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A Pragmatic Qualitative Approach for Juxtaposing Shapes

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Abstract: This paper presents a qualitative shape description scheme which has been defined in order to have a formal theory to allow the construction of new shapes from a set of given shapes by using a juxtaposition operation. Specifically, the qualitative shape description scheme defined is a pragmatic scheme since it has been defined in order to be applied in the automatic and intelligent assembling of *trencadís* mosaics.

Keywords: Qualitative Shape Description, Shape Description Scheme, *Trencadís*
Categories: I.2.4, I.2.1, I.4.10

1 Introduction

Shape description is now a major field of study in the disciplines of computer vision, robotics, pattern analysis and recognition, computer graphics, image processing and computer-aided design and manufacture.

Although there are studies for describing the shape of an object, shape description is still challenging. This is due to the fact that it is particularly difficult to describe computationally a shape, because when a three dimensional object is projected from the real world onto a two-dimensional image plane, one dimension of the object information is lost. Therefore, the shape extracted from a digital image of the object only represents the projected object partially. Moreover, digital images are often corrupted with noise, defects, arbitrary distortion and occlusion.

Because of the numerical properties of digital images, most of the image processing in computer vision has been carried out by applying mathematical models and other quantitative techniques to describe and identify the shape of the objects contained in an image ([Mehtre, Kankanhalli and Lee, 1997; Rucklidge, 1997; Peura and Iivarinen, 1997; Belongie, Malik, and Puzicha, 2001; Zhang and Lu, 2002; Lu and Sajjanhar, 1999; Morse, 1994]).

Most of these quantitative approaches for shape description have succeeded in the task for which they were designed. However, we have decided to tackle the problem of shape description and identification qualitatively because from a cognitive point of view visual knowledge about shape is qualitative in nature, and therefore we believe that, in this way, we are using a process closer to that which human beings use to describe shapes. Moreover, qualitative approaches describe the shape of objects by focusing only on its relevant information, and therefore they are able to manage uncertainty better than the quantitative ones. They also can speed up the description and recognition processes because they do not rely in complex mathematical functions to describe the objects.

In [Museros and Escrig 04a], a qualitative theory able to describe different types of shapes is presented. It can describe regular and non-regular closed shapes, whose boundaries can be completely straight, or curved, or a mixture of both. These shapes may also contain holes whose shape and location can also be described by our approach. To be precise, it is based on qualitative representations of: angles, types of curvature, length of the edges, convexities, and concavities. This theory has been applied to different domains, as the industrial domain [Museros 06] or the mobile robot navigation domain [Museros and Escrig 04b]. These applications provide evidence for the effectiveness of using qualitative information to describe and identify shapes.

But, given some new applications where for instance it is necessary to create new shapes from two other given shapes, we have found the need of extending the theory developed in [Museros and Escrig 04a]. To create a new shape we need to define the operations of juxtaposition, addition, intersection and difference of shapes. For instance, in order to create a cognitive map of the environment of a robot it is usually necessary the description of a new shape from other given shapes. Another example arises in the industrial domain, as in the case of *trencadís* ceramic mosaics, where it is necessary to create a design with prior unknown shapes.

Trencadís (figure 1) is a type of ceramic artistic mosaic which involves using broken tile shards to create the mosaics. The *trencadís* manufacturing process, accomplished manually, requires extensive labor to align and adjust the pieces of tile during the assembly. Currently, the pieces are barely placed onsite, but a set of predefined patches with different measures (so-called meshes or blankets, typically 0.5 m in size) are prefabricated and afterwards mounted onsite. The unstructured task of producing *trencadís* patches needs intelligent methods and sophisticated perceptual capabilities to be automated. Indeed, human operators apply a whole set of situation-sensitive rules to decide on the next piece to place, its position within the current mosaic border, and how to fit it. Moreover, the way in which the geometry of the piece is perceived is also situation dependent. The key to construct a good *trencadís* is to choose pieces with similar sizes, ordered in a way that they seem a puzzle leaving a space between each piece in order to fix them with cement.

Figure 1: Example of trencadís mosaic at Güell Park (Barcelona)

This paper presents an extension of the qualitative shape description theory in [Museros 06] in order that it will be able to construct new shapes from a set of given shapes by juxtaposing two of them. In fact, the new theory is a qualitative shape description scheme. Moreover, the approach presented here is our first approximation for juxtaposing shapes which will be suitable to be used in the *trencadís* industrial application, because in order to be able to assembly a *trencadís* it is necessary to define how we are going to juxtapose shapes.

The theory presented can be also applied in other applications, such as the self-assembling of structures in nano-technology and robotics.

Next section presents a brief state of the art about shape description schemes, section 3 describes an overview of the qualitative shape description theory presented in [Museros 06]. Then, the extension of this theory in order to be able to juxtapose two shapes in a suitable way to apply it on the *trencadís* assembling problem is described. Finally, our conclusions and future work are explained.

2 State of the art about shape description schemes

A shape description scheme is a *notational system* for expressing the shapes of objects, just as we use notation to express music or electronic circuitry. In the study of shape description schemes we can observe two trends:

- The *pure approach* in which the study of the subject is regarded as a self-sufficient exercise in pure thought. In this approach, the studies deal primarily with the question of "what" rather than "how". The studies are often very rigorous and logically well-connected with each other, but cannot be readily applied to solve practical problems.
- The *pragmatic approach* in which a method or scheme is conceived as a response to the needs of a particular problem (an application in the real world). In this approach, the studies deal primarily with the question of "how" rather than "what". The studies are useful for the purpose of practical application, but are highly intuitive and, therefore they often lack rigor.

Therefore, as the pragmatic approaches are *purpose-oriented*, the choice of a description scheme must be guided by the problem context.

In literature we can find shape descriptions schemes which have been used with much success to study and represent past and contemporary architectural and other designs [Ahmad and Chase 04; Andaroodi et al. 06; Chase 89; Knight 94; Lebigre 01; Stiny 80a], and they have been used also into the education and practice field [Halatsch et al. 08; Stiny 80b; Wang and Duarte 98].

In this paper we develop a new qualitative shape description scheme which, to the best of our knowledge, is the first pragmatic and qualitative approach which is able to juxtapose shapes in order to solve the problem of automatically assembling a *trencadís* mosaic. Therefore, it is not possible to carry out any comparative study with our approach.

3 Overview of the qualitative shape description theory

To make this paper self-contained we now explain the qualitative shape description theory presented in [Museros 06], where a fuller explanation can be found.

Shape description using reference-points information will have to make use of some reference points. As reference points we understand these points which completely specify the boundary. For polygonal boundaries we have chosen the vertices as reference points. For circular shapes and curvilinear segments in a shape we have chosen three points: the starting and the end point of the curve and its point of maximum curvature.

The qualitative description of a reference point, named *j*, is determined using the previous reference point, named *i*, and following reference point, named *k*. The order of the reference points is given by the natural cyclic order of the vertices of closed objects. We only have to determine the sense in which we visit or describe each reference point, which should be the same for the description of all the objects. We have chosen to visit the vertices in a counter-clock sense. The description of each reference point is given by a set of three elements (triple) which can differ if these elements are from straight segments of curvilinear ones:

- In the case of straight segments the triple is $<A_j,C_j,L_j>$, where A_j means the angle for the reference point j, C_j means the type of convexity of point j and L_j means the compared relative length of the edges associated to reference point j (edge formed by vertices i and j versus edge formed by vertices j and k), where:

  $A_j \in$ {right-angle, acute, obtuse};
  $C_j \in$ {convex, concave} and
  L_j belongs to the compared Length Reference System (LRS), where LRS = {smaller, equal, bigger}. The suitable label is associated to each reference point or vertex by comparing the Euclidean distance of the edges sharing the vertex.

- In the case of curvilinear segments the triple is $<Curve,C_j,TC_j>$, where the symbol Curve means that the node in the description string is describing a curve, C_j means the type of convexity of point j and TC_j means the type of curvature of the curve associated to the point j, where:

  $C_j \in$ {convex, concave} and
  $TC_j \in$ {plane, semicircle, acute}

To describe the objects with holes the topological concept of Completely Inside Inverse (CIi) is used [Isli, Museros et al. 00], due to the fact that the hole is always Completely Inside (CI) the boundary of the closed objects.

The cardinal reference system by Frank [Frank 91] is used in order to relate the orientation of the hole inside the object. Determining the orientation of each hole inside the container is necessary because we can have an object with a hole which the boundaries of containers are equal and boundaries of the holes too, but the hole is placed in other position of the container and then they are not the same object. The orientation is fixed using Frank's Cardinal Reference System (CRF). The CRF is defined by placing its origin into the centroid of the object.

Then once the CRF is placed in the centroid of the object the orientation of the hole with respect to the container is calculated, for instance figure 2 calculates the orientation of the hole with respect to the container, obtaining that the hole is [NE,E,SE] oriented inside the container. We call centre (C) to the orientation that occurs when the hole is placed around the centroid, and all orientations hold.

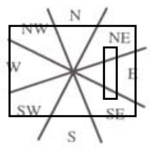


Figure 2: Example of the Orientation Calculation of a hole with respect to its container in which we observe the CRF placed in its centroid.

Color is also stored in the description as RGB coordinates for later comparisons when matching objects.

Therefore, the complete description of a shape of a 2D object is defined by the following tuple:

[type_hole, type_curve, [Color, [A_1, C_1, L_1 | Curve, C_1, TC_1] … [A_n, C_n, L_n | Curve, C_n, TC_n]], CIi, Orientation, [type_curve, [AH_1, CH_1, LH_1 | Curve, CH_1, TCH_1] … [AH_j, CH_j, LH_j | Curve, CH_j, TCH_j]],

where *n* is the number of vertices (reference points) of the container and *j* is the number of vertices (reference points) of the hole. The *type_hole* symbol may have one of the following values *[without holes, with holes]*, and the symbol *type_curve* may adopt one of the following values *[without curves, with curves, only curves]*; both symbols are introduced to accelerate the correspondence or recognition process. Color is the RGB color of the figure described by the three basic components, the Red, Green, and Blue coordinates. Each vector, [A_i, C_i, L_i | Curve, C_i, TC_i], represents a qualitative description node, which may be the description of a vertex of a straight line segment, then taking on the form [A_i,C_i,L_i], with $i=1,..,n$, where A_1, …, A_n, are the qualitative angles of each vertex; C_1, …, C_n are the types of convexity of each vertex and L_1, …, L_n are the relative lengths of each pair of adjacent edges to each of the vertices of the straight line segments of the container; or second tuple [Curve, C_i, TC_i] which represents the qualitative description of a curvilinear segment and is formed first by the *Curve* symbol to indicate that this is a curvilinear segment and then by the labels C_1, …, C_n, and TC_1, …, TC_n, which are the qualitative description of the type of convexity and the type of curvature, respectively. The same occurs with the hole container: AH_1, …, AH_j, CH_1, …, CH_j and LH_1, …LH_j, are the qualitative angle, type of convexity, and relative length of the adjacent edges of each of the vertices of the straight line segments of the hole and CH_1, …, CH_j, and TCH_1, …, TCH_j are the type of convexity and type of curvature of the curvilinear segments of the hole. *CIi* is the topological relation that relates the hole to its container, as already explained in the foregoing section. Finally, the *Orientation* is one or a set of orientation relations given by the CRF, in order to provide the relative position of the hole in regard to its container. CIi, the Orientation, and the description of the hole boundaries only appear when the object is of the type with holes and will appear as many times as is the number of holes in the figure

Figure 3 shows an example of the complete description of a shape with a hole, rectilinear and curvilinear segments and its qualitative shape description, formally named QualShape(S), being S the reference to the object described. Thus, the complete description of the shape in Figure 3 is:

QualShape(S)=[with-holes, with-curves, [[0,0,0], [right-angle, convex, bigger], [curve, convex, acute], [right-angle, convex, bigger], [right-angle, convex, smaller], [right-angle, convex, bigger]], CIi, C,[[right-angle, convex, smaller], [right-angle, convex, bigger],[right-angle, convex, smaller], [right-angle, convex, bigger]]].

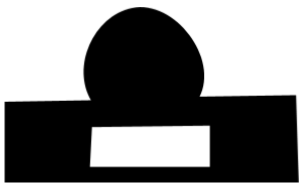


Figure 3: Example of the Orientation Calculation of a hole with respect to its container in which we observe the CRF placed in its centroid.

It is worth noting that an advantage of the theory is its lineal computational cost. The temporal computational cost for the worst case need for the construction of a qualitative description of the figures is of the order O(N+M*K), where N is the number of vertices of the container, M is the number of vertices of the holes and K is the number of holes ([Museros 06]).

4 The pragmatic qualitative scheme for juxtaposing shapes

For extending the theory presented previously we define a qualitative shape description scheme. A shape description scheme is a notational system for expressing the shape of objects.

Our approach uses the definition of shape description scheme given in [Ghosh and Deguchi 08] in order to create a qualitative shape description scheme. A shape description scheme is defined by a 4-tuple as follows:

$$(P, *, C, A),$$

where:

- P is a set of *primitive shapes*,
- * is a set of functions/operations, called *shape operators*,
- C is a set of *production rules*, which specifies how the shape operators are to be used to construct new shapes from the already existing shapes,
- And A is a set of *explicit axioms*, which specify conditions that each constructed shape must satisfy. In a sense, A is a set of *constraints* or *restrictions*. In a shape description scheme, the set A may or may not be present.

Figure 4 shows a graphical example of the process of juxtaposing two shapes.

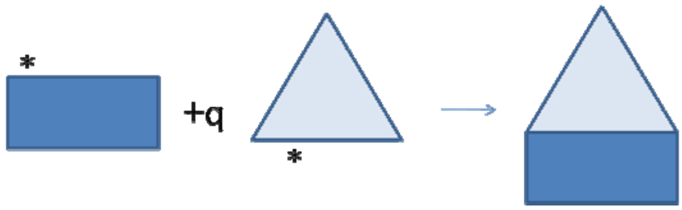


Figure 4: Graphic example of the juxtaposing operation between two shapes. The symbol "" in figures specifies the edges that should be considered during the juxtaposing operation.*

Next we define the 4-tuple for our qualitative shape description scheme. In section 4.1 the set of primitives is defined, section 4.2 establishes the shape operators, and in sections 4.3 and 4.4 the set of production rules and explicit axioms are respectively described. Finally, in section 4.5 an example of the application of the new qualitative shape scheme to juxtapose two shapes is presented and section 4.5 characterizes this scheme.

4.1 The set of primitives *P*

The set of primitives *P* is the set of the regular and non-regular polygonal shapes described by the qualitative shape description theory presented in section 3. Therefore the shapes to be juxtaposed can contain convex and concave segments.

In this approach we have not considered shapes with holes or shapes with curvilinear segments due to the application to which it is oriented. In fact, when assembling a *trencadís* mosaic, it is not lightly to have broken tiles with holes or with curvilinear segments.

4.2 The shape operators *

The set * contains the qualitative juxtaposition operator, named $+_q$. In order to juxtapose two shapes it is necessary to indicate the related edges in $+_q$. For instance if we want to juxtapose shape in Figure 5a) with shape in Figure 5b) we have to indicate that the juxtaposition has to be done considering the edge going from vertex 2 to 3 in

Figure 5a) and the edge going from vertex 4 to 1 in Figure 5b). Therefore we have defined next notation for the qualitative juxtaposition operation:

$$\mathbf{A(i) +_q B(m)} \quad (1)$$

Where A, B are shapes to be juxtaposed, and *i* and *m* indicates the first vertex of the edge (in a counter-clock sense) to be considered in the juxtaposition operation. For instance the result of A(2) $+_q$ B(4) is graphically shown in figure 6.

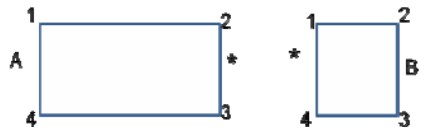


Figure 5: a) First operand in an adding operation, b) second operand.

Figure 6: Shape C, resulting of A(2) +q B(4), given figures A and B in figure 5.

The operator $+_q$ accomplish the commutative law. In fact A(2) $+_q$ B(4) = B(4) $+_q$ A(2).

Moreover, for $+_q$ the *identity element* is the empty set, which means that there is no shape to juxtapose.

4.3 The set of production rules *C*

The set of production rules *C* describes how to apply the juxtaposition operator. This set is defined by the following Extended Backus-Naur Form (EBNF) production rules:

Region_i::= Vertex+ (minimum 3 vertices)
Vertex::= <A, C, L>
A ::= *obtuse* | *right* | *acute*
C ::= *concave* | *convex*
L ::= *smaller* | *equal* | *bigger*

Then, when computing the $+_q$ operation of 2 vertices we obtain:

$<A_1,C_1,L_1> +q <A_2,C_2,L_2> = <A_3, C_3, L_3>$ where:
$A_3 \subseteq$ {obtuse, right-angle, acute, $\varnothing$}
$C_3 \subseteq$ {convex, concave, $\varnothing$}
$L_3 \subseteq$ {smaller, equal, bigger, $\varnothing$}.

When juxtaposing two vertices it can happen that in the resulting shape these vertices disappear (figure 6 is an example, where vertices 2 and 3 in figure A and 1 and 4 in figure B disappear). If this happens then the symbol ∅ is used.

To compute A_3, C_3, and L_3 we have to define three tables. The first table (table 1) is applied to calculate the angle A_3 knowing the angle A_1 and A_2. For instance, if A_1 is acute and A_2 is right, using table 1 we can determine that the new angle A_3 will be obtuse. In the case that there are several possibilities when adding A_1 and A_2 then the set of possible angles is given.

A₂ / **A₁**	**Acute**	**Right**	**Obtuse**
Acute	{Acute, Right, Obtuse}	Obtuse	Obtuse
Right	Obtuse	Obtuse	Obtuse
Obtuse	Obtuse	Obtuse	Obtuse

Table 1: +q angle table.

Table 2 is used to calculate the new convexity C_3. Convexity information depends on the angle information too, therefore table 2 computes C_3 considering the convexity of C_1 and C_2 and angles A_1, and A_2. Note that it is not possible to create a new shape by adding two concave vertices.

Table 3 is used to calculate the compared length L_3 of the new shape. To calculate this table, following the constraints defined in set A, we have considered that the edges being juxtaposed have approximately the same length.

Using this tables, a region R_3 is the $+_q$ of regions $R_1(i)$ and $R_2(j)$ if:

- It has a minimum of $|R_1| + |R_2|$ -4 vertices, and a maximum of $|R_1| + |R_2|$ -2, being $|R_1|$ the number of vertices in R_1 and $|R_2|$ the number of vertices in R_2;
- Each vertex v_i in R_3 must be:
  - Equal to one of the original vertices in R_1 and R_2,
  - or the result of applying the $+_q$ operation to one vertex of R_1 and another of R_2

C₂ / **C₁**	**Concave (any angle)**	**Convex + Acute Angle**	**Convex + Right Angle**	**Convex + Obtuse Angle**
Concave (any angle)	Not Possible	Concave	Concave	Concave
Convex + Acute Angle	Concave	Convex	Convex	{Convex, ∅}
Convex + Right Angle	Concave	Convex	∅	Concave
Convex + Obtuse Angle	Concave	{Convex, ∅}	Concave	Concave

Table 2: +q convexity table.

L_2 / L_1	**Smaller**	**Equal**	**Bigger**
Smaller	Smaller	Smaller	{Smaller, Equal, Bigger}
Equal	Smaller	Equal	Bigger
Bigger	{Smaller, Equal, Bigger}	Bigger	Bigger

Table 3: +q compared length table.

4.4 The set of explicit axioms *A*

The last element to define in the 4-tuple for our qualitative shape description scheme is **the set of explicit axioms *A*.** *A* specifies the restrictions or constraints that must be satisfied in order to apply the $+_q$ operator to obtain a correct new shape considering that we are developing the scheme to solve the *trencadís* automatic assembling problem. Therefore, the restrictions defined, some of them mentioned previously, are:

- It is not possible to overlap shapes when computing the $+_q$. In fact when assembling the broken tiles in a *trencadís* mosaic it is not possible to overlap them. If the edges involved in the operation will return an overlapping shape one of the figures would be rotated before applying the +q operation (figure 7). To ensure this we have defined two restrictions:
    - The area of the final figure constructed by the juxtaposition has to be the addition of the areas of the two basic areas considered in the operation.
    - The number of vertices of the final figure constructed by the juxtaposition is between *n+m-4* and *n+m-2*, where *n* is the number of vertices of the first basic figure juxtaposed and *m* the number of vertices of the second basic figure.
- The shapes considered as operands are only simply connected and closed 2D regions.
- In $+_q$ it is necessary to specify the edges involved in the operation, and these edges should satisfy that they have similar lengths, formally:

$$\textbf{Length(edge}_1\textbf{)} \approx \textbf{Length(edge}_2\textbf{)}$$

- Having figure F_1 and F_2, the vertices in figure F_3, resulting of juxtaposing F_1 and F_2 (F_3= F_1 +q F_2), are defined as follows:
    - Starting by vertex 1 in F_1, we copy it in the new figure, and repeat this step with the next vertex in a counter-clock sense, up to the vertex which is one of the vertices in the $+_q$ operation.
    - This vertex is replaced by the result of applying $+_q$ between this vertex and the corresponding one in figure F_2.
    - We continue copying the vertices in figure F_2 (in a counter-clock sense) up the next vertex in figure F_2 related with the $+_q$ operation. This vertex is replaced by the $+_q$ of this vertex and its corresponding one in figure F_1.

- If there are still vertices in figure F_1 which has not been considered during the juxtaposition, they are copied to the new figure F_3 too.

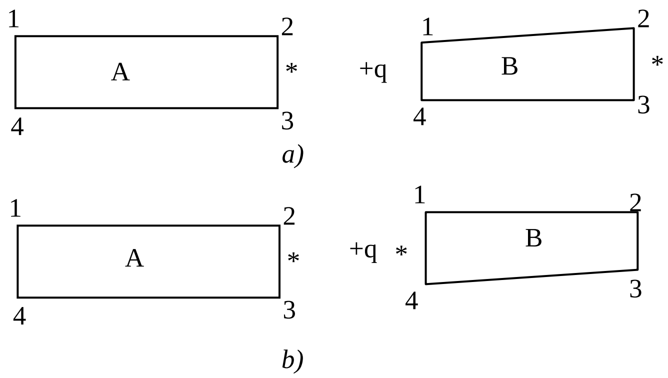


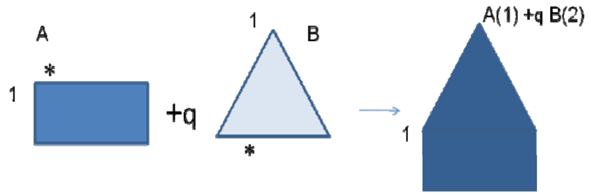
Figure 7: If we want to juxtapose figures A(2) +q B(2) as they are represented in a) the figures will overlap, therefore before calculating this operation figure B has to be rotated as it is shown in b), and we will calculate the operation A(2) +q B(4).

4.5 Describing a juxtaposition by our qualitative description scheme

This section shows an example of the qualitative juxtaposition ($+_q$) of two figures (Figure 8). It shows the graphic result and its qualitative description obtained using the qualitative shape description scheme presented in this paper.

The shape description scheme has been implemented in an application where, given two images, first we calculate the qualitative shape description of the shape in each image and then, the qualitative shape description of the shape obtained by the juxtaposition of both shapes.

The computational cost of this application is still lineal, and basically it is the cost of constructing the qualitative description of the figure. Specifically as we are considering only closed polygonal shapes the cost is of the order *O(N)*, where *N* is the number of vertices of the polygon. The calculus of the juxtaposing operation does not add any relevant computational cost, since it is only applied in the two vertices related with the juxtaposition. For each vertex of the basic figures, it is copied in the final description of the juxtaposed shape or it is substituted by the juxtaposing operation of this vertex and its corresponding one in the other basic figure using the composition tables presented in this paper.



QualShape(A)= { <right, convex, smaller>, <right ,convex, bigger>, <right, convex, smaller>, <right, convex, bigger>}

QualShape(B)= {<acute, convex, equal>, <acute, convex, equal>, <acute, convex, equal>}

A(1) $+_q$ **B(2)** = {<obtuse, convex, smaller>, <acute, convex, equal>, <obtuse, convex, bigger>, <right, convex, smaller>, <right, convex, bigger>}

Figure 8: Example of qualitatively juxtaposition A and B, showing the graphical result and the qualitative shape description of the new shape obtained, which has been constructed using the qualitative shape description scheme presented.

4.6 Characterization of our qualitative description scheme

Once we have defined the qualitative shape description scheme it is necessary to characterize it. It means to answer to next questions:

- **Description domain.** For what class of shapes is the scheme designed? The *description domain* in our qualitative description scheme has been defined by specifying that it is able to describe regular and non-regular polygonal shapes (closed shapes), without holes, and without curves. Moreover the shape can contain convex and concave parts.
- **Uniqueness in description.** In this system, is there more than one way to describe the same shape? As in the defined +q operation we have to determine the edges involved in the operation when juxtaposing two shapes by the same edges. Therefore, there is not more than one way to describe the same shape.
- **Geometric and topological properties.** What are the (simple) geometric and topological properties of the objects that are being constructed? Our qualitative shape scheme is able to describe shapes which are only simply connected shapes; therefore its *geometric and topological properties* have been also defined.
- The **physical validity** of the shapes. Is every shape constructed by the system physically valid? As it has been defined the set of production rules C, the resulting shape of a juxtaposing operation will be a regular and non-regular polygonal shape (closed shapes), without holes, and without curves. Therefore it will be a *physically valid* shape.
- If the scheme is a pragmatic approach: **Areas of application.** What are the most suitable areas of application for the system? The scheme is a **pragmatic approach** which tries to calculate the qualitative juxtaposition of two qualitative

descriptions of shapes in order to be applied during the *trencadís* mosaic intelligent and automatic assembling. Therefore, the main **area of application** has been defined.

5 Conclusions and future work

We have defined a pragmatic qualitative shape description scheme for the juxtaposition of two polygons, which cannot overlap. Its description domain, uniqueness in description, geometric and topological properties and physical validity are described.

Currently, it is being implemented in order to show experimental results which will exemplify the validity of the defined qualitative shape description scheme composed only by the qualitative juxtaposition operation.

In the near future we want to extend this scheme by defining also the intersection and difference operations.

On the other hand, in order to be able to apply this scheme to other domains, we want also to extend this scheme by defining it considering holes and curves in the original shapes. Nowadays, we are working in its extension considering shapes with holes, where the qualitative description of each hole will be the same in the resulting shape, as the containers cannot overlap. However, the new orientation of each hole in the resulting shape has to be calculated considering the original positions of the holes and the relative orientation of the edges considered for juxtapose the two original shapes.

Moreover, currently we are working on applying the scheme to the intelligent assembly of *trencadís* mosaics joining this approach to a multiagent system in order to decide the assembling strategy.

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References

[Ahmad and Chase 04] S. Ahmad, S. Chase, in: Design Generation of the Central Asian Cara-vanserai, First ASCAAD International Conference, e-Design in Architecture KFUPM, Dhahran, Saudi Arabia, 2004, pp. 43–58.

[Andaroodi et al. 06] E. Andaroodi, F. Andres, A. Einifar, P. Lebigre, N. Kandoa Ontology-based shape-grammar scheme for classification of caravanserais: a specific corpus of Iranian Safavid and Ghajar open, on-route samples, *Journal of Cultural Heritage 7* (2006) 312–328.

[Belongie, Malik, and Puzicha, 2001] S. Belongie, J. Malik, J. Puzicha, Matching shapes. In: *8th IEEE International Conference on Computer Vision (ICCV2001),* Vol. I, pp. 454–461, Vancouver, Canada, July, 2001.

[Chase 89] Chase S C, 1989, “Shapes and shape grammars: from mathematical model to computer implementation”, Environment and Planning B: Planning and Design, 16:215-242.

[Frank 91] Frank, A.U., Qualitative Spatial Reasoning with Cardinal Directions, in *Proc. of the Seventh Austrian Conference on Artificial Intelligence,* Wien, Springer, Berlin, pp. 157-167, 1991.

[Freksa 91] Freksa, C. Qualitative spatial reasoning. *In Mark, D. M. and Frank, A. U., editors, Cognitive and Linguistic Aspects of Geographic Space,* NATO Advanced Studies Institute, pages 361–372. Kluwer, Dordrecht, 1991

[Ghosh and Deguchi 08] Ghosh, P.J., and Deguchi, K. Mathematics of Shape Description: A Morphological Approach to Image Processing and Computer Graphics. Wiley ed. ISBN: 978-0-470-82307-1, May 2008.

[Halatsch et al. 08] Halatsch, J., Antje, K. , and Schmitt, G. Using Shape Grammars for Master Planning. In *Design Computing and Cognition '08, Proceedings of the Third International Conference on Design Computing and Cognition* (John S. Gero and Ashok K. Goel eds), ISBN978-1-4020-8727-1, Springer Netherlands, pp. 655-673, 2008.

[Isli, Museros et al. 00] Isli, A., Museros, L., Barkowsky, T. & Moratz, R: A topological calculus for cartographic entities. In C. Freksa, W. Brauer, C. Habel & K. Wender, eds, *Spatial Cognition II- Integrating Abstract Theories, Empirical Studies, Formal Methods, and Practical Applications*,Springer, Berlin, pp. 225–238. (2000).

[Knight 94] Knight T W, 1994, “Shape grammars and color grammars in design” *Environment and Planning B: Planning and Design 21(6) 705 – 735*

[Lebigre 01] P. Lebigre, Towards an inventory on internet and the creation of a digital network of silk roads countries, in: K. Ono (Ed.), *Proceedings of the 2001 Tokyo Symposium for Digital Silk Roads, National Institute of Informatics*, Tokyo, ISBN: 4-86049-007-X.

[Lu, Sajjanhar, 1999] G.J. Lu, A. Sajjanhar, Region-based shape representation and similarity measure suitable for content-based image retrieval. In: *Multimedia Syst*. No. 7 (2) (1999) 165–174.

[Mehtre, Kankanhalli and Lee, 1997] B.M. Mehtre, M.S. Kankanhalli, W.F. Lee. Shape Measures for Content Based Image Retrieval: A comparison. In: *Information Processing and Management,* Vol. 33, No. 3, pp. 319-337, 1997.

[Morse, 1994] B.S. Morse, Computation of object cores from grey-level images. In: Ph.D. Thesis, University of North Carolina at Chapel Hill, 1994.

[Museros and Escrig 04a] Museros L., Escrig M. T., “A Qualitative Theory for Shape Representation and Matching for Design*”, Proceedings ECAI 2004*, IOS Press ISSN 0922-6389, pgs. 858-862, 2004.

[Museros and Escrig 04b] Museros L., Escrig M. T., “A Qualitative Theory for Shape Matching applied to Autonomous Robot Navigation.”, *Frontiers in Artificial*

Intelligence and Applications. Recent Advances in Artificial Intelligence Research and Development. ISBN 1 58603 466 9, 2004.

[Museros 06] Museros, L. Qualitative Theories on Shape Representation and Movement. Application to Industrial Manufacturing and Robotics. In: Ph.D. Thesis, University Jaume I, Castellón, Spain, 2006.

[Peura and Iivarinen, 1997] M. Peura, J. Iivarinen, Efficiency of simple shape descriptors. In: *3rd International Workshop on Visual Form*, pp. 443–451, Capri, Italy, May, 1997.

[Stiny 80a] Stiny, G. Introduction to Shape and Shape Grammars. *Environment and Planning B: Planning and Design*, *7: 343-351.*

[Stiny 80b] Stiny, G. Kindergarten grammars: designing with Froebel's building gifts, *Environment and Planning B 3* (1980): 459-460

[Rucklidge, 1997] W.J. Rucklidge, Efficient locating objects using Hausdorff distance. In: *Int. J. Comput. Vision*, No. 24 (3) (1997) 251–270.

[Wang and Duarte 98] Y. Wang and J. P. Duarte, "Synthesizing 3D forms: shape grammars and rapid prototyping," Workshop on Generative Design, *Artificial Intelligence in Design '98 Conference*, Lisbon, Portugal, 1998, pp. 7-18.

[Zhang and Lu, 2002] D.S. Zhang, G. Lu, A comparative study of Fourier descriptors for shape representation and retrieval. In: *5th Asian Conference on Computer Vision (ACCV02)*, pp. 646–651, Melbourne, Australia, January 22–25, 2002.