	0.853	0.894	0.757	0.883	0.832	0.922	0.926	0.748	0.937	0.950	3.83
Ψ^7_{wMeGS}	0.722	0.584	0.855	0.895	0.758	0.880	0.841	0.922	0.937	0.748	0.938	0.949	4.83
Ψ^8_{wMeGS}	0.721	0.584	0.857	0.893	0.759	0.876	0.849	0.924	0.933	0.748	0.936	0.947	4.58
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**5.33**
Ψ^2_{MdGS}	0.726	0.569	0.841	0.898	0.764	0.892	0.782	0.926	0.931	0.748	0.934	0.948	4.08
Ψ^3_{MdGS}	0.725	0.574	0.874	0.896	0.771	0.893	0.776	0.925	0.937	0.747	0.934	0.949	4.87
Ψ^4_{MdGS}	0.724	0.569	0.848	0.891	0.769	0.896	0.787	0.924	0.911	0.748	0.935	0.951	4.00
Ψ^5_{MdGS}	0.725	0.575	0.876	0.891	0.772	0.895	0.801	0.925	0.937	0.748	0.935	0.948	5.46
Ψ^6_{MdGS}	0.724	0.573	0.863	0.891	0.769	0.895	0.784	0.924	0.922	0.748	0.939	0.949	**3.58**
Ψ^7_{MdGS}	0.725	0.577	0.884	0.893	0.769	0.890	0.761	0.922	0.938	0.748	0.940	0.948	4.75
Ψ^8_{MdGS}	0.724	0.577	0.869	0.891	0.771	0.893	0.748	0.923	0.926	0.748	0.942	0.946	4.33
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**4.92**

Table 3: ACC and mean rank for space division into 7 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.727	0.574	0.815	0.902	0.768	0.888	0.782	0.930	0.931	0.748	0.934	0.946	4.58
Ψ^3_{wMeGS}	0.726	0.575	0.830	0.899	0.768	0.892	0.784	0.929	0.931	0.748	0.935	0.949	5.33
Ψ^4_{wMeGS}	0.714	0.573	0.811	0.896	0.765	0.889	0.814	0.928	0.935	0.748	0.935	0.949	4.00
Ψ^5_{wMeGS}	0.724	0.574	0.829	0.896	0.759	0.885	0.823	0.926	0.909	0.748	0.936	0.949	**3.83**
Ψ^6_{wMeGS}	0.724	0.574	0.835	0.895	0.761	0.881	0.831	0.924	0.926	0.748	0.937	0.950	4.25
Ψ^7_{wMeGS}	0.723	0.577	0.845	0.895	0.760	0.878	0.846	0.924	0.937	0.748	0.938	0.949	4.83
Ψ^8_{wMeGS}	0.722	0.584	0.842	0.893	0.761	0.875	0.851	0.925	0.933	0.748	0.937	0.948	4.17
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**5.00**
Ψ^2_{MdGS}	0.726	0.576	0.828	0.902	0.771	0.890	0.769	0.930	0.931	0.748	0.932	0.946	4.67
Ψ^3_{MdGS}	0.725	0.574	0.868	0.896	0.773	0.893	0.760	0.928	0.937	0.748	0.933	0.948	4.79
Ψ^4_{MdGS}	0.724	0.575	0.827	0.891	0.773	0.895	0.781	0.928	0.911	0.748	0.936	0.951	4.58
Ψ^5_{MdGS}	0.725	0.579	0.865	0.892	0.774	0.894	0.790	0.926	0.937	0.748	0.935	0.948	**5.21**
Ψ^6_{MdGS}	0.724	0.577	0.861	0.891	0.772	0.895	0.777	0.925	0.922	0.748	0.939	0.949	4.17
Ψ^7_{MdGS}	0.725	0.577	0.867	0.893	0.772	0.889	0.758	0.923	0.938	0.748	0.940	0.948	4.83
Ψ^8_{MdGS}	0.724	0.575	0.868	0.891	0.771	0.893	0.748	0.924	0.926	0.748	0.941	0.946	3.50
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**4.25**

Table 4: ACC and mean rank for space division into 9 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.726	0.573	0.848	0.900	0.770	0.907	0.746	0.927	0.931	0.748	0.934	0.947	4.33
Ψ^3_{wMeGS}	0.725	0.576	0.859	0.898	0.771	0.911	0.754	0.926	0.931	0.748	0.936	0.950	4.75
Ψ^4_{wMeGS}	0.714	0.574	0.842	0.895	0.764	0.907	0.784	0.927	0.935	0.748	0.937	0.950	4.50
Ψ^5_{wMeGS}	0.724	0.574	0.849	0.894	0.757	0.902	0.799	0.925	0.909	0.748	0.938	0.950	**3.92**
Ψ^6_{wMeGS}	0.724	0.576	0.859	0.893	0.759	0.899	0.808	0.924	0.926	0.748	0.939	0.950	4.75
Ψ^7_{wMeGS}	0.722	0.582	0.863	0.892	0.758	0.895	0.825	0.923	0.937	0.748	0.939	0.950	4.50
Ψ^8_{wMeGS}	0.721	0.585	0.861	0.891	0.758	0.892	0.830	0.925	0.933	0.748	0.938	0.948	4.25
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**5.00**
Ψ^2_{MdGS}	0.726	0.576	0.841	0.900	0.772	0.891	0.726	0.927	0.931	0.748	0.932	0.946	4.08
Ψ^3_{MdGS}	0.725	0.586	0.877	0.894	0.776	0.893	0.726	0.925	0.937	0.747	0.933	0.949	5.04
Ψ^4_{MdGS}	0.724	0.577	0.829	0.890	0.771	0.895	0.748	0.926	0.911	0.748	0.935	0.951	4.25
Ψ^5_{MdGS}	0.725	0.579	0.865	0.890	0.774	0.894	0.766	0.925	0.937	0.748	0.936	0.948	5.04
Ψ^6_{MdGS}	0.724	0.574	0.858	0.890	0.773	0.895	0.766	0.924	0.922	0.748	0.938	0.949	3.92
Ψ^7_{MdGS}	0.725	0.577	0.867	0.892	0.771	0.889	0.754	0.923	0.938	0.748	0.939	0.948	4.92
Ψ^8_{MdGS}	0.724	0.578	0.860	0.891	0.769	0.892	0.746	0.924	0.926	0.748	0.940	0.946	**3.75**
Ψ_{MV}	0.723	0.582	0.901	0.891	0.770	0.839	0.743	0.923	0.938	0.748	0.940	0.952	**5.00**

Table 5: MCC and mean rank for space division into 3 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.330	0.130	0.658	0.775	0.257	0.755	0.682	0.462	0.019	0.000	0.863	0.828	5.04
Ψ^3_{wMeGS}	0.325	0.106	0.720	0.774	0.255	0.759	0.690	0.454	0.020	0.000	0.862	0.835	4.87
Ψ^4_{wMeGS}	0.304	0.109	0.734	0.771	0.179	0.757	0.722	0.417	0.053	0.000	0.865	0.833	4.87
Ψ^5_{wMeGS}	0.320	0.095	0.750	0.770	0.136	0.747	0.725	0.389	0.038	0.000	0.869	0.833	4.87
Ψ^6_{wMeGS}	0.324	0.107	0.729	0.770	0.095	0.740	0.715	0.379	0.031	0.000	0.872	0.837	4.46
Ψ^7_{wMeGS}	0.316	0.107	0.744	0.771	0.086	0.735	0.716	0.372	0.013	0.000	0.870	0.832	3.79
Ψ^8_{wMeGS}	0.310	0.112	0.743	0.772	0.078	0.730	0.707	0.374	0.006	0.000	0.867	0.828	**3.29**
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**4.79**
Ψ^2_{MdGS}	0.331	0.130	0.664	0.775	0.257	0.755	0.631	0.462	0.019	0.000	0.859	0.829	4.46
Ψ^3_{MdGS}	0.331	0.149	0.771	0.774	0.264	0.759	0.620	0.448	0.000	0.000	0.858	0.836	4.87
Ψ^4_{MdGS}	0.326	0.137	0.726	0.771	0.240	0.767	0.627	0.450	0.047	0.000	0.862	0.840	**4.96**
Ψ^5_{MdGS}	0.328	0.161	0.792	0.772	0.223	0.763	0.606	0.443	0.000	0.000	0.865	0.833	4.71
Ψ^6_{MdGS}	0.323	0.161	0.756	0.771	0.217	0.774	0.587	0.444	0.027	0.000	0.870	0.835	4.79
Ψ^7_{MdGS}	0.327	0.158	0.791	0.780	0.203	0.757	0.528	0.443	0.000	0.000	0.868	0.830	4.42
Ψ^8_{MdGS}	0.325	0.139	0.769	0.776	0.198	0.766	0.508	0.443	0.023	0.000	0.875	0.825	3.96
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**3.83**

Table 6: MCC and mean rank for space division into 5 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.333	0.123	0.684	0.795	0.204	0.763	0.575	0.462	0.018	0.000	0.855	0.830	4.79
Ψ^3_{wMeGS}	0.333	0.140	0.701	0.788	0.222	0.768	0.595	0.461	0.021	0.000	0.862	0.838	**5.71**
Ψ^4_{wMeGS}	0.309	0.115	0.673	0.784	0.181	0.763	0.637	0.434	0.053	0.000	0.863	0.837	4.12
Ψ^5_{wMeGS}	0.319	0.116	0.708	0.789	0.120	0.749	0.677	0.394	0.040	0.000	0.866	0.836	4.87
Ψ^6_{wMeGS}	0.322	0.113	0.718	0.786	0.114	0.738	0.699	0.388	0.031	0.000	0.865	0.836	4.37
Ψ^7_{wMeGS}	0.317	0.127	0.724	0.788	0.080	0.732	0.715	0.373	0.013	0.000	0.867	0.834	4.29
Ψ^8_{wMeGS}	0.312	0.120	0.726	0.784	0.077	0.722	0.730	0.394	0.006	0.000	0.865	0.829	3.62
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**4.21**
Ψ^2_{MdGS}	0.333	0.116	0.694	0.795	0.218	0.768	0.590	0.462	0.017	0.000	0.859	0.832	4.71
Ψ^3_{MdGS}	0.331	0.137	0.753	0.789	0.236	0.769	0.573	0.447	0.000	0.000	0.859	0.838	4.96
Ψ^4_{MdGS}	0.326	0.132	0.703	0.780	0.217	0.775	0.595	0.446	0.047	0.000	0.861	0.845	4.87
Ψ^5_{MdGS}	0.328	0.147	0.762	0.781	0.226	0.772	0.619	0.453	0.000	0.000	0.861	0.834	**5.46**
Ψ^6_{MdGS}	0.323	0.137	0.736	0.780	0.209	0.772	0.581	0.453	0.027	0.000	0.869	0.836	4.54
Ψ^7_{MdGS}	0.327	0.153	0.779	0.785	0.201	0.761	0.524	0.443	0.000	0.000	0.871	0.833	4.33
Ψ^8_{MdGS}	0.325	0.140	0.751	0.780	0.203	0.766	0.494	0.439	0.023	0.000	0.876	0.825	3.79
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**3.33**

Table 7: MCC and mean rank for space division into 7 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.332	0.116	0.662	0.802	0.223	0.754	0.579	0.485	0.018	0.000	0.859	0.823	4.96
Ψ^3_{wMeGS}	0.331	0.113	0.682	0.797	0.228	0.762	0.586	0.465	0.021	0.000	0.861	0.833	**5.62**
Ψ^4_{wMeGS}	0.310	0.102	0.640	0.791	0.202	0.752	0.650	0.445	0.053	0.000	0.862	0.832	4.21
Ψ^5_{wMeGS}	0.325	0.100	0.676	0.790	0.146	0.740	0.673	0.417	0.040	0.000	0.863	0.833	4.54
Ψ^6_{wMeGS}	0.320	0.098	0.691	0.787	0.121	0.731	0.691	0.399	0.031	0.000	0.865	0.835	4.46
Ψ^7_{wMeGS}	0.316	0.105	0.708	0.787	0.090	0.724	0.722	0.368	0.013	0.000	0.867	0.833	4.04
Ψ^8_{wMeGS}	0.312	0.118	0.705	0.784	0.081	0.717	0.732	0.380	0.006	0.000	0.866	0.827	3.79
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**4.37**
Ψ^2_{MdGS}	0.333	0.131	0.671	0.803	0.240	0.764	0.563	0.485	0.017	0.000	0.856	0.828	4.62
Ψ^3_{MdGS}	0.331	0.137	0.744	0.791	0.244	0.769	0.538	0.463	0.000	0.000	0.857	0.836	4.96
Ψ^4_{MdGS}	0.326	0.139	0.664	0.781	0.232	0.772	0.581	0.464	0.047	0.000	0.863	0.843	**5.29**
Ψ^5_{MdGS}	0.328	0.150	0.737	0.781	0.233	0.770	0.594	0.456	0.000	0.000	0.861	0.832	4.96
Ψ^6_{MdGS}	0.323	0.138	0.728	0.780	0.220	0.772	0.566	0.453	0.027	0.000	0.870	0.836	4.62
Ψ^7_{MdGS}	0.327	0.145	0.744	0.785	0.212	0.758	0.518	0.443	0.000	0.000	0.873	0.833	4.33
Ψ^8_{MdGS}	0.325	0.141	0.747	0.780	0.197	0.766	0.494	0.444	0.023	0.000	0.874	0.825	4.04
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**3.17**

Table 8: MCC and mean rank for space division into 9 regions of competence

	bio	bup	cry	dat	hab	ion	met	pop	sei	two	wdb	wis	Rank
Ψ^2_{wMeGS}	0.332	0.135	0.732	0.798	0.234	0.795	0.501	0.470	0.018	0.000	0.860	0.819	5.12
Ψ^3_{wMeGS}	0.334	0.120	0.740	0.795	0.233	0.803	0.523	0.454	0.021	0.000	0.863	0.829	**5.71**
Ψ^4_{wMeGS}	0.312	0.111	0.708	0.789	0.183	0.793	0.591	0.447	0.053	0.000	0.866	0.829	4.37
Ψ^5_{wMeGS}	0.330	0.098	0.727	0.786	0.129	0.781	0.622	0.446	0.040	0.000	0.867	0.830	4.54
Ψ^6_{wMeGS}	0.324	0.105	0.738	0.784	0.102	0.771	0.647	0.434	0.031	0.000	0.870	0.831	4.71
Ψ^7_{wMeGS}	0.315	0.107	0.755	0.782	0.072	0.763	0.682	0.408	0.013	0.000	0.870	0.829	3.96
Ψ^8_{wMeGS}	0.313	0.113	0.751	0.781	0.067	0.756	0.693	0.420	0.006	0.000	0.867	0.823	3.54
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**4.04**
Ψ^2_{MdGS}	0.333	0.140	0.700	0.798	0.236	0.765	0.475	0.470	0.017	0.000	0.854	0.827	4.54
Ψ^3_{MdGS}	0.330	0.165	0.761	0.787	0.252	0.769	0.462	0.446	0.000	0.000	0.857	0.836	**4.96**
Ψ^4_{MdGS}	0.326	0.140	0.664	0.779	0.217	0.772	0.507	0.456	0.047	0.000	0.861	0.844	4.79
Ψ^5_{MdGS}	0.328	0.152	0.737	0.779	0.231	0.771	0.542	0.453	0.000	0.000	0.864	0.834	4.87
Ψ^6_{MdGS}	0.323	0.139	0.725	0.777	0.227	0.771	0.538	0.457	0.027	0.000	0.868	0.837	4.71
Ψ^7_{MdGS}	0.327	0.151	0.744	0.783	0.201	0.757	0.506	0.449	0.000	0.000	0.871	0.834	4.50
Ψ^8_{MdGS}	0.325	0.147	0.732	0.779	0.190	0.764	0.490	0.446	0.023	0.000	0.873	0.825	3.79
Ψ_{MV}	0.319	0.109	0.810	0.779	0.208	0.630	0.458	0.435	0.000	0.000	0.867	0.894	**3.83**

geometric space Ψ_{MdGS} is significantly better than the reference classifier Ψ_{MV}. This situation occurs with the appropriate division of the feature space into regions of competence and selecting about half of the base classifiers. On the other hand the method Ψ_{wMeGS} can be statistically worse than the reference method Ψ_{MV}.

6 Conclusion

In this article the algorithm of classifier integration in geometric space was proposed. It means, that we used decision boundaries directly in the process of integrating base classifiers – instead of using the class labels or predicted probabilities in integration process of base classifiers as in the methods known so far.

Table 9: p-values obtained in Holm post-hoc test

	Ψ_{wMeGS}		Ψ_{MdGS}	
m	ACC	MCC	ACC	MCC
3	0.20	0.13	0.36	0.26
5	**0.07**	0.13	0.18	**0.03**
7	0.24	0.21	0.34	**0.03**
9	0.28	0.10	0.21	0.26

Table 10: Comparison pairs of classifier using p-values obtained in Bonferroni–Holm post-hoc test

measure	MCC	MCC	measure	ACC
no of regions	5	7	no of regions	5
Ψ_{MV} vs Ψ^2_{MdGS}	0.17	0.14	Ψ_{MV} vs Ψ^2_{wMeGS}	0.36
Ψ_{MV} vs Ψ^3_{MdGS}	0.10	0.07	Ψ_{MV} vs Ψ^3_{wMeGS}	0.93
Ψ_{MV} vs Ψ^4_{MdGS}	0.12	**0.03**	Ψ_{MV} vs Ψ^4_{wMeGS}	0.21
Ψ_{MV} vs Ψ^5_{MdGS}	**0.03**	**0.07**	Ψ_{MV} vs Ψ^5_{wMeGS}	**0.07**
Ψ_{MV} vs Ψ^5_{MdGS}	0.23	0.14	Ψ_{MV} vs Ψ^6_{wMeGS}	0.13
Ψ_{MV} vs Ψ^7_{MdGS}	0.32	0.24	Ψ_{MV} vs Ψ^7_{wMeGS}	0.62
Ψ_{MV} vs Ψ^8_{MdGS}	0.65	0.38	Ψ_{MV} vs Ψ^8_{wMeGS}	0.45

The weighted mean or median method was used in order to get one decision boundary, which is the final stage of building EoC. In addition, the proposed method uses the selection of base classifiers in the defined regions of competence. The selection phase can be used to assure an odd number of base classifiers although it is not required, what makes the proposed method more general than majority voting, which for the two-class data set requires an odd number of base classifiers.

Twelve open-source benchmarking data sets were used in the experimental part to perform the statistical analysis of results concerning two classification measures. Bonferroni–Holm test showed, that in two cases the proposed algorithm provided statistically significant better results than the results obtained using the reference classifier. These cases concern in particular the use of the median method to obtain a decision boundary of EoC and the use of about half of the base classifiers from all base classifiers. The obtained results clearly indicate that in the case of the proposed method of combining decision limits in the geometrical space, better results are obtained when a median is used rather than an weighed average to determine the final EoC decision boundary.

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