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Color Image Restoration Using Neural Network Model

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Abstract: Neural network learning approach for color image restoration has been discussed in this paper and one of the possible solutions for restoring images has been presented. Here neural network weights are considered as regularization parameter values instead of explicitly specifying them. The weights are modified during the training through the supply of training set data. The desired response of the network is in the form of estimated value of the current pixel. This estimated value is used to modify the network weights such that the restored value produced by the network for a pixel is as close as to this desired response. One of the advantages of the proposed approach is that, once the neural network is trained, images can be restored without having prior information about the model of noise/blurring with which the image is corrupted.
Key Words: Ill Posed Problem, Regularization, Color Image Restoration, Neural Networks
Category: I.2, I.2.6, I.4, I.4.4, G.1.8

1 Introduction

The usage of visual information is continually playing an increasingly important role in our lives as more and more multimedia information is getting generated every day. It means that the images are becoming widespread day by day but unfortunately, many images represent scenes in an unsatisfactory manner. This is because many physical imaging systems are imperfect, and the conditions under which images are obtained are frequently less than ideal hence a recorded image often represents a degraded version of the original scene. The imaging process, the atmosphere, and the recording medium all introduce degradations into the captured image so that the image that is actually recorded often fails to represent the scene adequately. Image restoration addresses the problem of unsatisfactory scene representation. The goal of image restoration is to manipulate an image in such a way that it will, in some sense, more closely depict the scene that is intended to be represented. The image restoration problem appears in many fields like astronomy, medicine, military, photo processing. All disciplines in which images are acquired under less than ideal conditions find restoration techniques useful. These fields have diverse aims for image restoration, but certain fundamentals are common to all image restoration problems. Image restoration in

general is an ill posed problem [Hadamard 1923] [Tikonov and Arsenin 1977] [Andrews and Hunt 1977].
A problem is said to be well posed when the following three conditions are satisfied[Poggio et al. 1985]

- Its solution exists
- Its solution is unique
- Its solution depends continuously on the initial data

An ill posed problem, fails to satisfy one or more of the above three conditions [Miller 1970] [Blanc Feraud et al. 1995][Charbonnier et al. 1994]. For a well posed problem also, even though the third condition ensures numerical stability, it does not imply that the solution is robust [Anderson and Netravali 1976]. Many researchers have concluded that ill posed problems [Nashed 1981] can be solved by converting them to a possible well posed problems. This conversion restricts the class of admissible solutions by introducing suitable apriori knowledge. This apriori knowledge can then be used to apply variational principles that improve constraints on the possible solutions or as statistical properties of solution space. Restoration of blurred and noisy images belong to the category of inverse problems, which requires the adoption of many different approaches and one among them is to use regularization approach [Banham and Katsaggelos 1997] [Jain 2001][Kang and Katsaggelos 1992].In the presence of both blur and noise, the restoration process is ill conditioned and requires the specification of additional smoothness constraints on the solution[Miller 1970]. This is achieved by adding a regularization term in the associated cost function along with usual least square error term[Wong and Chan 2001][Chickerur and Kumar 2010]. Regularization parameter controls the relative contribution of these two terms. Adaptive processing techniques have been applied to images degraded by noise and blur to have adequate preservation of important image features such as edges and textures. It has been generally seen that a technique suited for restoring edges is not efficient for restoring texture portion of the image and vice versa. In this paper, a neural network based approach for solving adaptive regularization problem for color images is presented. The local regularization parameters are regarded as neural network weights which are modified through the supply of training set data. The updated pixel value in the current iteration is regarded as the network output. This is then considered as a desired network output, which forms part of a training example, and is then used to adjust the network weight. As a result, instead of choosing this parameter by trial and error, we can now explicitly incorporate neighboring pixel value information to guide its selection. This makes it possible to automatically adjust the regularization parameters under different degradation conditions. The salient feature of our appraoch is the local estimation of the restored image based on feed forward backpropogation gradient descent approach. The next section introduces neural network based color image restoration, followed by the section, which discusses experimental setup. This is followed by results and discussion. The last section concludes this paper.

2 Neural Networks based Color Image Restoration

Various researchers have been working on solving image restoration problems using different techniques. One among them is neural networks[Zhou et al. 1988] [Perry and Guan 2000] [Anand et al. 1995]. In this section, the neural network based approach for solving the image restoration problem is explored. The cost function for image restoration, regularization approach and its relationship to neural networks will be investigated.

2.1 Regularization using Neural Network approach

Restoring an original image, when given the degraded image, with or without knowledge of the degradation function, degree and type of noise present is an ill posed problem and can be approached in a number of ways as discussed in chapter 1. Generally the image degradation model suitable for most practical purposes is formed as a linear process with additive noise for which the matrix form is

$$g = Hf + n \tag{1}$$

where g and f are degraded and original vectors respectively, H is the degradation matrix and n represents the noise. Regularized image restoration methods aim to minimize the constrained least-square error measure

$$E = \frac{1}{2}||y - Hx||^2 + \frac{1}{2}\lambda||Dx||^2 \tag{2}$$

Where y vector represents blurred image pixels, x vector is the restored image pixel estimate, λ represents the regularization parameter and D is the regularization matrix which is generally a high pass operator.

2.2 Adaptive Learning rate Neural Network based image restoration

The key to solve this problem is to minimize the degradation measure which is represented by $E_{i,j}$ defined for each pixel (i, j) in an M X N image and is given by

$$E_{i,j} = \frac{1}{2}[y_{i,j} - h * x]^2 + \frac{1}{2}\lambda_{i,j}[d * x]^2 \tag{3}$$

where $h * x$ denotes the convolution between a blur filter h centered at point (i, j) and the restored image x, $d * x$ represents the convolution between a high pass filter d centered at a point (i, j) and the restored image x. The Variable $\lambda_{i,j}$ represents the local regularization parameters. In general large values of λ are required for smooth regions and small λ for edges. Iterative update approaches are generally used to minimize this cost function by gradient optimization approaches as

$$\frac{\partial E_{i,j}}{\partial x_{p,q}} = (h * x - y_{i,j})\, h_{-a,-b} + \lambda(d * x)h_{-a,-b} \tag{4}$$

where
$p = i + a$,
$q = j + b$,
$p \in \{i - W/2, ..., i + W/2\}$,
$q \in \{j - H/2, ..., j + H/2\}$
and considering that the convolution mask is W X H. To minimize this cost function, we need to equate the above derivative to zero for the updated pixel in which the previous pixel value $x_{p,q}$ is replaced by $x'_{p,q}$. Thus the required amount of update for the pixel is given as

$$\varDelta x_{p,q} = x'_{p,q} - x_{p,q} \tag{5}$$

It is thus seen that the required amount of update depends on the local regularization parameter. To train the neural network, two sets of data are required. One set of data is the input vector of the blurred /corrupted image and the other set denotes the individual gray/color level change for each pixel for satisfactory restoration. In other words the training example is of the form

$$\left(x_{p,q}, \varDelta x^d_{p,q}\right) \tag{6}$$

The training data given in Equation.6 is used to train the neural network model and thus, we can update the parameter $\lambda_{i,j}$ by using a supervised learning approach. Once the neural network is trained the noisy/blurred image can be given to the neural network which generates the deblurred/denoised image. The above approach gives good results for the gray scale images. The modified model based neural network approach for color image restoration is as given below. To minimize the cost function, we need to equate the above derivative to zero for the updated pixel $x'_{p,q}$ in which the previous color value $x_{p,q}$ is replaced by $x'_{p,q}$. Thus the required amount of update for the pixel is given as

$$\varDelta x_{p,q} = x'_{p,q} - x_{p,q} \tag{7}$$

In addition, this can be further enumerated as,
for red value of the pixel

$$\varDelta x_{(p,q)_\mathrm{r}} = x'_{(p,q)_\mathrm{r}} - x_{(p,q)_\mathrm{r}} \tag{8}$$

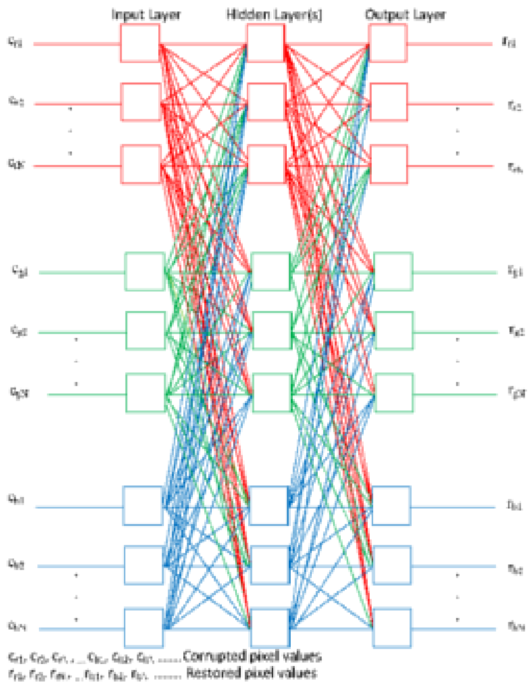

Figure 1: Neural network design for color image restoration

for green value of the pixel

$$\Delta x_{(p,q)_{\mathrm{g}}} = x'_{(p,q)_{\mathrm{g}}} - x_{(p,q)_{\mathrm{g}}} \tag{9}$$

and for blue value of the pixel

$$\Delta x_{(p,q)_{\mathrm{b}}} = x'_{(p,q)_{\mathrm{b}}} - x_{(p,q)_{\mathrm{b}}} \tag{10}$$

In this approach the complexity of the neural network increases as each of the red, green and blue value of each pixel is considered as input to the neural network. This increases the number of nodes and interconnections as shown in Figure 1.

3 Experimental Setup

To test the approach discussed above, a series of experiments were structured. In the first experiment, the performance of the approach discussed in this paper was compared to the other approaches available in literature. The performance in this experiment was evaluated in terms of visual perception of the user for the obtained results. The second experiment was done with images degraded by Gaussian noise, Speckle Noise and Blur. The performance in this experiment was evaluated based on Signal to noise ratio(SNR), Peak Signal to Noise Ratio(PSNR), Root Mean Square Error(RMSE) and Mean Absolute Error(MAE).

3.1 Test Images

A common source of test images for image processing techniques can be obtained from various repositories. The images for the experiments have been taken from USC-SIPI Image Database, Standard test images from R C Gonzalez and R E woods home page of Digital Image processing textbook and medical images at MIDAS Image Gallery. In all the experiments many images from the following databases were used for testing. The images were chosen since these images have been widely used in literature and have a mix of regions of texture and edges.

3.2 Simulating Degradation

The images used in the experiments were subjected to three types of degradations mentioned above.

1. Blurring:The first step was creating a point spread function to add blur to an image. Point Spread Function (PSF) is the degree to which an optical system blurs (spreads) a point of light. The blur was implemented by first creating a PSF filter that would approximate linear motion blur. This PSF was then convolved with the original image to produce the blurred image. The amount of blur added to the original image depended on two parameters of the PSF: length of blur in Pixels (P), and the angle of the blur(T).

2. Speckle Noise:Speckle noise is defined as multiplicative noise, having a granular pattern and is mostly seen in ultrasound image and SAR image. It degrades the quality of the active radar and synthetic aperture radar (SAR) images. These images are corrupted with uniformly distributed multiplicative noise with different levels of standard deviation (σ) of the noise. Values lie in the range $[0.1 to 1]$, with $(\sigma) = 0.1$ representing an almost noise-free image, $(\sigma) = 0.95$ representing a very noise image.

3. Gaussian Noise: Gaussian Noise is one in which the pixels are not affected uniformly - the effect decrease from kernel center to edges according to a bell-shaped curve. The blurring is dense in the center and feathers at the edge. Gaussian noise depends on the Size and Alfa.

3.3 Neural Network Design

The back propagation neural network used is pictured as having three layers or at the most four layers. The input layer is given input after the normalization of the pixel values. The number of nodes in the input layer is equal to the number of pixels in the image. If the image being considered is represented as M x N pixels, we have M x N input nodes. The images which have been considered are 128 x 128 and 256 x 256 pixels wide. Considering the second case then the number of input nodes are 65536 for gray scale image and 65536 x 3 for the color images. Since each of the pixel value need to be regularized using the regularization approach previously discussed, the number of output nodes in the output layer is equal to the number of input nodes.
The decision of the number of nodes in the hidden layer depends on various factors like:

1. Number of input and output nodes.
2. Number of training cases.
3. The amount of noise in the targets.
4. Regularization

In most of the cases, there is no standard way to determine the best number of nodes in the hidden layer without training the network and estimating the generalization error. The decision of the number of hidden nodes should be balanced. If too few hidden nodes are used, training error is high so also the generalization error due to under fitting of the data. On the other hand if the hidden nodes are more, even though the training error achieved is less, high generalization error is a possibility due to over fitting and high variance. Some books and articles offer " rules of thumb" for selecting the number of nodes. The proposed experiment set up considers the following for selecting the number of nodes:

– The number of nodes should be between the input layer size and output layer size.

– Specify as many hidden nodes as needed to capture $70 - 90\%$ of the variance of the input data set.

After considering the above, the number of hidden nodes have been kept between 10 and 100 during experimentation. It is also because the number of hidden nodes directly impacts the computation time required to train the network. Also the number of hidden layers is limited to maximum of two. This is because the generalization of the network is reduced and memorization increases which is not of interest. The transfer function used for both the hidden layer and the output layer is logsig. This adjusts the values that are in the middle and force values which are low to be even lower and values which are high to be even higher. Since we have normalized all the values between 0 and 1 this was well suited for the experiments.

3.4 Performance Parameters

The statistical values for Signal to Noise ratio(SNR), Peak Signal to Noise Ratio(PSNR), Root Mean Square Error(RMSE) and Mean Absolute error(MAE) of images obtained after the restoration were used as parameters to judge the performance of various filters [Hadamard 1923].

4 Results and Discussion

This section discusses the results obtained during experimentation for both gray scale and color image restoration using back propagation neural network with adaptive learning. Various images have been tested and few of the very famous images and their results are discussed. The results obtained after restoring the corrupted image using the proposed approach is compared with other approaches like Mean Filter, Median Filter and Wiener filter.

4.1 Neural Network Training

Once the back propagation neural network is created and the network weights and biases are initialized, the network can be trained. The batch mode or epochs mode of training has been followed throughout the experiments. Incremental mode was not preferred since stability - plasticity dilemma is a major problem. Also training for the problem being considered can be done off-line, batch mode was considered more appropriate. The image data set used for training was divided into three subsets. This was done randomly such that 60% of the data set is assigned to the training set, 20%to the validation set and 20% to the test set. The training set is used for computing the gradient and updating the network weight and biases. The error obtained as a result of validation set data is continuously monitored during the training process. If the error on the validation set rises, it means that the network is overfitting the data. In the design proposed if the validation error increases for five iterations, the training is stopped and the weights and biases at the iteration before the validation error started to rise is reatined. Various other parameters which have been used with their brief description and ranges are given below.

1. Learning rate: The minimum learning rate was set as 0.01 and has been varied till 0.6 for various cases. The ratio to increase learning rate has been varied and tested for various cases from 0.05 to 1.05. The ratio to decrease learning rate was set to 0.7.
2. Maximum number of epochs to train: This represents the maximum number of iterations for training. This parameter was set as 6000.
3. Performance goal: The performance goal has been varied and tested for ranges between 0 and 1E-5.
4. Momentum: Momentum is used since a network can get struck in a local minimum. Momentum helps the network to pass through such minimum. Momentum parameter has been varied from 0.2(less momentum) to 0.9 (more momentum) for various cases.

Table 1 shows the data set used for experiments. Images used were corrupted by Gaussian noise, speckle noise and blur to generate the training set data for the particular image. The training was carried out for different values of parameter discussed above. One such case of training with 55 images of Lena corrupted by various factors and types is presented. Figure 2 shows how the training has evolved over various epochs. Figure 2(a) shows that during epoch 62 the learning rate is 1.0297, gradient is 0.0011866 and Figure 2(h) shows that during epoch 332 the learning rate is 236373.7316, gradient is 2.4259e-006 and the network has converged. The Figure 2(b-f) shows that the learning rate is adaptive and changes depending upon the gradient during the intermediate epochs. There is no error generated when the validation set sample is tested for various epochs. Table 2 gives parameters obtained during training the neural network for peppers image with various test cases. [h] The results obtained after applying test images which were corrupted by different values from that used in the training sets have been discussed next.

4.2 Color images

The performance of the proposed approach for various color images for various types of corrupted images are shown in Figure 3, Figure 4 and Figure 5. The results for the statistical values of SNR, PSNR, RMSE and MAE for Lena Color Image corrupted by Gaussian noise is as shown in Table 3, Table 4, Table 5 and Table 6. Here also the results indicate that our approach outperforms all the other compared approaches.

Though we have got good results for Images corrupted with Gaussian Noise, we would like to present a case were the performance of the neural network based filter will be either constant or may not perform as well as the other filters

Table 1: Data Sets Used in Experiments

Image	**Type**	**Number of Images in Training set**	**Number of Images in Test set**
Peppers	Color	125	25
Cat	Color	75	15
Boat	Color	108	21
Lena	Color	120	24
Mandrill	Color	105	21
Tree	Color	102	20
Jelly Beans	Color	98	19
Splash	Color	95	19
Sailboat on lake	Color	101	20

Table 2: Parameter value obtained during training neural network for Pepper images

No. of images	**Epoch (6000)**	**Time**	**Performance (1.00e-05)**	**Gradient (1.00e-10)**	**Learning rate**
5	352	0:10:15	9.85e-06	6.65e-07	1303841.1359
10	355	0:10:36	9.82e-06	6.33e-07	1437484.8523
20	351	0:09:54	9.85e-06	6.65e-07	1303841.1359
30	357	0:10:04	9.91e-06	6.05e-07	1584827.0497
50	352	0:09:51	9.85e-06	6.65e-07	1303841.1359
70	350	0:09:45	9.85e-06	6.65e-07	1303841.1359
100	357	0:10:11	9.82e-06	6.33e-07	1437484.8523
120	354	0:10:15	9.85e-06	6.65e-07	1303841.1359

Table 3: SNR Results of Various Methods on Lena Color Image Corrupted by Gaussian Noise.

Noise Value	**Noisy Image**	**Mean Filter**	**Median Filter**	**Weiner Filter**	**NN based Approach**
20	12.441	14.185	12.318	14.339	**15.779**
40	12.314	12.764	12.318	16.365	**20.284**
60	14.411	15.826	14.412	15.635	**17.141**
80	10.867	11.078	10.884	15.362	**16.685**

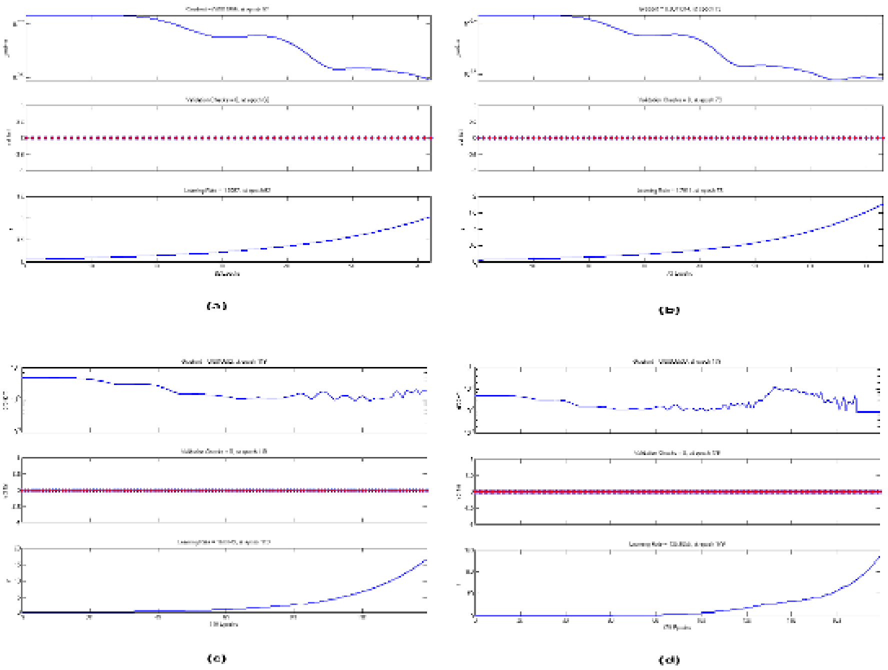

Figure 2: Plot of various parameters during training of Lena Image

Table 4: PSNR Results of Various Methods on Lena Color Image Corrupted by Gaussian Noise.

Noise Value	Noisy Image	Mean Filter	Median Filter	Weiner Filter	NN based Approach
20	19.372	20.116	18.249	20.27	**21.71**
40	18.245	18.695	18.249	21.068	**26.215**
60	20.342	21.757	20.343	21.566	**23.072**
80	17.07	17.28	17.086	21.565	**22.887**

discussed above if the training data supplied to the neural network does not have enough information or if it is not trained with sufficient number of test cases. In the experiment shown below we test the neural network trained which is not trained as good as in the previous cases. In this case we show the results of neural network which is saturated and/or has found a local minima instead of a global minima. As seen from the Figure 10, Figure 11, Figure 12 and Figure 13, the performance of the neural network with insufficient training is not as

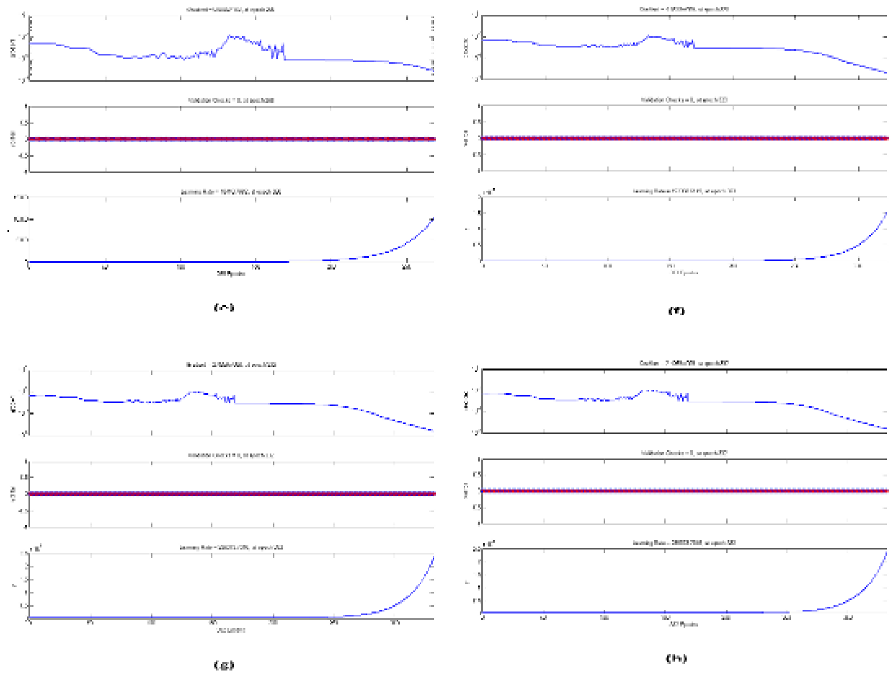

Figure 2(continued)

Table 5: RMSE Results of Various Methods on Lena Color Image Corrupted by Gaussian Noise.

Noise Value	**Noisy Image**	**Mean Filter**	**Median Filter**	**Weiner Filter**	**NN based Approach**
20	21.124	22.2	27.526	21.81	**18.478**
40	27.539	26.147	27.526	18.321	**11.001**
60	21.63	18.379	21.63	18.789	**15.797**
80	35.731	34.876	35.664	21.297	**18.288**

Table 6: MAE Results of Various Methods on Lena Color Image Corrupted by Gaussian Noise.

Noise Value	**Noisy Image**	**Mean Filter**	**Median Filter**	**Weiner Filter**	**NN based Approach**
20	21.219	17.036	22.809	13.189	**11.308**
40	22.783	22.761	22.809	10.806	**8.367**
60	13.811	12.392	13.833	11.709	**11.158**
80	22.883	27.23	27.786	14.101	**12.74**

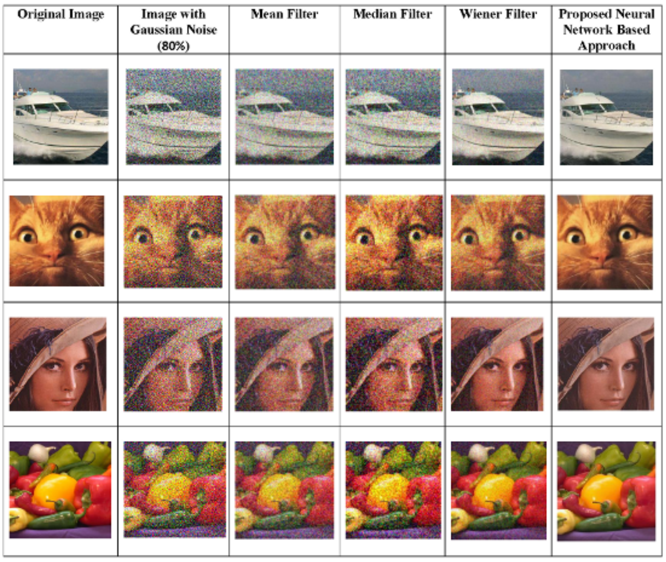

Figure 3: Comparison of the results of proposed approach with other approaches on Color Images corrupted by Gaussian Noise.

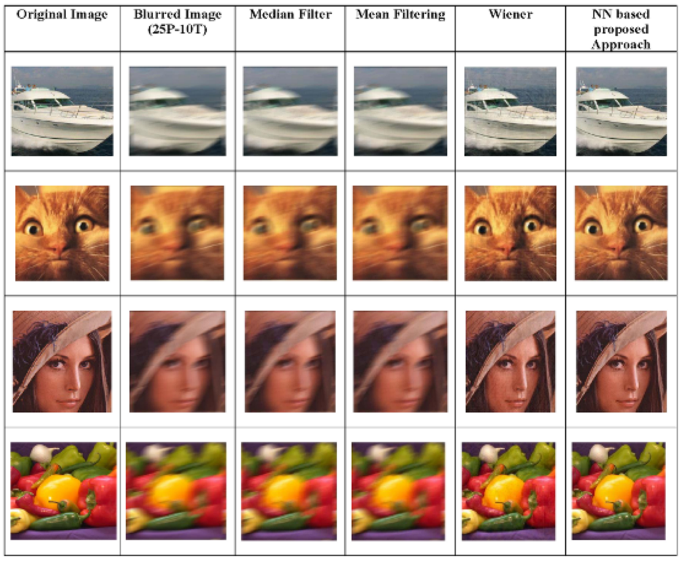

Figure 4: Comparison of the results of proposed approach with other approaches on Blurred Color Images.

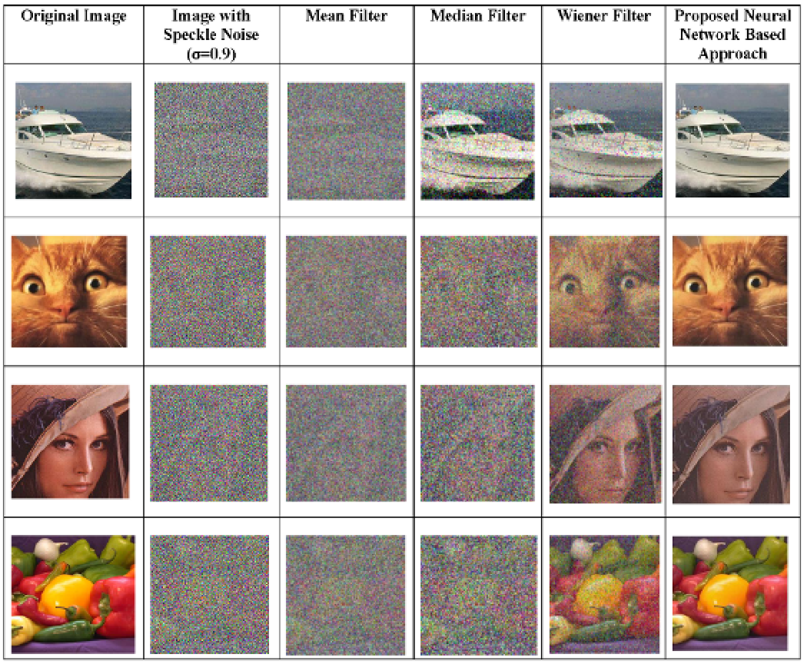

Figure 5: Comparison of the results of proposed approach with other approaches on Color Images corrupted by Speckle Noise.

expected and the same neural network when trained correctly performs well and better as shown in the previous cases.

5 Conclusion and Future work

This paper presents a novel technique for restoring colour images. The results show that the technique restores the blurred/degraded images better than the other methods. It performs extremely well once the neural network is trained properly. However, currently we have developed algorithms that work very effectively with the existing model for test images and other images for solving image inpainting problem. Since the neural network used here is complicated and uses many nodes, one of the promising areas to work in future will be to exploit multicore/manycore for parallelizing the operation and working of the neural network discussed.

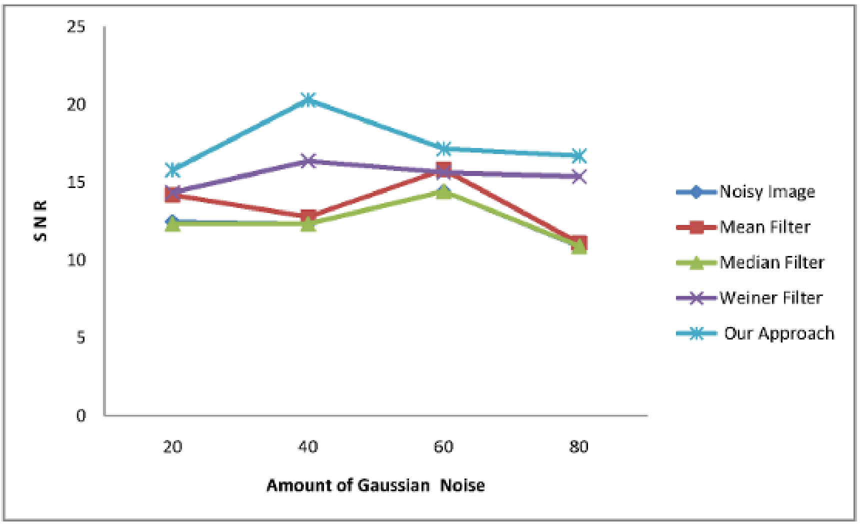

Figure 6: Representation of the SNR result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise.

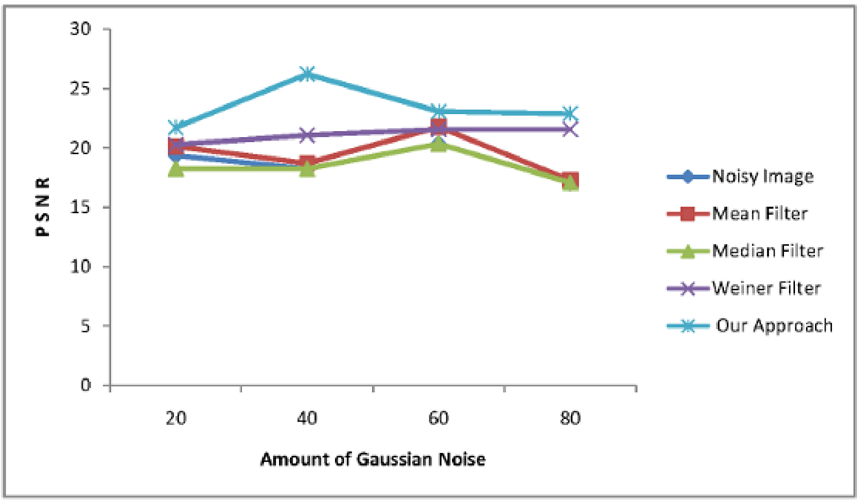

Figure 7: Representation of the PSNR result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise.

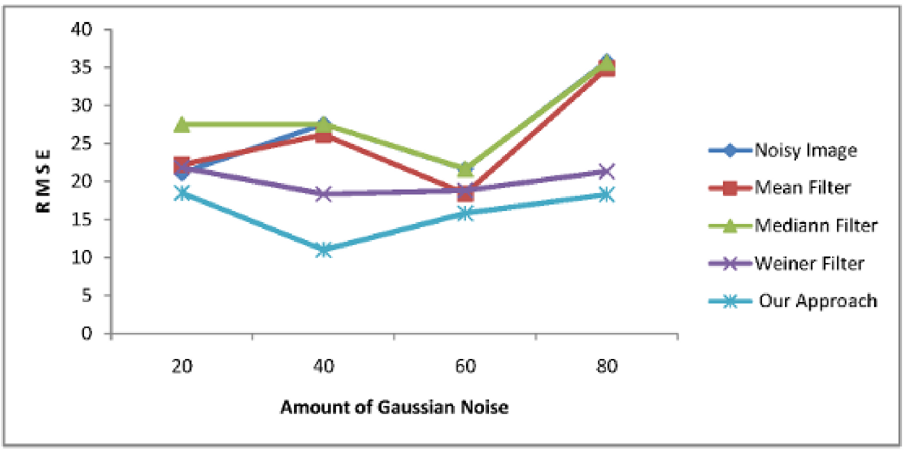

Figure 8: Representation of the RMSE result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise.

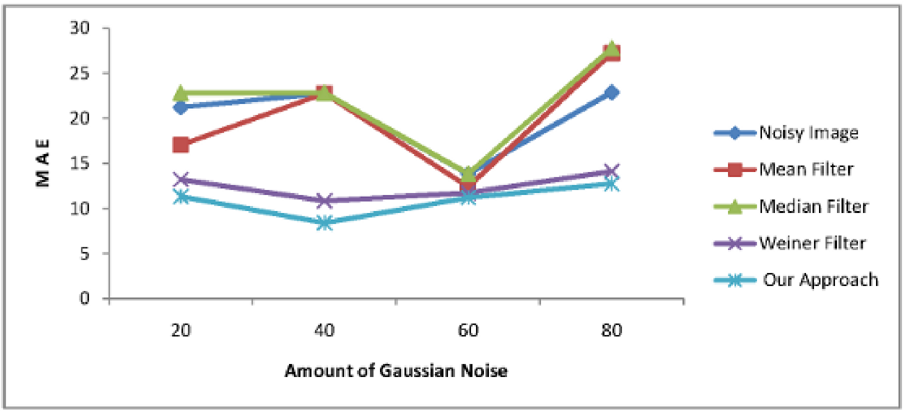

Figure 9: Representation of the MAE result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise.

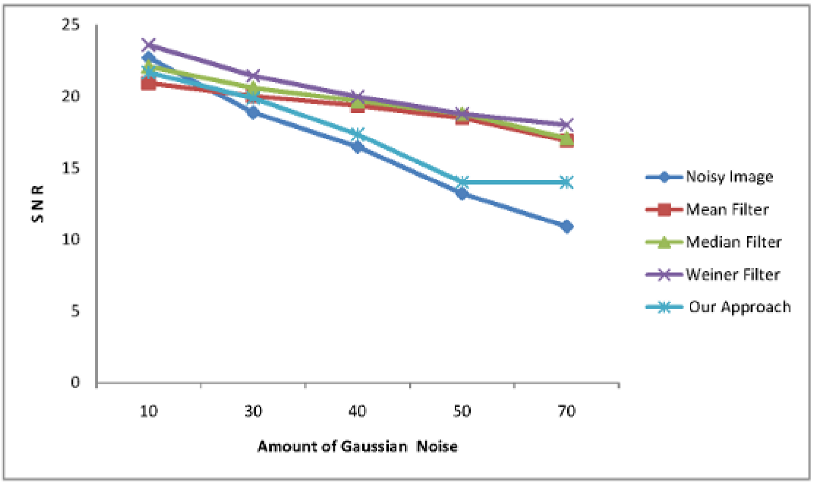

Figure 10: Representation of the SNR result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise when Neural Network is not Trained Properly.

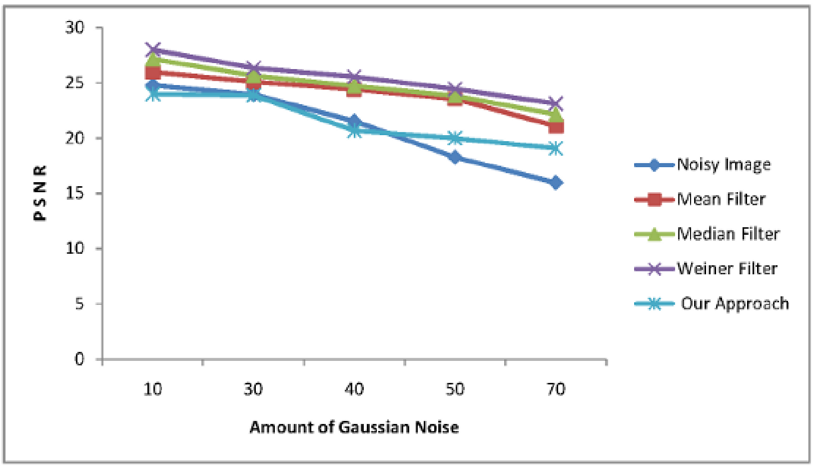

Figure 11: Representation of the PSNR result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise when Neural Network is not Trained Properly.

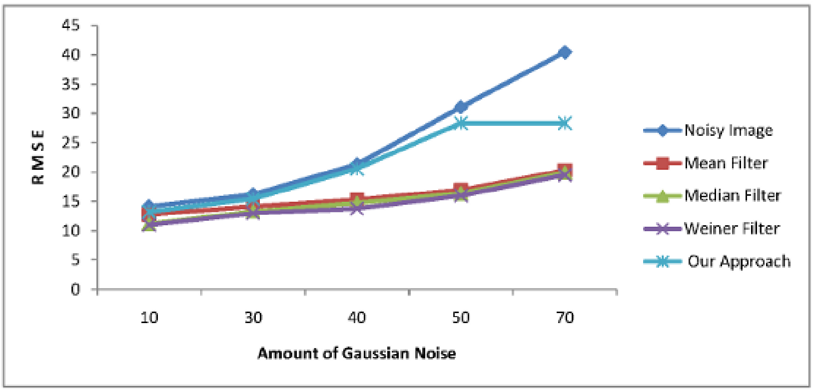

Figure 12: Representation of the RMSE result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise when Neural Network is not Trained Properly.

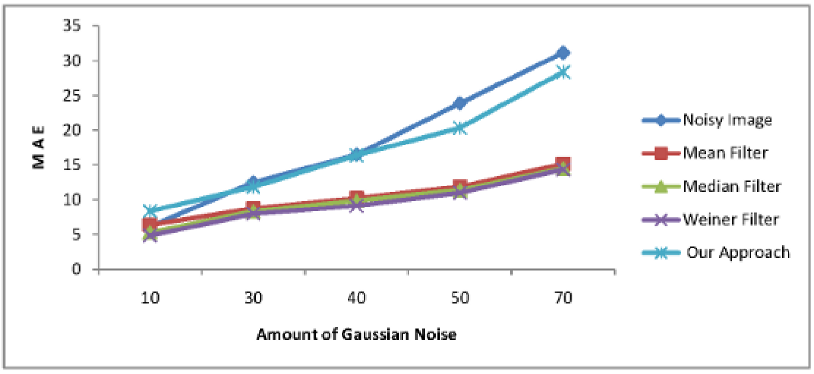

Figure 13: Representation of the MAE result obtained by applying various methods on Lena Color Image corrupted with Gaussian Noise when Neural Network is not Trained Properly.

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