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Identifying Fuzzy Controllers Parameters by Fuzzy Clustering Technique

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Abstract: In fuzzy control, there is a large amount of parameters involved in the system design. Due to their interdependency, these parameters are sometimes conflicting causing an unavoidable trade-off among performance indices. It is difficult to discern the best combination of fuzzy parameters with respect to a given range of some performance indices. In this case, a clustering technique represents a powerful tool to deal with the problem. Main clusters of fuzzy controllers having similar behavior with respect to some performance indices are discovered. In order to precisely characterize rule bases and transform them to a quantifiable entity, transition between topological and numerical form of fuzzy rule bases is studied. Formulating a vector space structure and a base of relationships between fuzzy sets represents one of the main foci of the research. Adding logic parameters and defuzzification procedures to the formulated vectors is required to apply the clustering technique. In fact, this latter requires the existence of quantifiable fuzzy controllers. The obtained vectors are then treated by a fuzzy-neural clustering algorithm. Membership nuance to a cluster allows better legibility to evaluate relevance and relative interest of fuzzy controller parameters according to performance indices.

Keywords: Fuzzy logic, Fuzzy Logic Controllers, Vector Space, Classes of Fuzzy Controllers, Clustering and Learning
Categories: I.2, I.5

1 Introduction

Mamdani's Fuzzy Logic Controllers (FLCs) [Mamdani, 74] consisting of a collection of linguistic fuzzy rules, are the most common fuzzy rule based systems (FRBSs). The structure analysis of these systems is an active research field in the area of fuzzy control theory. FLCs can be specified by three families of parameters: "K" the knowledge base parameters, "L" the logical parameters and "D" the defuzzification procedures. The "K" family includes fuzzy rules, membership functions (MFs), fuzzy partition, shape of the membership functions and some parameters related to the size of the fuzzy system: number of membership functions, number of rules and number of condition part in a rule. The "L" family includes the fuzzy logic operators applied for AND, OR and implication. The "D" family includes aggregation operations and defuzzification methods. An optimal control depends on a combination and a judicious choice of these parameters according to the specified performance indices (overshoot%, response time, rise time, etc.). The carelessness of the interdependence between parameters leads to a temporary and an instable choice [Chenaina and Chouigui, 00]. In fact, an eventual performance evolution facing designers, leads to the control degradation. This latter is a consequence of reasoning non-conformity ("L" family) with knowledge ("K" family). In the fuzzy control framework, the parameters, adjustment is a critical point. A fundamental question faced by designers concerns the right choice of fuzzy parameters class.

By analyzing the literature of the fuzzy modeling, it seems that the choice of these parameters is based on FRBSs tuning component [Alcalá et al., 07a] and/or fixed by experimentation on real applications [Chavez et al., 12], [Gacto et al., 12]. Most of the works that are characterized by their ability to self-learn their structures, have solved part of the problem. However, most of them focus on non-transparent optimizations of parameters by using neural networks or genetic algorithms. The optimal adjustment of fuzzy partition by learning techniques characterizes this tendency. This partly explains the evolution of the fuzzy control to the "all numerical" evolution discrepant with the main motivation of the fuzzy sets theory's development: the visibility.

Works on artificial neural networks has contributed significantly to the field of knowledge engineering. The knowledge, however, is represented at a sub-symbolic level in terms of connections and weights. Neural networks act like black boxes providing little insight into how decisions are made. They have no explicit, declarative knowledge structure that allows the representation and generation of explanation structures. Thus, knowledge captured by neural networks is not transparent to users and cannot be verified by domain experts. To solve this problem, researchers are interested in developing a humanly understandable representation for neural networks. Multi-objective constrained optimization models in which criteria such as accuracy, transparency and compactness have been taken into account are proposed [Gacto et al., 12] [Perez et al., 13].

This paper attempts to deal with these issues by using a clustering technique of Mamdani's FLCs parameters. An alternative identification method of fuzzy parameters is proposed, in order to first allow relating fuzzy parameters to performance indices and then placing them in the nearest cluster. This synthesis provides FLCs' designers with more efficient and transparent means to assess the relevance and the relative interest of parameters with regards to some performance indices. The aim of clustering is to find the best setting for FLCs' parameters and not only to fine independently specific parameters.

This paper is organized as follows: Section 2 presents different approaches of Fuzzy Systems Modeling; their advantages and disadvantages are highlighted. Section 3 focuses on the passage from topological form of a fuzzy rule base (RB) to its numerical form, in order to carry out accurate identification rule bases, able to characterize different FLCs. From a computing point of view, this passage relates symbolic and numerical data. The specificity of our approach is the development of a vector space structure and relations base between fuzzy sets needed for quantifying fuzzy partition. We put in relation trapezoidal fuzzy sets and Allen's intervals [Allen and Koomen, 83]. Then realizable relations between fuzzy sets are determined and a vector space base of relations is constructed. Finally, parameters characterizing a fuzzy partition for a given rule base are determined. The trapezoidal model as shown in Fig. 1, presents the advantage of an easy adjustment of the membership functions in a computerized treatment of data. Furthermore, the choice of this model is justified by the fact that cores and supports are identified as intervals. They fall within Allen's interval algebra. In Section 4 we proceed to a classification of these vectors (FLCs) according to some performance indices. A clustering algorithm of FLCs is proposed, its principle relies on a non-supervised learning method of a neural network [Simpson, 93]. This latter considers the activation function of a neuron taking into account the membership degree calculation of a parameters' combination to classify in the cluster. Section 5 presents the experimental results and describes their significance. In section 6 soundness and justification of the work are discussed. Finally Section 7 summarizes the paper.

2 Fuzzy Systems Modeling

The basic objective of fuzzy systems modeling is to identify the parameters of a fuzzy inference system in order to reach a desired behavior. Fuzzy control systems modeling involve at least two basic parts: parameters identification and structure identification. This latter is related to; the variables' identification, the determination of MFs' number for each variable and the determination of discourse universes. The following are some approaches of fuzzy systems modeling:

1. *Fuzzy modeling* based knowledge engineering is inspired by the knowledge engineering methods used in expert systems. The first based knowledge approach, proposed by Zadeh [Zadeh, 73], tries to build a fuzzy model directly from the expert knowledge. However, there is no general methodology for the implementation of this approach, which involves heuristic knowledge and intuition. The magnitude of the problem space has motivated the use of automatic approaches to fuzzy modeling.
2. *Approaches based on classic identification algorithms* [Schiavo and Luciano, 01] deal with an iterative estimation of MFs, which are applied to a pre-defined model structure in order to approximate an expected behavior. In some fuzzy modeling techniques, the pre-defined parameters do not guarantee that a desired behavior can be reached.
3. *In constructive learning approaches* [Rojas et al., 00], a priori expert knowledge is used to guide the search process instead of being used to directly construct the fuzzy system. After an expert-guided definition of logic parameters, relevant variables and universes of discourse, a sequence of learning algorithms was progressively applied to construct an adequate final fuzzy model.
4. The *hybridization of fuzzy systems* with genetic algorithm and neural networks known as *Genetic Fuzzy Systems* (GFSs) and *Neural Fuzzy Systems* (NFSs) are applied to improve the automatic design of Fuzzy Logic Systems (FLSs). A recent example of GFS is presented by Chavez et al. [Chavez et al., 12] to improve laser spot system detection by means of MFs' tuning. In NFSs approach the Fuzzy-rule extraction technique extracts from the knowledge embedded in trained neural networks a set of fuzzy rules [Duch et al., 01]. The advantage of this kind of representation is that such hybrid systems can be optimized via powerful, well-known neural-network learning algorithms. The main disadvantage of this technique is that

the access to the knowledge requires a previous rule-extraction phase and that they are intended to maximize accuracy, ignoring human interpretability.

5. *Multi-objective Evolutionary Algorithms* (MOEAs) which appeared only in the last decade, are used to search on large and complex search spaces [Coello et al., 07]. Fuzzy modeling can be considered as an optimization process where the parameters of a fuzzy system constitute the search space. Works investigating the application of MOEAs have been divided into two subcategories: MFs tuning and inference parameters tuning [Alcalá et al., 07a]. Recent example of MOEAs is presented by Gacto et al. [Gacto et al., 12] to improve the performance of Heating, Ventilating and Air Conditioning System.

The main objective of FLCs theory is to obtain fuzzy models with good interpretability. The interpretability of fuzzy control systems depends on several parameters; especially the fuzzy partition, the number of input variables, the number of rules, the number of condition part in a rule, etc. Some works have attempted to define objective criteria that facilitate the automatic modeling of interpretable fuzzy systems. Alcalá et al. [Alcalá et al., 11] conclude on the importance of completely determining appropriate granularities (number of fuzzy sets) and fuzzy partition. Gacto et al. [Gacto et al., 11] has analyzed and classified the universalities of interpretability measures. To carry out the trade-off between interpretability and accuracy, Gacto et al. have proposed a taxonomy with four levels:

i. Complexity at the rule base level
ii. Complexity at the level of fuzzy partitions
iii. Semantics at the rule base level
iv. Semantics at the level of fuzzy partitions

They conclude that there are well-known measures to quantify complexity such as the number of rules, the number of condition part in a rule, etc. However, well-established definitions for interpretability of fuzzy systems at the level of rule base or fuzzy partitions can't be defined. Indeed, the interpretability that expresses the behavior of the real system in an understandable way remains a subjective property depending on the designer's requirements.

The optimal adjustment of fuzzy partition by learning techniques characterizes several works of fuzzy modeling. This justifies our interest to provide an accurate measure to evaluate the fuzzy partition.

3 Vector Space of Relationships between Fuzzy Sets

Tuning approaches use symbolic translation of a fuzzy set [Alcalá et al., 07a], a lateral displacement and the amplitude variation of the fuzzy set support [Alcalá et al, 07b]. In fuzzy interpolation, many works [Chen and Ko, 08], [Yang and Shen, 11] perform geometric manipulation to define the representative value of a trapezoidal fuzzy set. They apply geometric operations on MF's supports and cores to capture the overall location of the fuzzy set. In [Yang and Shen, 11] the representative value of fuzzy set is defined by:

$$\mathrm{R}ep\,(A) = w_0\frac{a+d}{2} + w_1\frac{b+c}{2} \tag{1}$$

Where "a", "b", "c" and "d" represent the parameters of the trapezoidal fuzzy set A (Fig. 1), w_0 and w_1 are the weights of the support and the core of fuzzy set A. [Chen and Ko, 08] and [Yang and Shen, 11] use respectively (2) and (3):

$$\mathrm{R}ep(A) = \frac{a+b+c+d}{4} \tag{2}$$

$$\mathrm{R}ep(A) = \frac{b+c}{2} \tag{3}$$

Looking on these propositions it can be concluded that the representative value of a trapezoidal fuzzy set depends on the relationship between supports and cores. In the present contribution, in order to define the representative value of fuzzy sets distributions associated with linguistic variables we will consider a study of relationships that can exist between cores and supports of fuzzy sets.

3.1 Representation of Fuzzy Sets by Intervals

The trapezoidal model as shown in Fig. 1, presents the advantage of an easy adjustment of the membership functions in a computerized treatment of data.

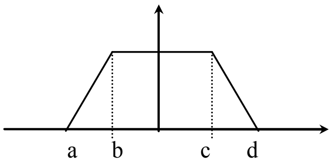


Figure 1: A fuzzy set T represented by four parameters. T = (a, b, c, d) $\in [-1,1]^4$

The study of the relative position between fuzzy sets requires relationships between four dimensions space. The representation by intervals presents two advantages. First, it allows working in two dimensions space. Second, it permits the unification of fuzzy sets relationships with Allen's interval algebra [Allen and Koomen, 83]. Temporal relationships between two time intervals can be expressed by one of the 13 relations [Allen and Koomen, 83] as shown in Table 1.

Note 1.

1) B is the set of the 13 relations as shown in Table 1.

2) 2^B is the set of composed relations; "°" is the composition relation. Table 4 shows the composition of some relationships.

3) The addition of two sets S_1 and S_2 is defined by: $S_1+S_2 = (S_1-S_2) \cup (S_2-S_1) = (S_1 \cup S_2) - (S_1 \cap S_2)$. The addition operation coincides with the exclusive disjunction.

(2^B,+, °) is an algebra over the Boolean body [Allen and Koomen, 83]. The set of the 13 relations (Table l) forms a base of elementary binary relations [Allen and

Koomen, 83]. Let E be a vector space of B. Taking into account this result, the purpose of the following sections is to build a vector space $\mathbb{E} \subseteq E \times E$ of relations between fuzzy sets.

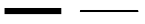
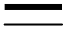
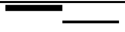
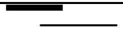
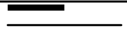
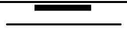
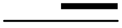
Symbolic Constraints	*Reverse Constraints*	*Time*
b:I before J	*b' : J after I*	
e:I equals J	*e : J equals I*	
m:I meets J	*m' : J met-by I*	
o:I overlaps J	*o' : J overlapped-by I*	
s:I starts J	*s': J started-by I*	
d: I during J	*d': J contains I*	
f:I finishes J	*f' : J finished-by I*	

Table 1: Relationships between time intervals

Definition 1. The set $\mathbb{T}$ of fuzzy sets is defined by:

$$\mathbb{T} = \{(I_1, I_0) \subseteq I \times I, / I_1 \subseteq I_0\}$$

I is the discourse universes, I_1 and I_0 are respectively the core and the support of the fuzzy set $T \in \mathbb{T}$. if T_1 and $T_2 \in \mathbb{T}$, it can be written:

$T_1 = (I_1, I_0) / I_1 = [B_1, C_1]$ and $I_0 = [A_1, D_1] / I_1 \subseteq I_0$ (i.e.) $A_1 \leq B_1 \leq C_1 \leq D_1$.

$T_2 = (J_1, J_0) / J_1 = [B_2, C_2]$ and $J_0 = [A_2, D_2] / J_1 \subseteq J_0$ (i.e.) $A_2 \leq B_2 \leq C_2 \leq D_2$.

A fuzzy set can be considered as a couple of two intervals: support and core. Let $T_1 = (I_1, I_0)$ and $T_2 = (J_1, J_0)$ are two elements of $\mathbb{T}$. The idea is to combine the relative positions of supports and cores, in order to determine the realizable relations between T_1 and T_2. In fact, a relation is designed by a couple (x, y) of BxB, where x and y represent respectively the relation between cores (I_1, J_1) and supports (I_0, J_0).

Example 1.

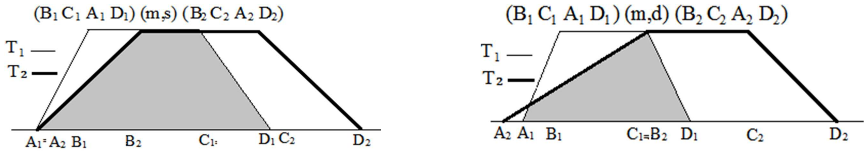

(B₁ C₁ A₁ D₁) (m,s) (B₂ C₂ A₂ D₂)
T₁
T₂
A₁= A₂ B₁
B₂
C₁=
D₁ C₂
D₂
(B₁ C₁ A₁ D₁) (m,d) (B₂ C₂ A₂ D₂)
T₁
T₂
A₂ A₁ B₁
C₁=B₂ D₁
C₂
D₂

Figure 2: The relationships (m ,s) and (m ,d)

T_1 (m, s) T_2 and T_1 (m, d) T_2 produce two different fuzzy sets. In the following, a formal approach to reduce the large number of relations between fuzzy sets is presented.

Theorem 1. The Boolean value set {0, 1} with addition and multiplication, defined respectively by Table 2 and Table 3 has a body structure.

+	0	1
0	0	1
1	1	0

Table 2: Exclusive Disjunction

•	0	1
0	0	0
1	0	1

Table 3: Logic Conjunction

Lemma 1. Let (E, +) be a commutative group with an additive law "+", "0_E" is its neutral element. (E, +) is a vector space over ({0, 1}, +, •), if and only if: $\forall x \in E$, $x + x = 0_E$.

Note 2. The proof of theorem1 is evident. The body ({0, 1}, +, •) defines the Boolean coefficients of linear combinations of relationships between fuzzy sets. Thus, "1» represent the existence of relationship, "0" the non-existence. The lemma will be useful for the calculus simplification.

3.2 Realizable Relationship

In this section, we seek to find realizable relations. In fact, some irrelevant couples can be formed from the set B ={e, b, m, o, d, s, f, b', m' o', d', s', f'}, such as the relation (d, b) which is not realizable on **T**.

Definition 2. $(x, y) \in B\times B$ is realizable on **T** is equivalent to $\forall\ T_1 \in$ *T*, if $T_1(x, y)\ T_2$ then $T_2 \in$ **T**.

Theorem 2. The relationships (x, x) and (x, d) are realizable for any $x \in B$.

Proof. a) Let's show $\forall\ x \in B$, the elementary relationship (x, d) is realizable between two elements of **T**. Let's suppose that (I_0 d J_0) (the support of T_1 is during the support of T_2). Let's show any relationship among the 13 base primitives may relate the cores I_1 and J_1, as T_1 and $T_2 \in$ **T**. Now let us consider that:

$A_1 \le B_1 \le C_1 \le D_1 \Leftrightarrow I_1\ (s+d+f+e)\ I_0$
$A_2 \le B_2 \le C_2 \le D_2 \Leftrightarrow J_0\ (s'+d'+f'+e)\ J_1$

we have**:** $\{I_1(s+d+f+e)I_0$ and $I_0 d J_0$ and $J_0\ (s'+d'+f'+e)\ J_1\} = \{I_1((s+d+f+e)°d°(s'+d'+f'+e))J_1\}=((s+d+f+e)°d)°(s'+d'+f'+e)=((s°d)+(d°d)+(f°d)+(e°d))°(s'+d'+f'+e) = (d+d+d+d)°(s'+d'+f'+e) = \phi°(s'+d'+f'+e)$.

Refer to table 4 and Lemma 1. ($\forall x\in B$, $x+x = \phi$, ϕ is the neutral element of +) = $(\phi°s')+(\phi°d')+(\phi°f')+(\phi°e) = \phi$.

This leads to the conclusion regarding the feasibility of the relationship $\forall x \in B$.

q°r	**B**	**d**	**m**	**o**	**s**	**f**	**e**
s	B	d	b	bmo	s	d	s
f	B	d	m	osd	d	f	f
d	B	d	b	bmosd	d	d	d
e	B	d	m	o	s	f	e
sfde	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	sfde

q°r	**b’**	**m’**	**o’**	**s’**	**f’**	**d’**
s	b’	m’	fdo’	ses’	bmo	bmof’d’
f	b’	b’	b’m’o’	b’m’o’	fef’	b’m’o’s’d’
d	b’	b’	fdb’m’o’	fdb’m’o’	bmosd	ϕ
e	b’	m’	o’	s’	f’	d’
sfde	ϕ	ϕ	ϕ	fdse	sdfe	bmob’m’o’f’s’d’

Table 4: The composition law "°" is a deduction law. Let I, J and K be three intervals such that " I b J" (I before J) and " J o K" (J overlaps K), the only thing that can be deduced is (J before K) or (J meets K) or (J overlaps K), i.e. (J bmo K) or (J b+m+o K), where + is the exclusive disjunction. The relationship ϕ expresses an indeterminacy; if "I d J "and "J d’ K" then we can deduce nothing about the relative position of J and K, d ° d’ = ϕ.

b) Let's show ∀x∈B, the couple of elementary relations (x, x) is realizable between two elements of T. According to (a), the relation (d, d) is realizable. It remains to prove the feasibility of (x, x), ∀x∈ B - {d}.

Consider that {I_1(s+d+f+e) I_0 and I_0 x J_0 and J_0 (s’+d’+f’+e) J_1} = {I_1 ((s+d+f+e)°x°(s’+d’+f’+e)) J_1}. We need to calculate ((f+d+s+e)°x°(f’+d’+s’+e) ∀ x ∈ B-{d}, for x ∈ {b,m,o,s,f,b’,m’,o’}:((s+f+d+e)°x)°(s’+f’+d’+e) = ϕ°(s’+f’+d’+e) = ϕ.

Refer to table 4 so (I_1xJ_1) is realizable, then ∀ x ∈{b,m,o,s,f,b’,m’,o’}, (x, x) is realizable.

For x = e: ((s+f+d+e)°e°(s’+f’+d’+e)) = ϕ
for x=s': ((f+d+s+e)°s’°(f’+d’+s’+e)) = ϕ
for x = f': ((f+d+s+e)°f’°(f’+d’+s’+e)) = ϕ
for x = d': ((f+d+s+e)°d’°(f’+d’+s’+e)) = ϕ

Hence (e, e), (s', s'), (f', f'), (d’, d’) are realizable.

Theorem 3. The set B = {(x, x), (x, d) / x ∈ B} is a free system of the product vector space E×E over the Boolean body ({0, 1}, +, •). (The proof is provided in the Appendix A).

Note 3.

1) Any relationship can be written as a linear combination with Boolean coefficients of elementary realizable relationships.

2) Let "r"$\in 2^B$, length(r) is the cardinal of the subset "r".

Theorem 4.

a) The vector space $\mathbb{E}$ of realizable relationships between two fuzzy sets of *T* over the body ({0, 1}, +, •) is equal to the following set: $\mathbb{R} = \{(r_0, r_1)\in E\times E$ / length (r_0) + length (r_1) is even$\}$.

b) The set $\mathbb{B}$ = {(x, x); (x, d) / x∈B} is a base of the realizable elementary relationships of the vector space $\mathbb{E}$. (The proof is provided in the Appendix A).

Example 2.

i) Length (dmo + efd) = length (d+m+o+e+f+d) = length (moef) = 4. Consequently length (dmo) + length (efd) - 2 × number of simplifications = 3 + 3 - 2×1 = 4.

ii) Length (mbo + m'b'bo) = length (mm'b') = 3. Length (mbo) + length (m'b'bo) - 2 × number of simplifications = 3 + 4 - 2 × 2 = 3.

Showing that $\mathbb{E} \subseteq \mathbb{R} = \{(r_0, r_1) \in E\times E$ / length (r_0) + length (r_1) is even$\}$. Let (x,y) $\in \mathbb{E} \subseteq E\times E$

1st case: if (x, y) is an elementary relation i.e. (x, y) ∈ B×B then length(x) + length(y) =1+1=2 is even then (x, y) ∈ $\mathbb{R}$. Therefore, $\mathbb{E} \subseteq \mathbb{R}$.

2nd case: if (x, y) is not an elementary relation then the relation (x, y) is written as a disjunction of realizable elementary relations. Therefore, length (x) + length (y) = (2 x number of terms of the sum) – (2 x number of simplifications). This number is even, thus (x, y)∈ $\mathbb{R}$, it can be concluded that $\mathbb{E} \subseteq \mathbb{R}$.

Note 4.

∀x ∈ B, the relations (x,ϕ) and (ϕ,x) $\notin \mathbb{E}$ since their length = 1.

Example 3.

i) The relationship (mdo, m) is written as a linear combination with Boolean coefficients of elementary realizable relations: (mdo, m) = (m, m) + (d, d) + (0, d), so it belongs to $\mathbb{E}$. On the other hand, length (mdo) + length (m) = 3 + 1= 4 = 2x3 - 2x1 is even. Consequently, the relation (mdo, m) ∈ $\mathbb{R}$.

ii) Let (bb ', d) = (b, d) + (b', d) + (ϕ, d). Since (ϕ, d) is not an elementary relation, (bb', d) $\notin \mathbb{E}$. On the other hand, length (bb') + length (d) = 2+1 = 3 is odd.

Note 5. The relation (x, y) of E, is written in the base $\mathbb{B}$ as: (x, y) = (x, d) + (y, d) + (y, y)

3.3 Evaluation of Fuzzy Partition over the Universe of Discourse

The fuzzy sets partition is based on the realizable relationships study. The base $\mathbb{B}$ allows to restrict the study to the following 25 relationships:

Every product relationship between sets is written as a disjunction of these relationships. The relationships analysis allowed to retain two main proprieties: the overlapping and the spacing proprieties. Therefore, it is possible to define numerical parameters able to characterize a given fuzzy partition.

(d, d)	(0, d)	(m, d)	(e, d)	(b, d)
(0', d)	(m', d)	(b', d)	(f, d)	(s, d)
(e, e)	(b, b)	(f', d)	(s', d)	(d', d)
(b', b')	(f, f)	(s, s)	(0,0)	(m, m)
(f', f')	(s', s ')	(d', d')	(0', 0')	(m', m')

Table 5: The relationships base

Definition 3. The function $\cap_{(x,y)}$ associates with each relation (x, y) of $\mathbb{E}$, a fuzzy set $T_{ij} \in \boldsymbol{T}$

$\cap_{(x,y)}: \mathbb{E} \rightarrow \boldsymbol{T}$

$(x, y) \rightarrow \cap_{(x,y)} = T_{ij} = T_i \cap T_j$

$\forall (x,y) \in \mathbb{E}, (x, y) = (x, d) + (y, d) + (y, y)$

$\forall T_i, T_j \in \boldsymbol{T} / T_i(x, y)T_j$, we have:

$T_i (x, y) T_j = T_i ((x, d) + (y, d) + (y, y)) T_j = (T_i (x, d) T_j) + (T_i (y, d) T_j) + (T_i (y, y) T_j) =$

$\cap_{(x,y)} \Delta \cap_{(y,d)} \Delta \cap_{(y,y)}$

(Δ is the symmetric difference between two sets)

Example 4.

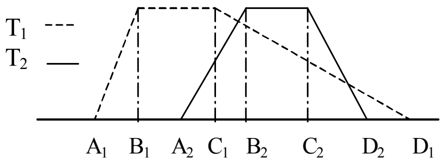


Figure 3: The relation (b, d')

$(b, d') = (b,d) + (d',d) + (d',d') \cap_{(b,d')} = \cap_{(b,d)} \Delta \cap_{(d',d)} \Delta \cap_{(d',d')} =$
$\{A_1, A_1B_1 \cap A_2B_2, C_1D_1 \cap A_2B_2, D_1\} \Delta \{A_1, A_1B_1 \cap A_2B_2, B_2, C_2, C1D1 \cap C_2D_2, D_1\} \Delta \{A_2, B_2, C_2, D_2\} = \{A_2, C_1D_1 \cap A_2B_2, C_1D_1 \cap C_2D_2, D_2\}$

Note 6. The point $(A_1B_1 \cap A_2B_2)$ which is not defined for the case (b, d'), is simplified in calculation.

3.4 Overlapping Degree

The overlapping (o_{ij}) between two fuzzy sets T_i and T_j is calculated by the following formula:

$$O_{ij} = 2 \times AO_{ij} / (A_i + A_j) \qquad (4)$$

where: AO_{ij} is the area of $T_i \cap T_j$ and $(A_i + A_j)$ is the area sum of T_i and T_j.

Let us note that $O_{ij} \in [0,1]$, if $T_i = T_j$ then $O_{ij} = 1$. The overlapping degree (O_{LV}) of a fuzzy partition associated with a linguistic variable is defined by:

$$o_{LV} = \frac{\sum_{i \neq j} o_{ij}}{C_n^2} \tag{5}$$

where "n" is the number of fuzzy sets associated with the linguistic variable. The overlapping degree of a rule can be defined by the same formula where "n" is the number of premises and conclusion parts of the fuzzy sets. The rule base overlapping can be defined by an arithmetic average on the overlapping degrees of each rule.

Example 5. Let the rule base:
if X is A_1 and Y is B_1 then Z is C_1
if X is A_2 and Y is B_2 then Z is C_2
if X is A_3 and Y is B_3 then Z is C_3

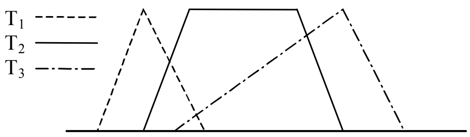


Figure 4: The fuzzy sets partition of a linguistic variable

$O_{12} = O_{21} = 2\ A_{12} / (A_1 + A_2)$
$O_{13} = O_{31} = 2\ A_{13} / (A_1 + A_3)$
$O_{23} = O_{32} = 2\ A_{23} / (A2 + A_3)$
$O_X = (O_{12} + O_{13} + O_{23}) / 3$

3.5 Spacing measure

An evaluation of the spacing S_{ij} of two fuzzy sets T_i and T_j describing the distance between T_i and T_j is carried out as shown in Fig. 5.

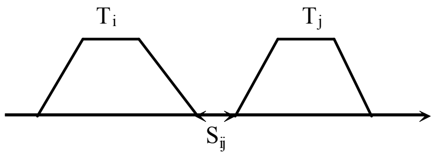


Figure 5: Spacing between two fuzzy sets

S_{ij}= inf(J_0) - sup(I_0) if ($T_i \cap T_j$)=∅ and (I_0 b J_0) else S_{ij}= 0, where I_0 and I_1 are respectively supports of T_i and T_j

The spacing between two fuzzy sets of a linguistic variable or a rule is calculated by the following formula:

$$S = \frac{\sum S_{ij}}{n-1} \tag{6}$$

S_{ij} is the distance between T_i and T_j. "n" is the fuzzy sets' number of the linguistic variable. The rule base spacing can be calculated by an arithmetic average on the spacing degrees of each rule.

Distance is obvious when a membership function is interpreted in terms of similarity. This has been very often done in clustering [Precup et al., 13] and more generally in many applications of fuzzy sets for the definition of fuzzy numbers [Rao and Shankarb, 12]. The most widely used distances for fuzzy sets are the Euclidean distance and the Hamming distance.

Furthermore, to impose constraints on fuzzy sets at the fuzzy partition level, other many works use a distance between the centers of fuzzy sets. Gacto et al. affirm that when the value of the distance is smaller, the number of acceptable fuzzy sets per domain will increase, increasing the number of rules and also increasing the complexity of the model. On the other hand, as the value of the distance increases the number of fuzzy sets per domain decreases, reducing the number of rules [Gacto et al., 11].

In this work, the base of relationships between fuzzy sets as a result of the previous analysis leads to two characteristics; the overlapping degree (4) and the distance (6) between the closest extremities of fuzzy sets' supports. The modeling process of an accurate FLC could lead to complex fuzzy partitions, which could make the interpretability of the system by a designer difficult. This latter can vary in the overlap and the distance between fuzzy sets respecting the fuzzy partition semantics by preserving some properties as: Completeness, Normalization, Distinguishability and Complementarity [Gacto et al., 11].

4 Fuzzy Control Systems Clustering

As a result of the previous analysis, the rule base can be considered as vector (7) with coordinates the degree of overlapping (ov) and of spacing (sp). Other coordinates indicating the number of rules (nr), number of linguistic variables (nv), sets associated with each variable (sv) and two Booleans reflecting the partition's symmetry (sy) and equidistance (eq) can be added.

$$(ov, sp, nr, nv, sv\ sy, eq)^T \tag{7}$$

Assuming that it is possible to classify a given FLC as having a characteristic behavior model, then it can be associated with the corresponding vector (7). In other words it is an association of a particular configuration of fuzzy parameters. The obtained numerical model of a rule base provides a realistic framework that is able to classify FLCs according to performance indices.

The presented study in this paper is based on the "Fuzzy min-max neural network clustering" method [Simpson, 93]. It clearly illustrates the association between neural networks and the fuzzy sets. The neural network compares a sample of inputs with a set of examples. If neurons represent separated categories (clusters) then, the more the input introduced inside the cluster, the more the output value of this neuron is raised. Without relearning, the used method allows parallel processing to provide new individuals classification, again. The cluster's sizes are dynamically determined during the learning process. This method does not need a similarity measure between individuals. Such measure strongly affecting results is hardly evaluated in the FLCs' case. In order to achieve certain flexibility between clusters' barriers, fuzzy clustering technique is used. It is interesting to know the FLC belonging degree to one cluster. In other words, we can determine parameters combination which responds to performance indices better than others.

4.1 The Membership Function

The main objective of using fuzzy logic in the clustering algorithm is to refine the membership of fuzzy systems (FLCs) to different clusters. The neural network is a powerful framework for aggregation with efficiency and speed of calculation. The neural network compares an input with a sample of fuzzy systems (FLCs) stored in the memory. The activation function of a neuron calculates the membership degree of a fuzzy system to the cluster associated with that neuron. If neurons represent clusters, then the more the fuzzy system is introduced into the cluster, the greater the value of the output corresponding to this neuron is high. Each neuron (cluster) is considered as a fuzzy set and its activation function is identified with the membership function of this fuzzy set.

"Fuzzy min-max neural networks clustering" method provides hypercubes clusters form (($[0,1]^n \subseteq \Re^n$). In this work, two performance indices are used to regroup FLCs: overshoot percentage (″ov %″) and response time (″R_T″). Hence, a square will be used instead of an hypercube (Fig. 6). The fuzzy sets are completely defined by the "min" and "max" points. Therefore, it is possible to describe the degree with which a FLC belongs to a cluster (square) or to another.

The aim of the clustering is to regroup fuzzy systems "s_j" according to a set of performance indices (c = 1 for "ov %", c=2 for "R_T", ...), and to interpret these clusters with a synthesis of results. Each index corresponds to a dimension of space. Since it is difficult to visualize these systems in spaces greater than two dimensions, we seek to represent these systems in subspaces (planes) so that the representation is simple and easily interpretable. It should be noted that when the clustering is performed on large spaces, the graph can be obtained by projections on different planes that constitute the subspace.

Let s_j be the j^{th} FLC of the Σ set (j = 1...n), n is the number of FLCs. $s_j = (s_{j1}, s_{j2}) \in \Re^2$, s_{jc} is the c^{th} component of the j^{th} FLC, on each dimension (c =1, 2). Let i be the i^{th} square definite by (8).

$$C_i = \{m_i, M_i, \mu_i(s_j, m_i, M_i)\} \tag{8}$$

where $m_i = (m_{i1}, m_{i2}) \in \Re^2$, the minimum of C_i and $M_i = (M_{i1}, M_{i2}) \in \Re^2$, the maximum of C_i.

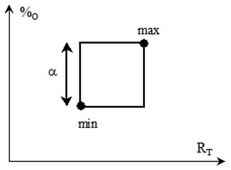


Figure 6: Cluster Structure

$$\mu_i\,(s_j, m_i, M_i) = 1/2 \sum_{c=1}^{2} [1 - f\,(s_{jc} - M_{ic}, \beta) - f\,(m_{ic} - s_{jc}, \beta)] \qquad (9)$$

where : $0 \le \mu_i\,(s_j, m_i, M_i) \le 1$

$$f(x, \beta) = \begin{cases} 1 & \text{if } x * \beta > 1 \\ x * \beta & \text{if } 0 <= x * \beta <= 1 \\ 0 & \text{if } x * \beta < 0 \end{cases}$$

Membership function μ_i (s_j, m_i, M_i) measures membership degree indicating the proximity between the FLC s_j and the cluster C_i formed by the couple of points min-max (m_i, M_i). This is considered as a measure of frontier distance of the square, indicating, how much each component is greater (less) than the value from the "max" ("min") point. The closer the FLC we get to square, the more the membership degree approaches 1. If the FLC is inside the square the membership degree equals 1.

β is a sensitivity parameter measuring the decrease rate of the membership function as far as a FLC "s_j" is separated from the cluster core. β regulates the speed with which membership function values decrease when a FLC "s_j" is separated from the cluster's core. When β is small, the fuzzy set becomes less contracted, however, when β is big, the fuzzy set becomes more contracted. The value of β (sensitivity parameter) and α (square size) are provided by the user at the beginning of the program, α changes during learning.

4.2 The Learning Process

Membership degree of the FLC to be classified in the corresponding cluster is calculated by the neuron activation function. Clusters containing the provided FLCs are constructed by the learning process. If a cluster cannot be found, that is, it does not verify the extension criteria, then new cluster will be formed and added to the system. Overlap between the formed clusters can be generated by extension. Membership degree of 1 implies that the FLC belongs to several clusters causing ambiguity. To avoid this problem (if it exists), a contraction procedure will be used.

- *Extension test*: Considering a FLC "s_j", in order to find the cluster C_i with the highest membership degree and allowing the extension, the following test should be verified:

$$(\max\,(M_{ic}, s_{jc}) - \min\,(m_{ic}, s_{jc})) \le \alpha \qquad (10)$$

$$\overset{new}{m_{ic}} \leftarrow \min \left(\overset{old}{m_{ic}} , s_{jc} \right) \; i = 1,2 \tag{11}$$

$$\overset{new}{M_{ic}} \leftarrow \max \left(\overset{old}{M_{ic}} , s_{jc} \right) \; i = 1,2 \tag{12}$$

If (10) is verified, the couple min-max is then adjusted by (11) and (12). If all squares are examined without any possibilities of extension then a new square should be built.

- *Overlapping test*: Suppose that the square C_i is to be extended. A comparison between C_i and previously found squares, noted C_k is to be led. The extension creates an overlapping between C_i and C_k if and only if one of the following four conditions is verified, for c = 1, 2:

  a. $m_{ic} < m_{kc} < M_{ic} < M_{kc}$
  b. $m_{kc} < m_{ic} < M_{kc} < M_{ic}$
  c. $m_{ic} < m_{kc} <= M_{kc} < M_{ic}$
  d. $m_{kc} < m_{ic} <= M_{ic} < M_{kc}$ (13)

- *Contraction procedure*: The four overlapping cases previously described, are respectively eliminated by the following instructions :

  a. $m_{kc} = M_{ic} \leftarrow (m_{kc} + M_{ic}) / 2$
  b. $m_{ic} = M_{kc} \leftarrow (m_{ic} + M_{kc}) / 2$
  c. if $(M_{kc} - m_{ic}) < (M_{ic} - m_{kc})$ then $m_{ic} \leftarrow M_{kc}$ else $M_{ic} \leftarrow m_{kc}$
  d. if $(M_{ic} - m_{kc}) < (M_{kc} - m_{ic})$ then $M_{ic} \leftarrow m_{kc}$ else $m_{ic} \leftarrow M_{kc}$ (14)

In order to take into account inherent parallelism of extension and contraction processes, the implementation of the learning with the help of a neural network is interesting, although the clustering approach is not necessarily neuronal. The chosen architecture is composed of two layers. The input layer contains "c" neurons; a neuron for each performance indice. The output layer contains "g" neurons as shown in Fig.7. Each neuron of the output layer represents a cluster and uses the membership function described by (9).

The ith membership function $\mu_i(s_j, m_i, M_i)$ (9) associated with the cluster C_i, represents the activation function of the ith neuron of the output layer. The connections m_{ic} and M_{ic} between the cth neuron of the input layer and the ith neuron of the output layer, respectively represent minimal and maximal values with respect to "c" criteria.

5 Experimental Results

After obtaining a numerical model of a rule base (7), and adding logical parameters ("imp", a t-norm "t-n", a t-conorm "t-c") and defuzzification procedures ("defuz"), we can therefore completely define a FLC "sj" by vector (15). It can be associated vector to each FLC having a characteristic behavior; in other words it can be associated rule base, logical operators and defuzzification procedures to a FLC.

Table 6.1 and 6.2 show results related to eleven FLCs. The first is an example of back-up steering angle control system, taken from [Kong and Kosko, 92] (details are provided in the Appendix B). Respecting the symmetry property, the ten others are obtained by widening fuzzy set support associated with the control variables. In fact, variation in fuzzy sets overlapping allows distinguishing several FLCs as formulated by (16) and reported in Table 6.1 and 6.2.

The parameter "sp" is set to zero since there is no spacing between fuzzy sets, "nr=35", "nv=3", "sv= (5, 7, 7) ". The parameter "sy" is set to one which indicate that the fuzzy set partition is symmetric, "eq" is set to zero which indicate that the fuzzy set partition is not equidistant. The parameter "t-c" is set to the "Max" operator. The parameters "t-n" is set to the "Min" or "Product" operator, "defuz" is set to the center of gravity defuzzification method. The parameter "imp" varies during simulations. It is noteworthy that universal approximation framework is adopted, since FLCs can be considered universal approximators; they can approximate any real continuous function in a compact set to arbitrary accuracy [Herrera et al., 11].

		Mamdani's implication		*Larsen's implication*	
s_i	ov%	R_T	ov%	R_T	ov%
1	7 0.05968	46.00000	0.07778	42.00000	0.02222
2	7 0.06984	45.00000	0.06667	42.00000	0.02222
3	7 0.08045	45.00000	0.07778	42.00000	1.20000
4	7 0.09114	45.00000	1.34444	42.00000	1.20000
5	7 0.10192	46.00000	2.51111	42.00000	1.92222
6	7 0.11284	47.00000	3.61111	42.00000	2.44444
7	7 0.12390	48.00000	4.43333	43.00000	2.83333
8	7 0.13515	42.00000	0.11111	41.00000	1.75556
9	7 0.14642	42.00000	0.00000	41.00000	0.88889
10	7 0.15760	42.00000	0.06667	41.00000	0.40000
11	7 0.16861	42.00000	0.00000	41.00000	0.16667

Table 6.1: The 2^{nd} column presents parameters "ov%" *associated with the different fuzzy control systems. Columns 3 and 4 present the* overshoot percentage «ov%" *and the response time "R_T" associated with FLCs, respectively simulated by Mamdani's implication, Larsen's implication.*

The clustering results illustrated in Fig. 8 which clearly show that systems (s5, s6, s7), (s1, s2, s3, s4) and (s8, s9, s10, s11) form three distinct clusters. In the same cluster, systems are equivalent in a given interval of performance. It can be noticed that systems having an important overlapping degree present a small "ov%" and a short "R_T". The clustering results shown in Fig. 9 illustrate bad performance (relatively high "ov%" and "R_T"), achieved by systems 5, 6 and 7 with Mamdani's implication.

		Lukasiewicz's implication		*max-min implication of Zadeh*	
s_j	ov%	R_T	ov%	R_T	ov%
1	7 0.05968	-1.00000	22.45556	-1.00000	0.00000
2	7 0.06984	-1.00000	21.77778	-1.00000	0.00000
3	7 0.08045	-1.00000	20.41111	-1.00000	0.00000
4	7 0.09114	-1.00000	20.44444	-1.00000	0.00000
5	7 0.10192	93.00000	20.70000	-1.00000	0.00000
6	7 0.11284	87.00000	19.77778	-1.00000	0.00000
7	7 0.12390	86.00000	19.47778	-1.00000	0.00000
8	7 0.13515	78.00000	19.52222	-1.00000	0.00000
9	7 0.14642	75.00000	19.81111	-1.00000	0.00000
10	7 0.15760	73.00000	19.66667	-1.00000	0.00000
11	7 0.16861	72.00000	19.56667	-1.00000	0.00000

Table 6.2: Columns 3and 4 present the overshoot percentage *"ov%" and the response time "R_T" associated with FLCs, respectively simulated by Lukasiewicz's implication and the max-min implication of Zadeh. The negative value (-1.00000) means that the "R_T" of the considered system is not defined.*

$$s_j = (ov^j, sp^j, nr^j, nv^j, sv^j, sy^j, eq^j, imp^j, t\text{-}n^j, t\text{-}c^j, defuz^j) \tag{15}$$

$$s_j = (ov^j, 0, 35, 3, (5,7,7), 1, 0, imp^j, Min, Max, center\ of\ gravity) \tag{16}$$

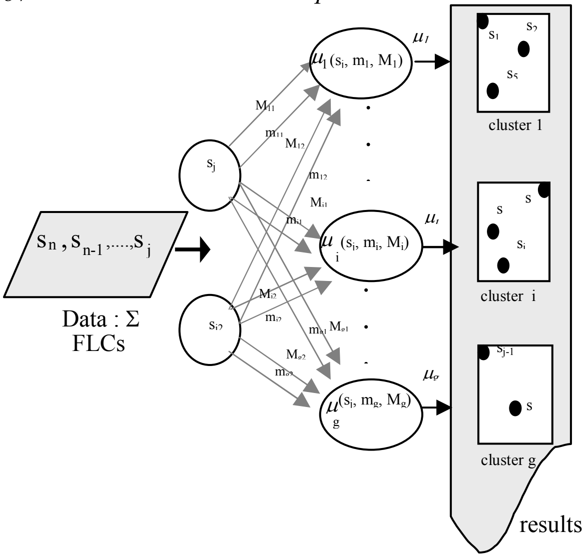

$S_n, S_{n-1}, \dots, S_j$
Data : Σ FLCs
s_j
s_{i2}
M_{11}
m_{11}
M_{12}
m_{12}
M_{i1}
m_{i1}
M_{i2}
m_{i2}
m_{g1} M_{g1}
M_{g2}
m_{g2}
$\mu_1(s_i, m_1, M_1)$
$\mu_i(s_i, m_i, M_i)$
$\mu_g(s_i, m_g, M_g)$
μ_1
μ_i
μ_g
s_1
s_2
s_5
cluster 1
s
s
s_i
cluster i
s_{j-1}
s
cluster g
results

Figure 7: Implementation of the Clustering Algorithm

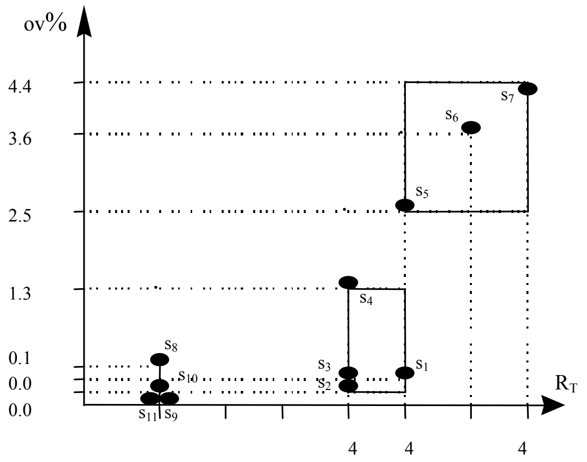


Figure 8: Clustering of FLCs (16), with Mamdani's implication

To reach a short "R_T" and a low "ov%", it is necessary to try another implication. It is worthy to remark that systems 8, 9, 10 and 11 using Mamdani's implication, and systems 1, 2, 3, 4, 5, 8, 9, 10 and 11 using Larsen's implication are equivalent with respect to "R_T" ($41 \le R_T \le 42$) and "ov%" ($0 \le o\% \le 1.92$). The superiority of Mamdani's implication is not always verified; systems s_{25}, s_{26} and s_{27} of Fig. 9 show poor performance ("ov%" relatively high).

Algorithm:

```
Let $\Sigma = \{ s_j \setminus j = 1, n\}$
be the set of FLCs
Begin
    read ($\alpha$, $\beta$) ;  $m_i \leftarrow M_i \leftarrow s_j$
    Repeat
        research $C_i$ square containing $s_j$ ;
            If $C_i$ exists then $s_j$ is added to $C_i$
            Else
              If there exists a square verifying the
              extension criteria Then extension;
                While overlapping do
                    contraction;
              Else
                create a new square such that
                $m_i \leftarrow M_i \leftarrow s_j$
              End if
            End if
    Until  all clusters' min-max couples
           do not change during the successive
           presentations of FLCs
End.
```

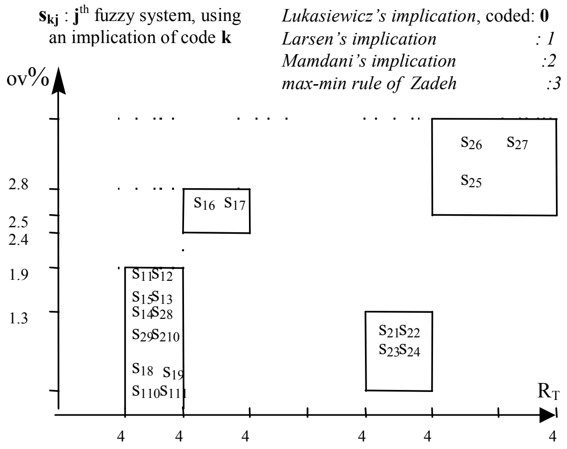


Figure 9: Clustering of FLCs "s_j" (16) with several implication. The 5th row of Table 6.1 and the Mamdani's implication (number 2) is the corresponding configuration of s_{25}

The obtained results show that systems using the Mamdani's implication and having a high overlapping degree in the fuzzy rules present a small "ov %" and a short "R_T". Systems using Mamdani's implication and Larsen's implication (t-norms class) have similar performances compared to those using other implications.

Comparative studies of fuzzy implications operators combined with the center of gravity defuzzification method have been treated in several works. It has been shown that the t-norms class has the best static behavior [Sakly and Benrejeb, 02]. The t-norms class guarantees better performances, having much better accuracy than R-implication [Cupertino et al., 02]. Lukasiewicz implications (R-implication) take very little control action and is therefore of little use in fuzzy control [Iancu (12)]. Compared with these studies, similar results were obtained with respect to the efficiency of fuzzy implications, as shown in Table 6.1 and 6.2. This justifies the proposed method by transformation of FLCs to a quantifiable entity and the use of the clustering approach.

6 Discussion

The aim of this study is to identify FLC's parameters of nonlinear control systems characterized by second-order models to achieve an optimal control performance. These systems are encountered in many industrial and non-industrial process applications. The quality of control FLC depends on performance indices (response time, rise time, etc.). Our main challenge is to find the best setting for all FLCs' parameters as a whole, while reducing such indices and avoiding undesirable overshoot.

It was necessary to establish the vectorization of FLC, particularly its knowledge base before carrying out the clustering. The transition between the topological and the numerical form of FLC's rule bases is presented in Section 3. The vector space base of relationships between fuzzy sets leads to two characteristics qualified to specify a given fuzzy partition - the overlapping degree (4) and the distance (6) between the closest extremities of fuzzy sets' supports. As a result of the analysis of section 3, the FLC's rule base can be considered as a vector (7). Then a FLC (rule base, logical operators and defuzzification procedures) has been completely defined by a vector (15). The obtained numerical model of a FLC provides a realistic framework capable of classifying FLCs according to performance indices.

The design of a FLC depends on the number and shape of the input-output membership functions. Generally, it is very difficult to examine all the input-output data from a system to find the optimal membership function for a given FLC. That is why some recent works [Pelusi, 11b], [Meza et al., 09] based on Genetic Algorithms have been applied (with good results), to achieve the membership functions which improve the control system performances. Pelusi has carried out some works to improve the overshoot and the settling time of a FLC by optimizing membership functions, using Genetic Algorithms [Pelusi, 12b]. However, these systems are a time-consuming adjusting process. In Precup's work [Precup et al., 13] a model base from input–output data has been built using an online clustering procedure. Recent studies [Pelusi, 11a] and [Pelusi, 12a] propose genetic-neuro-fuzzy techniques able to improve some performance indices of second order control systems. In [Pelusi and Mascella 13], an optimization procedure of the membership functions has been accomplished through a genetic procedure, in order to improve the overshoot. Although, these techniques remain important issues in fuzzy modeling, the accuracy of their resulting model is highly dependent on the quality of training data used in its identification. In a parallel path, evolutionary algorithms [Riid and Rüstern, 11] [Sanz et al., 10] have become popular in recent years; they have proved their efficiency in the optimal tuning of fuzzy control systems and have shown an improvement in the FLCs interpretability. However, these algorithms which work with a family of potential solutions [Riid and Rüstern, 11], are computationally expensive and require several iterations to converge. This is often disapproved in many applications.

The main contributions of this paper with respect to the current literature are:

1. The passage from the topological form of a fuzzy rule base to its numerical form, is carried out in order to reach an accurate rule bases identification. This contribution is important and advantageous with respect to the current works because:

- The FLCs' representation in the form of vectors allows to study how FLCs reach the desired performance indices. It also allows to study the various parameters and their interaction with respect these indices.
- The designer can easily individualize FLCs by modifying parameters, and accurately locate a FLC with respect to another.

2. It is seen theoretically that the assessment measure of the fuzzy partition depends only on (4) and (6), which justifies the tuning approaches that use either symbolic translation of a fuzzy set or a lateral displacement of the fuzzy set support [Alcalá et al., 07a] [Alcalá et al, 07b].

3. The proposed approach can be seen as an alternative of a FLC's identification method that allows relating fuzzy parameters to performance indices. The advantage of this contribution is:

- Providing FLCs' designers with more efficient and transparent means of assessing the relevance and the relative interest of parameters with regards to some performance indices.
- Providing FLCs' designers with a library of clusters reference by identifying parameters of the fuzzy controller with those of the nearest cluster.
- The proposed approach can become complementary to the learning methods. Indeed, it is often difficult for a designer to imagine the influence of parameters' interactions on the output. It will be easier to construct well adjusted fuzzy sets. For example, it may be noted that the fuzzy set "around zero" is too specific and not enough fuzzified. Therefore, the designer will be able to personalize the performance indices to his preferences, by adjusting the overlapping degree (4) and the spacing measure (6). He should impose some constraints to the MF's definition in order to preserve some properties as Completeness, Normalization, Distinguishability and Complementarity [Gacto et al., 11].

4. The interpretability is the capacity to express the behavior of the real system in an understandable way, which depends on several parameters, especially the fuzzy partition. The results of the example included to accompany the theoretical finding confirm that the clustering technique (section 4) is an efficient tool for minimizing the gap between the accuracy and transparency. Indeed, it allows to handle adequately all aspects of an optimal control performance (both the FLCs' parameters and performances indices), taking into account the interdependence between parameters and making the analysis of the system's behavior easier by studying the system's sensitivity to one or more parameters. This technique's advantages are evident in terms of computational complexity compared with other methods such as computational evolutionary algorithms or genetic algorithms which are considered as grafted foreign objects on the FLC.

5. The optimal choice of parameters was based on the universal approximation propriety which reduces the number of choices and refines the search problems. Universal approximation propriety has at least two advantages. First, it provides FLCs with the ability to simulate a multitude of fuzzy control systems without being attached to a particular context of the controlled process. Second, it reduces the parameters space. It also sets some of them, such as t-norms, defuzzification procedures, etc. For modeling, the question is whether a FLC is capable of uniformly approximating any continuous, nonlinear physical system, without being attached to a specific context of controlled process. The responses to this vast question are provided in the following:

- First, when taking into account the context, it is interesting to observe dissimilarity between the expert systems approach and the fuzzy control approach. This difference is caused by a divergence in the objectives embedded in the two approaches. Fuzzy expert systems seek to describe the linguistic knowledge meaning in order to calculate automatically imprecise conclusions [Zadeh, 79]. In fact, these artificial intelligence systems are characterized by the tasks they realize, and not by the programs that they accomplish. These latter are characterized by the knowledge that these programs express and the meaning that they possess in the real task's context [Newell, 82]. In other words, making-decisions should have a sense with regard to the context. Consequently, the fuzzy parameters choice involved in the evaluation of decisions has to depend on the context. The fuzzy control systems do not seek to describe the knowledge meaning during inferences but to interpolate functions by constructing control laws from rules.

- We can consider that the context is represented by the sequences of rules that a fuzzy control system uses to control a variable around performances indices. The rule base is not considered as a universal affirmation sort, but only applied to a specific context in which the model of fuzzy control has been defined. The context is represented by the meaning of symbols (fuzzy sets) included in the rules: a membership function μF of a fuzzy subset F depends on the context. For example, "the temperature is raised" has not the same meaning if it concerns the temperature of a room or a gas stove, etc. For two experts the phrase "the temperature is raised" does not always mean exactly the same thing.

7 Conclusion

Inspired by Allen's interval algebra, a product vector space of relationships between cores and supports of fuzzy sets was formulated. A base of relationships specifying fuzzy sets partition of a rule base was therefore deduced. Clustering approach based on the obtained numerical model of a rule base provides a realistic framework able to classify FLCs according to some performance indices. This synthesis provides FLCs' designers with a library of clusters reference by identifying parameters of the fuzzy controller to design with those of the nearest cluster. This contribution proposes a specific classification that can help better understand which FLC's parameters can lead to a range of some performance indices. A simulation analysis of FLCs and comparison of results with those existing in the literature justify the used methodology.

Rather than fully automated learning techniques (black-boxes), fuzzy modeling should be seen as an interactive method, facilitating the active participation of the designer in the modeling process. The designer is able to personalize the performance indices to his preferences, by adjusting the fuzzy partition and some other parameters. The clustering technique provides a mean to quantify the interpretability of Mamdani's FLCs – which is still an open problem – with respect to the preferred reference fuzzy partitions provided by a designer. The first limit of this work is related to the study of the intrinsic parameters, including scaling factors which have a contextual component. It is therefore interesting to explore this path. Another interesting way is to analyze how we could combine parameters of the different families.

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References

[Alcalá et al., 07a] Alcalá, R., Alcalá-Fdez, J., Herrera, F.: "A proposal for the genetic lateral tuning of linguistic fuzzy systems and its interaction with rule selection"; IEEE Trans. Fuzzy Syst., 15, 4, (2007), 616–635.

[Alcalá et al, 07b] Alcalá, R., Alcalá-Fdez, J., Gacto, MJ. Herrera, F.: "Rule base reduction and genetic tuning of fuzzy systems based on the linguistic 3-tuples representation "; Soft Computing, 11, 5, (2007), 401–419.

[Alcalá et al., 11] Alcalá, R., Nojima, Y., Herrera, F., Ishibuchi, H.: "Multi-objective genetic fuzzy rule selection of single granularity-based fuzzy classification rules and its interaction with the lateral tuning of membership functions"; Soft Computing, 15, (2011), 2303- 2318.

[Allen and Koomen, 83] Allen, J.F., Koomen, J.: "A Planing Using a Temporal World Model"; Joint. Artificial Intelligence, NY (1983).

[Chavez et al., 12] Chavez, F., Fernandez, F., Gacto, M.J, Alcalá, R.: "Automatic Laser Pointer Detection Algorithm for Environment Control Device Systems Based on Template Matching and Genetic Tuning of Fuzzy Rule-Based Systems "; International Journal of Computational Intelligence Systems, 5, 2, (2012), 368-386.

[Chen and Ko, 08] Chen, S.M., Ko, Y.K.: "Fuzzy interpolative reasoning for sparse fuzzy rule-based systems based on α-cuts and transformations techniques"; IEEE Trans. Fuzzy Syst., 16, 6, (2008), 1626–1648.

[Chenaina and Chouigui, 00] Chenaina, T., Chouigui, N.: "Interdependence of Fuzzy Controllers Parameters "; Proc. of International Conference on Artificial and Computational Intelligence for Decision, Monastir, (2000)

[Coello et al., 07] Coello, C.A.C., Lamont, G.B., Veldhuizen, D.A.V.: "Evolutionary Algorithms for Solving Multi-Objective Problems"; Springer-Verlag, (2007).

[Cupertino et al., 02] Cupertino, F., Dotoli, M., Giordano, V., Maione, B., Salvatore, L.: "Fuzzy Control Experiments on DC Drives Using Various Inference Connectives"; Fuzzy Systems, 1, (2002), 52-57.

[Duch et al., 01] Duch, W., Adamczak, R., Grabczewski, K.: "A new methodology of extraction, optimization and application of crisp and fuzzy logical rules"; IEEE Transactions on Neural Networks, 12, 2, (2001), 277–306.

[Gacto et al., 11] Gacto M.J., Alcalá, R., Herrera, F.: "Interpretability of linguistic fuzzy rule-based systems: An overview of interpretability measures "; Inf. Sci., 181, 20, (2011), 4340–4360.

[Gacto et al., 12] Gacto, M.J., lcalá, R., Herrera, F.: "A Multi- Objective Evolutionary Algorithm for an Effective Tuning of Fuzzy Logic Controllers in Heating, Ventilating and Air Conditioning Systems "; Applied Intelligence, 36, (2012), 330-347.

[Herrera et al., 11] Herrera, L.J., Pomares, H., Rojas, I., Guillen, A., Valenzuela, O., "The TaSe-NF model for function approximation problems: approaching local and global modeling"; Fuzzy Sets Syst., 171, 1, (2011), 1–21.

[Iancu (12)] Iancu, I.: "A Mamdani Type Fuzzy Logic Controller"; Prof. Elmer dadios (Ed.) / Croatia (2012)

[Kong and Kosko, 92] Kong, S.G., Kosko, B.: "Adaptive Fuzzy System for Backing up a Truck-and-Trailer"; IEEE Trans. on Neural Networks, 3, 2, (1992).

[Mamdani, 74] Mamdani, E.H.: "Application of fuzzy algorithms for control of simple dynamic plant "; Proc. of IEEE, 121, 12, (1974), 1585–1588.

[Meza et al, 09] Meza, J.L., Soto, R., Arriaga, J.: "An Optimal Fuzzy Self-Tuning PID Controller for Robot Manipulators via Genetic Algorithm"; Eighth Mexican International Conference on Artificial Intelligence, (2009) 21- 26.

[Newell, 82] Newell, A.: "The Knowledge Level "; Artificial Intelligence", 18, (1982) , 87-127.

[Perez et al., 13] Perez, J., Milanes, V., Godoy, J., Villagra, J., & Onieva, E. : "Cooperative controllers for highways based on human experience"; Expert Systems with Applications, 40, 4, (2013), 1024–1033.

[Pelusi, 11a] Pelusi, D.: "On designing optimal control systems through genetic and neuro-fuzzy techniques": IEEE International Symposium on Signal Processing and Information Technology, Bilbao (2011), 134-139.

[Pelusi, 11b] D.: "Optimization of a Fuzzy Logic Controller using Genetic Algorithms"; IEEE International Conference on Intelligent Human- Machine Systems and Cybernetics, 2, Hangzhou (2011) 143-146.

[Pelusi, 12a] Pelusi D.: "Improving Settling and Rise Times of Controllers Via Intelligent Algorithms": IEEE International Conference on Modelling and Simulation, Cambridge (2012), 187-192.

[Pelusi, 12b] Pelusi, D.: "PID and Intelligent Controllers for Optimal Timing Performances of Industrial Actuators" IJSSST, 13,2, (2012), 1473-8031.

[Pelusi and Mascella 13] Pelusi, D., Mascella, R. : "Optimal Control Algorithms for Second order Systems:; Journal of Computer Science, 9, 2, (2013), 183-197.

[Precup et al., 13] Precup, R.E., David, R.C., Petriu, E.M., Radac, M.B., Stefan Preitl, Fodor, J.: "Evolutionary optimization-based tuning of low-cost fuzzy controllers for servo systems"; Knowledge-Based Systems 38, (2013), 74–84

[Rao and Shankarb , 12] Rao P.P.B., Shankarb, N.R.: "Ranking Generalized Fuzzy Numbers using Area, Mode, Spreads and Weights"; Int. J. Appl. Sci. Eng.,. 10, 1, (2012), 41-57.

[Riid and Rüstern, 11] Riid, A., Rüstern, E.: "Identification of transparent, compact, accurate and reliable linguistic fuzzy models", Information Sciences 18, (2011), 4378–4393

[Rojas et al., 00] Rojas, H., Pomares, J., Ortega, Prieto, A.: "Self-organized fuzzy system generation from training examples"; IEEE Transactions on Fuzzy Systems, 8, 1, (2000), 23– 36.

[Sakly and Benrejeb, 02] Sakly, A., Benrejeb, M.: "On the Choice of the Adequate Fuzzy Implication Operator with the Center of Gravity Defuzzification Method Based on Precision Criterion in Fuzzy Control"; 11th IEEE Int. Conf. on Fuzzy Systems, Honolulu, (2002).

[Sanz et al., 10] J.A. Sanz, A. Fernandez, H. Bustince, F. Herrera,: "Improving the performance of fuzzy rule-based classification systems with interval-valued fuzzy sets and genetic amplitude tuning"; Inform. Sci. 180, 19, (2010), 3674–3685.

[Schiavo and Luciano, 01] Schiavo, L., Luciano, L.: "Powerful and flexible fuzzy algorithm for nonlinear dynamic system identification "; IEEE-Transaction on Fuzzy Systems, 9, (2001), 828– 835.

[Simpson, 93] Simpson, P.: "Fuzzy Min-Max Neural Networks Part 2: Clustering "; IEEE Trans. on Fuzzy Systems, 1, 1, (1993).

[Yang and Shen, 11] Yang, L., Shen, Q.: " Adaptive Fuzzy Interpolation"; Fuzzy Systems, IEEE Transactions on, 19,6, (2011), 1107-1126.

[Zadeh, 73] Zadeh, L.A.: "Outline of a new approach to the analysis of complex systems and decision processes"; IEEE Trans. on Sys., Man, and Cybern., 3, (1973) , 28-44.

[Zadeh, 79] Zadeh, L.A.: "A Theory of Approximate Reasoning, Machine Intelligence"; Elsevier, 9, (1979), 149-174.

Appendix A

Proof of theorem 3

Let $\{\alpha_x, \beta_x, \gamma \mathbin{/} x \in B\text{-}\{d\}\}$ be the Boolean value set verifying the equation:

$$\sum_{x\neq d}\alpha_x(x,x)+\sum_{x\neq d}\beta_x(x,d)+\gamma(d,d)=0 \Leftrightarrow$$

$$\begin{cases}\sum\limits_{x\neq d}\alpha_x x+\sum\limits_{x\neq d}\beta_x x+\gamma d=0\\ \sum\limits_{x\neq d}\alpha_x x+\sum\limits_{x\neq d}\beta_x d+\gamma d=0\end{cases}$$

$$\Leftrightarrow \sum_{x\neq d}(\alpha_x+\beta_x)x+\gamma d=0 \tag{17}$$

$$\Leftrightarrow \sum_{x\neq d}\alpha_x x+\sum_{x\neq d}\beta_x+\gamma d=0 \tag{18}$$

B is a base of the intervals vector space E, so B is a free system. Respectively (19) and (20) are deduced from (17) and (18):

$$\forall\ x\in B\text{-}\{d\},\ \gamma=0 \text{ and } \alpha_x+\beta_x=0 \tag{19}$$

$$\forall\ x\in B\text{-}\{d\},\ \alpha_x=0 \text{ and } \sum_{x\neq d}\beta_x+\gamma=0 \tag{20}$$

Combining (19) and (20), it can be deduced that $\forall\ x \in B\text{-}\{d\}$, $\gamma = 0$, $\alpha_x = 0$ and $\beta_x = 0$. It results that $\mathbb{B}$ is a free system of the product vector space ExE.

Proof of theorem 4

a) Showing that $\mathbb{E} = \mathbb{R}$ $\{(r_0, r_1) \in E\times E$ / length (r_0) + length (r_1) is even$\}$ is a vector subspace of ExE.

- *Stability by addition*

Let (r_0, r_1) and $(r_0', r_1') \in \mathbb{R}$, we have: $(r_0, r_1) + (r_0', r_1') = (r_0 + r_0', r_1 + r_1')$.

Length $(r_0 + r_0')$ = length (r_0) + length (r_0') - 2× number of simplifications (simplification results from $\forall x\in B$, $x+x=\phi$). Length $(r_1+ r_1')$ = length (r_1) + length (r_1')

- 2× number of simplifications. Therefore: length $(r_0 + r_0')$ + length $(r_1 + r_1')$ = length (r_0) + length (r_0') + length (r_1) + length (r_1') - 2 × number of simplifications.

$(r_0, r_1) \in \mathbb{R}$, so length (r_0) + length (r_1) is even.

$(r_0', r_1') \in \mathbb{R}$, so length (r_0') + length (r_1') is even.

It results that length $(r_0 + r_0')$ + length $(r_1 + r_1')$ is even, so $((r_0, r_1) + (r_0', r_1')) \in \mathbb{R}$, thus the stability is proved.

- *Stability by the external law '•'*

Let $(r_0, r_1) \in \mathbb{R}$: $0.(r_0, r_1)=(0\bullet r_0, 0\bullet r_1)=(\phi, \phi) \in \mathbb{R}$ since length ϕ+length ϕ= 0 is even. $1\bullet(r_0, r_1) = (1\bullet r_0, 1\bullet r_1)=(r_0, r_1) \in \mathbb{R}$. Thus the stability is proved. Therefore, $\mathbb{R}$ is a vector space of $\mathbb{E}$.

b) Let's show that $\mathbb{B} = \{(x, x), (x, d) \mid x\in B\}$ is a base of realizable relationships vector space $\mathbb{E}$.$\mathbb{E}$ contains the vector subspace $<\mathbb{B}>$ generated by $\mathbb{B}$, this latter is a part of the realizable relationships set. We have:

$<\mathbb{B}> \subseteq \mathbb{E} \subseteq \mathbb{R} \subset E\times E.$ (21)

Dimension of E×E = 26 (22)

Dimension of $<\mathbb{B}>$ = 25 (23)

According to (21), (22) and (23) we obtain:

$25 \leq \dim \mathbb{E} \leq \dim \mathbb{R} < 26$ (24)

According to (24), dimension of $\mathbb{E}$ = 25, $\mathbb{B}$ is a free system of $\mathbb{E}$ (theorem 3) with dimension 25, $\mathbb{B}$ is a base of $\mathbb{E}$

Explanation of definition 3

$\forall (x,y) \in \mathbb{E}$, $\cap_{(x,y)}$ is defined by:

$\cap_{(b, b)} = \phi$

$\cap_{(e, e)} = \{A_i = A_j, B_i = B_j, C_i = C_j, D_i = D_j\}$

$\cap_{(m, m)} = \{C_i = D_i = A_j = B_j\}$

$\cap_{(f, f)} = \{A_i, B_i, C_i, D_i\}$

$\cap_{(o, o)} = \{A_j, B_j, C_i, D_i\}$

$\cap_{(s, s)} = \{A_i, B_i, C_i, D_i\}$

$\cap_{(d, d)} = \{A_i, B_i, C_i, D_i\}$

$\cap_{(b', b')} = \phi$

$\cap_{(m', m')} = \{C_j = D_j = A_i = B_i\}$

$\cap_{(f', f')} = \{A_j, B_j, C_j, D_j\}$

$\cap_{(o', o')} = \{A_i, B_i, C_j, D_j\}$

$\cap_{(s', s')} = \{A_j, B_j, C_j, D_j\}$

$\cap_{(d', d')} = \{A_j, B_j, C_j, D_j\}$

$\cap_{(b, d)} = \{A_i, A_iB_i \cap A_jB_j, C_iD_i \cap A_jB_j, D_i\}$

$\cap_{(e, d)} = \{A_i, B_i, C_i, D_i\}$

$\cap_{(m, d)} = \{A_i, A_iB_i \cap A_jB_j, B_j = C_i, D_i\}$

$\cap_{(f, d)} = \{A_i, B_i, C_i, D_i\}$

$\cap_{(o, d)} = \{A_i, A_iB_i \cap A_jB_j, B_j, C_i, D_i\}$

$\cap_{(s,\, d)} = \{A_i, B_i, C_i, D_i\}$
$\cap_{(b',\, d)} = \{A_i, A_iB_i \cap C_jD_j, C_iD_i \cap C_jD_j , D_i\}$
$\cap_{(m',\, d)} = \{ A_i, B_i = C_j, C_jD_j \cap C_iD_i, D_i\}$
$\cap_{(f',\, d)} = \{A_i, A_iB_i \cap A_jB_j, B_j, C_i = C_j, D_i\}$
$\cap_{(o',\, d)} = \{A_i, B_i, C_j, C_jD_j \cap C_iD_i, D_i\}$
$\cap_{(s',\, d)} = \{A_i, B_i, C_j, C_jD_j \cap C_iD_i, D_i\}$
$\cap_{(d',\, d)} =\{A_i, A_jB_j \cap A_iB_i, B_j, C_j, C_jD_j \cap C_iD_i, D_i\}$

Appendix B

Figure 10 shows the geometry of the simulated truck and loading dock. The three state variables ϕ, x, and y determine the truck position with ϕ specifying the angle of the truck with the horizontal. The coordinate pair (x, y) specifies the position of the rear center of the truck. We wanted the truck to arrive at the loading dock at a right angle ($\phi f = 90°$) and to align the position (x, y) of the truck with the desired loading dock (x_f, y_f) equaled (50,100). The truck moved backward a fixed distance at every stage until the truck hits the border of the loading zone. The loading zone corresponded to the plane [0,100] x [0,100]. At every stage the fuzzy controller produce the steering angle θ that backs up the truck to the loading dock from any initial position and from any angle in the loading zone. The coordinate x ranges from 0 to 100, ϕ ranges from -90 to 270, and θ ranges from -30 to 30.

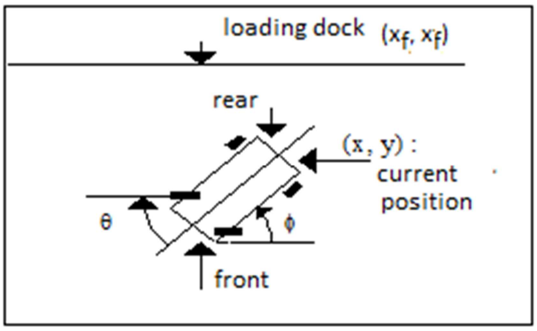

Figure 10: Simulated truck

Figure 11 shows fuzzy membership functions for each linguistic variable x, ϕ, and θ. **LE** being on Left, **LC** on the Left Center, **CE** on the Center, **RC** on the Right Center, **RI** on the Right, **RB** on the Right Below, **RU** on the on the Right Upper, **RV** on the Right Vertical, **VE** on the Vertical, **LV** on the Left Vertical, **LU** on the Left Upper, **LB** on the Left Below, **NB** on the Negative Big, **NM** on the Negative Medium, **NS** on the Negative Small, **ZE** on the Zero, **PS** on the Positive Small, **PM** on the Positive Medium and **PB** on the Positive Big.

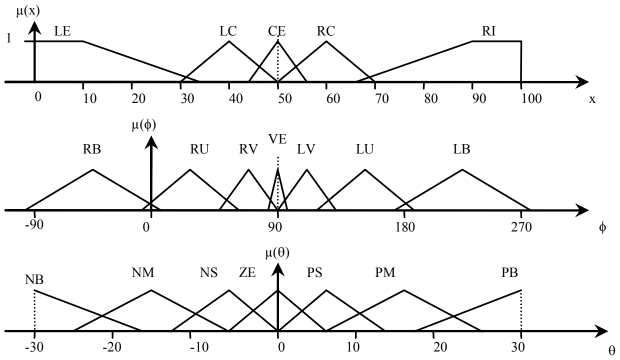

Figure 11: Membership functions of x, ϕ and θ

The rule base contains 35 rules as shown in table 7.

X ϕ	LE		LC		CE		RC		RI	
RB	1	PS	2	PM	3	PM	4	PB	5	PB
RU	6	NS	7	PS	8	PM	9	PB	10	PB
RV	11	NM	12	NS	13	PS	14	PM	15	PB
VE	16	NM	17	NM	18	ZE	19	PM	20	PM
LV	21	NB	22	NM	23	NS	24	PS	25	PM
LU	26	NB	27	NB	28	NM	29	NS	30	PS
LB	31	NB	32	NB	33	NM	34	NM	35	NS

Table 7: The rule base