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A Graph Model for Spatio-temporal Evolution

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Abstract: Evolving entities in space and time generate complex networks whose structural properties require the development of formal models. The research presented in this paper introduces a graph-based model whose objective is to retain the semantics of these networks. Entities are related at a given time, through space according to the locations they occupy, and across time according to some dependency relations. We propose an approach that characterises these different properties using several graphs, and where emerging properties are analysed at the local and global levels. This allows for a manipulation of these spatial, spatio-temporal, and temporal graphs using neighbourhood, descendant and ancestor operations at the local level. Global properties are studied according to the way two given entities in one of these graphs are related according to the possible routes between them. The principles of the modelling approach are illustrated by a case study of the propagation of brambles.

Key Words: spatio-temporal information theory, graph theory
Category: H.1.1

1 Introduction

The study of the world and interactions of humans and entities in the environment have long been the scope of geographical studies. Early works in time geography have used space-time paths to describe an individual's trajectory or to depict the space-time extent that can be accessed by an individual under certain constraints [Hägerstrand 1970]. With recent advances in Geographical Information Systems (GIS) there has been a renewed interest in the development of spatio-temporal theories for modelling environmental and urban phenomena, changes, events and processes, while studying the past or predicting and forecasting future trends [Frank 1993, Peuquet 2003]. Given the complexity of land phenomena, there is still a need for improved abstraction mechanisms and concepts in order to develop appropriate spatial and temporal models. This is intimately related to, and dependant on how people perceive their environment, and the phenomena considered to be of interest. As Helen Couclelis so elegantly put it: "people cultivate fields but manipulate objects" [Couclelis 1992]. Objects or entities in space are concepts perceived and identified in the environment, and usually modelled using a discrete and relative view of space [Goodchild 1993]. Space can also be considered as a continuous framework where temporal dynamics can be studied using process-based or transition-based models, for example by [Couclelis and Liu 2000].

When considering evolving entities the notions of identity and change have led to the development of numerous conceptual models including [Peuquet 1994, Claramunt and Thériault 1995, Hornsby and Egenhofer 1997, Cheng and Molenaar 1998, Yuan 1998]. Event-based formalisms and languages have been introduced for the representation of changes in space [Worboys 2005]. Qualitative reasoning provides abstract constructs and mechanisms to reason about events and processes [Frank 1994, Galton 2000], thus generating mathematical formalisms to develop theories of change and build computational implementations. Events, processes and changes generate complex networks in space and time that underlie the detailed features needed to record the evolution of geographical systems. The spatial and temporal structures that emerge are relational and topological in nature.

The world can be described as populated by entities which evolve under the action of processes. Modelling the dynamics of the world requires that we model not only the entities themselves but also several relations between them. Three key relations are the spatial (how two entities at the same time are related), the spatio-temporal (how spaces occupied by entities at distinct times are related) and filiation (how entities at distinct times are related by descent or transmission). While relations of each of these types have been studied before, their combination into a single model is necessary in order to fully represent a changing world but does not appear to have been achieved before. In this paper we present a formal model which supports these three relations and we illustrate the capabilities of the model by means of a botanical example.

The research presented in this paper combines these complementary relations into an integrated modelling approach. The network structures that emerge from these spatial, spatio-temporal and filiation relations generate complex networks whose properties are studied at different levels of granularity. The model is general enough to support different classes of relations which can be defined at the application level and for each modelling dimension. The space relations are qualitative and can represent topological relations (e.g. spatial connection), the spatio-temporal relations can be any of the usual topological relation considered over time, and finally filiation relation model the way entities are associated with one sort of dependency relation which is also application dependent. The network structures that emerge can be studied using graph-operators at the local level (e.g. using neighbouring and ancestor/descendent function), or more generally by a study of the way entities are related thoughout the graph. This allows us to suggest a concept of route that represents how chains of arrows in a given relation may be followed, in other words the way a given entity is connected to another in a given network, either spatial, spatio-temporal or filiation-based. The properties of these concepts of route are studied formally. The overall potential of the approach is illustrated by a case study of the propagation of bramble plants.

The remainder of the paper is organised as follows. The next section introduces an overview of related work. The motivation and modelling background of our approach are presented in the following section. The principles of our approach are introduced in the fourth section that introduces the graph-based representation of spatial, spatio-temporal and filiation relations; the notion of route applied to these modelling dimensions is presented and formalised. The modellling appoach is then illustratived by a case study oriented to the propagation of bramble plants. The final section of the paper presents conclusions and outlines further work.

2 Related work

Early works in spatial analysis were mainly oriented to the quantitative cross-comparison of snapshots, that is, successive layers of spatial information commonly compared using quantitative spatial operators [Armstrong 1988] or interpolated using fuzzy approaches [Dragicevic and Marceau 2000]. Such techniques are, however, inappropriate for tracking entity evolutions. The necessary extension of GIS towards the temporal dimension was considered at the conceptual and logical levels in order to provide methodological foundations [Langran 1992, Peuquet and Wentz 1994, Peuquet 1994]. Some early proposals were oriented to the categorisation of spatio-temporal processes and events in space and time [Claramunt and Thériault 1995, Hornsby and Egenhofer 1997]. More recently, there has been a growing interest in the representation of moving points as database objects [Güting and Schneider 2005], and in the study of the evolution of spatial configurations and topological relationships [Kurata 2009].

When studying the evolution of entities in space, the notion of identity is essential as this allows for a distinction between changes that maintain the identity of a given entity, and the ones that generate a change with a loss of identity [Hornsby and Egenhofer 1997]. This permits a clear distinction between the form of evolution and the way entities are spawned over time through continuation and derivation [Hornsby and Egenhofer 2000]. Processes and events can be considered at different levels of granularity in space and time, taking into account different levels of detail when observing evolving entities [Hornsby and Egenhofer 1999]. However, while categorising events and processes in space and time, these representations do not provide any mechanism to represent the intertwined networks that characterise and relate the evolutions of spatial entities. The evolution of spatial entities generate networks of transitions that can be abstracted as spatio-temporal trajectories [Stefanakis 2003]. These modelling concepts are necessary in the study of emerging networks relating entities in space and time [Grenon and Smith 2004].

[Sriti et al. 2005] introduced a conceptual and logical modelling approach where networks are generated by the evolution of spatial entities, and structured and analysed according to a model based on graph theory. However, the properties exhibited by the emerging networks are limited to the co-location of spatial entities in space and time, thus denying the integration of additional thematic relationships over time. [Stell 2003] introduced a spatio-temporal model based on posets providing formal support for reasoning about heterogeneous spatial data sets, whether thematic or spatial. The formal approach in [Stell 2003] used a dependance relation, called the *support* relation, and allowed the semantics to be generic so it might be specialized either to the spatial or the thematic case. Many applications need to distinguish several types of dependancy, making a distinction between filiations and spatial relations over time. Based on the latter, the research introduced in this paper extends this modelling approach, considering different categories of relations over time, either thematic, spatial or a combination of the two. This favours derivation of several complementary levels of network in space and time. Accordingly, the *support* relation is specialised towards three relations: a filiation relation modelled using thematic properties, a spatial relation defined for a given time, and a spatio-temporal relation combination of the two. The interest of the approach relies in the combination of some thematic properties, and the way these properties are distributed in space and time over the networks formed by the evolution of the represented spatial entities, and this according to some application-dependent criteria. The whole modelling approach is completed by the application of graph-based operators applied to the different relations identified.

3 Spatio-temporal model principles

The issues surrounding the ability of entities, such as people or cities, to maintain their identities over time, while not being physically identical are well-known in philosophy. These issues include many famous puzzles, such as the Ship of Theseus, but for our purposes criteria for the continuance of identity are dependent on specific applications and are not determined by our model. The model we present can accommodate different conventions about identity; it provides a filiation relation but does not say how to choose what is related to what. The filiation relation can model one person being the same person at two distinct times, it can also model one or more entities giving rise to new entities (for examples parents of children). It can also handle the entities being crowds of people which split and merge. In this case the conditions under which a crowd maintains its identity would be determined by the needs of the application. Possible criteria might include the proportion of people in the crowd remaining the same, or the presence of key people within the crowd. In the details below we focus on two types of filiation (continuation and derivation), but the relation in the underlying model provides the basis to distinguish other varieties of filiation as necessary.

To model evolving entities requires some notion of time. Any physical observations of the world take place at some granularity which may be based on time instants or time intervals. Our model is neutral about details such as whether time consists of instants or intervals or a combination of both, but we do assume time to consist of discrete elements at which entities may be observed. Even if time is required to be dense (so between any two distinct times a third time distinct from both may be located) an information system can only store data in a finite way. This means the representation of continuously varying entities must ultimately be based on snapshots together with a means of recording the variation between snapshots. Thus our basis of discrete times is grounded in the constraints to which any actual database or information system is subject.

We do not assume any particular model of space, nor do we explicitly model space itself. The entities will occupy space and thus a spatial relation between entities is required. For example, at a given time two houses may be adjacent, a house may be inside a city, or two lakes may be unconnected to each other. The spatial relation in the model can be interpreted in different ways according to different applications. It provides a notion of spatial connection without saying whether this means connection in the sense of being adjacent or overlapping or some other sense. There is an important distinction in the model between the spatial relation, which records how entities at the same time are related spatially, and the spatio-temporal relation, which records how entities at distinct times are related. This can best be understood in terms of the example which we present in Figure 1.

3.1 Informal example

The example in the figure 1 shows entities A, B, C, D, E, F, G at three consecutive times. The circle and the triangle will be used shortly when we discuss the possibility of interpreting the entities as containers, but initially these two

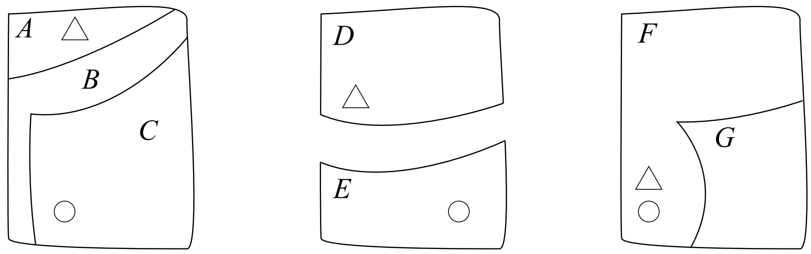


Figure 1: Example illustrating spatial and spatio-temporal relations

symbols may be ignored. At the first time three entities A, B, C are present. Interpreting the spatial relation as one of adjacency, we see that A is spatially related to B and B is spatially related to C. The entities here could be fields which are separated by fences which by the second time are moved into different positions. At this second time we see entities D and E which are not spatially related to each other. The gap between D and E could have appeared because a temporary road is made across the land, and the information system is concerned with modelling fields and not with modelling other types of entities. At the third time all the land is used as two fields F and G, separated by a fence along their common boundary, but still allowing F and G to be spatially related.

The spatio-temporal relation does not have to use the same kind of notion of connection as the spatial one. In the example of Figure 1 where entities arise from division of land we cannot have overlapping entities at a given time, but the locations of fields at two distinct times may overlap. This shows it is important to distinguish the space of a single time from the space of distinct times since the principles of reasoning about these two contexts can be quite different. The spatio-temporal relation ρ_{st} is shown in Figure 2.

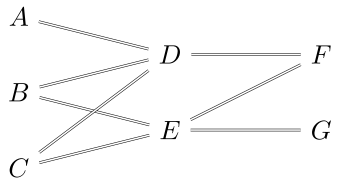


Figure 2: Spatio-Temporal Relation ρ_{st} - example of Figure 1

The spatio-temporal relation ρ_{st} here does not relate D to G even though their locations are spatially adjacent. The ability to have different kinds of relation for the spatial and the spatio-temporal case is not the only reason why we contend that these notions do need to be carefully distinguished. It is also on account of the compositional behaviour of these relations that the difference is important. By this we mean the ability to construct chains of observations such as B is spatio-temporally related to E which is spatio-temporally related to G. Clearly the spatio-temporal relation will not in general be transitive, and the same observation has often been made in the spatial case. However, unlike the spatial case, knowing that we have entities x, y, z where $x \; \rho_{st} \; y$ and $y \; \rho_{st} \; z$ does actually convey useful information relating x to z quite independently of whether or not $x \; \rho_{st} \; z$. This can be justified by considering the possibility that our entities might be containers and that objects within these containers might move over time. This container view is not an essential aspect of our model, but we include it in the current example to stress the distinction between the spatial and the spatio-temporal.

Regarding the entities shown in Figures 1 and 2 as fields, these fields may be inhabited by animals possibly farming stock such as cattle, but also potentially by large wild animals (e.g. deer, wild horses) which are unable to cross the boundaries. In another scenario, our entities might be crowds of people taking part in a political demonstration and boundaries between separate crowds might be enforced by the police. In yet another case we could be modelling entities which are organs in the human body and we could be concerned with certain organisms which might move from one place to another and able to inhabit particular regions. In all of these case we can conceptualize entities as containers, and in Figure 1 we have provided two particular objects one (shown as triangle) in A and one (shown as circle) in C. Now suppose we know the initial location (containing entity) of these objects, and we know the spatio-temporal relation between entities at distinct times. Can we then determine whether the two objects might inhabit the same container at the end? If we assume that objects may move between spatio-temporally related entities then it is possible to derive the possibility, in this particular example, that the objects do end up in the same place (i.e. the same entity). This is indicated schematically in Figure 1.

The formal aspects of our model that permit this kind of reasoning are explained in detail in section below, but we note here how this is dependent on distinguishing the spatial from the spatio-temporal case. One point to note is that movement, or indeed change of any type, requires time to happen. To reason about potential movement necessarily thus requires knowledge about connection at distinct times and not at a single time. Another point, which we alluded to above, is the different compositional behaviour of the two relations. Knowing that $B \; \rho_{st} \; E$ and $E \; \rho_{st} \; G$ tells us that an inhabitant of B could possibly move from B to G, whereas, in this example, knowing that $A \; \rho_s \; B$ and $B \; \rho_s \; C$ does not convey this kind of information at all.

The above example has shown the necessity to distinguish the three relations in order to deduce significant properties of the situation under study. We next introduce the modelling background of our approach, and a graph representation of several levels of relations. These relations are concerned with space, with space and time together, and with time alone.

3.2 The Time Domain

We assume there is a finite set $T = \{t_1, \ldots, t_n\}$ of time points under consideration and that these points are partially ordered. In our examples we use a linear order with $t_i < t_{i+1}$ for $1 < i < n$ but in general the model allows for branching time to accommodate uncertainties about past events or multiple predictions of future developments. Given time points $t_i, t_j \in T$ we use $[t_i, t_j]$ to denote $\{t \in T \mid t_i \leqslant t \leqslant t_j\}$. The structure of time we assume means that we are able to talk about one time being immediately before, or immediately after, another time.

First, let us introduce the basic principles of the initial model of [Stell 2003]. We use X to denote a *dynamic set* on a time domain T. This means that for each time $t \in T$ we have a set $X(t)$ which contains the entities that exist at this time t. For all $t \leqslant u$, there is a transitive binary relation $X(t, u)$ between the sets $X(t)$ and $X(u)$. This relation is defined as follows :

- If $t = u$ then $X(t, t)$ is the identity relation on $X(t)$. We can remark that the spatial entity e exists at time t if and only if $e \in X(t)$.
- If $t < u$ then an entity $a \in X(t)$ is related to an entity $b \in X(u)$ if a exists before b, and if b depends of the existence of a.

This modelling approach is generic, the dependancy relation in the second case above is general and unspecific, we do not know *a priori* the type of dependance, which is application dependant and might be based on biological or environmental considerations for example. When specifically applied to space and time, additional relations can enrich the model. This is the case for instance when there is a need to distinguish several types of dependancy, and in particular filiations and spatial relationships over time. For example, the study of virus transmission requires knowing who is infected and also where the contamination originated. For such sorts of application, several categories of dependancy should be capable of being represented.

Let us now refine the dependancy relation as defined by the modelling approach in order to distinguish spatial and spatio-temporal aspects, while keeping filiation relations.

3.3 Spatial connection

Let us consider a set σ of entities which relate spatially to each other. These spatial relationships might be derived by associating a region in space with every entity at each time, but our model does not require that this is done.

Between entities we take a relation C of connection as fundamental. The semantics of connection will depend on the application and the nature of any underlying space, but C will always be symmetric and reflexive. Connection can be refined into more specialized relations, as for example in the RCC8 classification of [Randell et al. 1992] where there are eight cases once disconnection is admitted as an option. In the following classification, our relation C would include the last seven cases without DC. The spatial connection relation will be indicated visually by a solid line as in Figure 3.

Let x and y denote two entities $\in \sigma$,

x DC y means x is disconnected to y

x EC y means x is externally connected to y

x TPP y means x is a tangential proper part of y

x NTPP y means x is a non-tangential proper part of y

x PO y means x is partially overlaps y

x EQ y means x is equal to y

x TPPi y, means y is a tangential proper part of x

x NTPPi y, means y is a non-tangential proper part of x

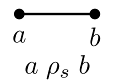


Figure 3: Schematic representation of the spatial relation between a and b

3.4 Neighbourhood functions

Whenever we have a relation R on a set X we can define a function (denoted by the same letter) $R : X \to \mathcal{P}(X)$ by

$$R(x) = \{y \in X \mid x \; R \; y\}$$

for all $x \in X$. Visually $R(x)$ consists of the set of elements of X which are one step away in the relation R. This is illustrated in Figure 4 which also shows the d-neighbourhood function $R^d : X \times \mathbb{N} \to \mathcal{P}(X)$, where R is in this example, considered as a assymetric relation. This function is defined by

$$R^d(x) = \{y \in X \mid x \; R^d \; y\}$$

where R^d on the right hand side denotes the d-fold composition of R with itself. that is, $R^1 = R$ and $R^d = R;R^{d-1}$. Here we are using the semi-colon (;) to denote composition of relations in the diagrammatic order. That is, for relations R and S on the set X, we write $x \; (R \,;S) \; y$ when there is some z for which $x \; R \; z$ and $z \; S \; y$. This order of composition is opposite to that which is commonly written as $R \circ S$ where $x \; (R \circ S) \; y$ when there is some z for which $x \; S \; z$ and $z \; R \; y$. The sense of ; we use is widely employed, see for example [Hirsch and Hodkinson 2002, p3]. It is significant that when R is transitive we have $R \,; R \subseteq R$ so that $y \in R(x)$ if and only if $y \in R^d(x)$ for some $d \geq 1$.

We can apply these functions to the case of the spatial relation ρ_s in our model. For an entity a, the value of $\rho_s(a)$ will be the set of all entities which are spatially related to a. Depending on the choice made for ρ_s in a particular application this set could consist of entities which overlap a, or which are adjacent to a, or which satisfy some particular kind of accessibility constraint such as there being a road from a.

The d-neighbourhood can also be considered for the spatial relation. Because ρ_s is not transitive in general we cannot expect that knowing, for example, that $b \in {\rho_s}^2(a)$ will provide much useful spatial information about the relation of a to b. Knowing that a overlaps c and that c overlaps b allows both the possibility that a overlaps b and that a does not overlap b.

Although the spatial d-neighbourhood function yields little information in general, the situation is quite different with the next relation that we introduce in our model: the spatio-temporal relation ρ_{st}. This difference, as we explain in the next section, is a major factor behind our decision to distinguish between spatial relations at the same time (the spatial relation) and at different times (the spatio-temporal relation).

3.5 Spatio-temporal relation

3.5.1 Definition of the relation

Let $a, b \in \sigma$ and $t, u \in T$ be two times where $t < u$. We say that a spatial entity a valid at time t is in spatio-temporal relation with a spatial entity b at time u, if the place occupied by a at time t is in spatial connection ρ_s with the place occupied by b at time u. We denote this spatio-temporal relation by ρ_{st}. The relation is illustrated in Figure 5.

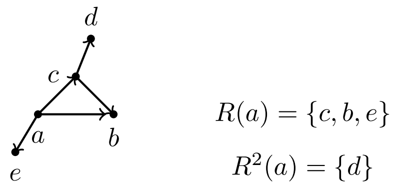


Figure 4: Neighbourhood functions of relation R

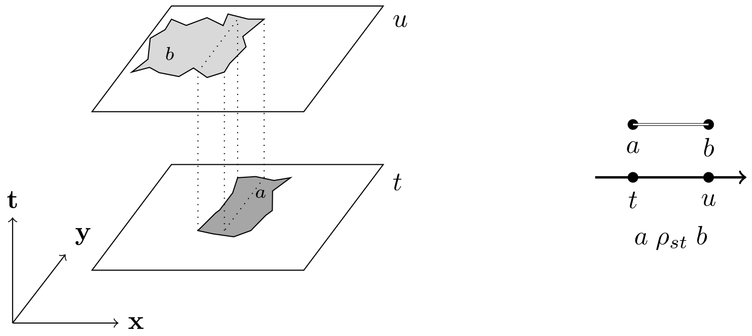


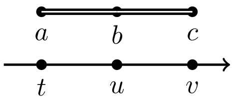
Figure 5: A region a at time t in relation of spatial connection ρ_s with a region b at time u, and (on the right) the type of diagram used to show the spatio-temporal relation in this situation

3.5.2 Spatio-temporal neighbourhoods

The concepts of neighbourhood and d-neighbourhood can be applied to the relation ρ_{st} and this is illustrated in Figure 6. We say that an entity a is a **spatio-temporal parent** of an entity b if $b \in \rho_{st}(a)$. As the spatio-temporal relation is directed in the same way as time, this means that b is valid at the time immediately preceding a and that their locations are related. We can express exactly the same relationship by saying that b is a **spatio-temporal child** of a, which can also be denoted formally as $a \in {\rho_{st}}^{-1}(b)$, where ${\rho_{st}}^{-1}$ is the converse relation to ρ_{st}.

Using d-neighbourhoods in the relation ρ_{st} allows us to obtain two further notions. We say that a is a **spatio-temporal ancestor** of b if for some $d \in \mathbb{N}$ we have $b \in {\rho_{st}}^{d}(a)$. The relation of being a spatio-temporal ancestor in this will will be denoted $b \in {\rho_{st}}^{+}(a)$. We can also express this relationship by saying that a is a **spatio-temporal descendant** of b, for which we use the notation $a \in {\rho_{st}}^{-}(b)$.

Sometimes we need to explicitly refer to the distance between two entities with respect to the relation ρ_{st}. When $b \in {\rho_{st}}^{d}(a)$ we say that b is a d-**step**



$$\rho_{st}{}^{-}(b) = \{a\} \qquad \rho_{st}{}^{-}(c) = \{a, b\}$$
$$\rho_{st}{}^{+}(b) = \{c\} \qquad \rho_{st}{}^{+}(a) = \{b, c\}$$

Figure 6: Spatio-temporal ancestry and spatio-temporal line of descent functions

decendant of a or that a is a d-**step ancestor** of b. The d-neighbourhood $\rho_{st}{}^{d}(a)$ gives the set of entities which are in the relation of spatial connection with the spatial entity a and that are subsequent to a in time, d steps later.

3.5.3 Routes in a relation

The relation ρ_{st} holds when entities at different times are related in the underlying space. If we know that $a \ \rho_{st} \ b$ and $b \ \rho_{st} \ c$ then there is in general no reason to suppose that $a \ \rho_{st} \ c$. However, this should not be taken to mean that nothing useful can be said about the relationship of a to c. To see this let us consider a specific example Figure 7.

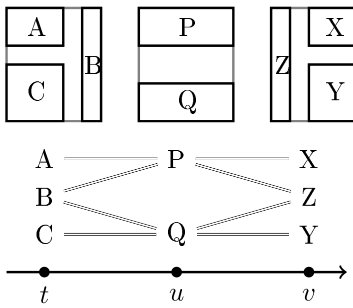


Figure 7: Routes in spatio-temporal relation - Example

The key point to note from Figure 7 is that although B and Z are not related by ρ_{st} they do have a meaningfully different relation to that of A and Y which are also spatio-temporally unrelated. This can be understood by viewing the entities as containers, as in the informal example provided in the Introduction.

The solution is to use what we call the **route extension** of the relation ρ_{st}. That is, we are able to define a relation $\star\rho_{st}$ which records not the relation ρ_{st} itself but the ways in which routes or chains of arrows in the relation may be followed. We shall see that the relation $\star\rho_{st}$ is a five-valued relation. We present the construction for a general relation R, rather than only considering ρ_{st}, since we require the same construction again when we consider filiation later.

Suppose we have a relation R on a set X. For $a \in X$ we introduce the notion of a **route out of** a. This is a sequence $a = a_0, a_1, \ldots, a_n$ of elements of X such that $a_{i-1}\ R\ a_i$ for $i = 1, \ldots, n$. We say that such a route **passes through** b if $b = a_i$ for some i. We can now define define two futher relations $\Box R$ and $\Diamond R$ on X by the following conditions for all $A, B \in X$.

$A \Box R\ B$	iff	every route out of A passes through B.
$A \Diamond R\ B$	iff	there is a route out of A which passes through B.

From these we define the relation $\boxplus R$ on the same set X.

$$A \boxplus R\ B \ \text{ iff } \ (A \Box R\ B) \wedge (A \Diamond R\ B).$$

Note that $\Diamond R$ is just the transitive closure of R. Since $A \Box R\ B$ holds vacuously in the case that there are no routes out of A, we have introduced $\boxplus R$ to express formally the idea that by taking the steps modelled by the relation R we are forced to proceed from A to B. By considering also the relations defined from the converse R^{-1} of R we are able to express additional concepts, which we emphasize have descriptions on the right-hand side below concerning routes in R and not routes in R^{-1}. In these we write $\Box R^{-1}$ to mean $\Box(R^{-1})$, and it is necessary to beware that $(\Box R)^{-1} \neq \Box(R^{-1})$ in general, although $(\Diamond R)^{-1} = \Diamond(R^{-1})$ and $(\boxplus R)^{-1} = \boxplus(R^{-1})$.

$B \Box R^{-1}\ A$	iff	every route (in R) arriving at B comes from A.
$B \Diamond R^{-1}\ A$	iff	there is a route (in R) arriving at B which leaves A.

We now observe that between A and B various situations may arise. For example, it is possible that some route does lead from A to B but that not all the routes leaving A pass through B, while it is true that every route arriving at B does pass through A. An example of this situation is shown in the following diagram, Figure 8.

In this case it holds that $(A \Diamond R\ B) \wedge \neg(A \boxplus R\ B) \wedge (B \boxplus R^{-1}\ A)$. In general we can measure the strength of the connection available using routes in R between A and B by noting which of the four statements $A \Diamond R\ B$, $A \boxplus R\ B$, $B \boxplus R^{-1}\ A$, and $B \Diamond R^{-1}\ A$ is true. These four are not independent of each other, and there are implications as in the following diagram.

$$\begin{tikzcd}
A \boxplus R\ B \arrow[r] \arrow[d] & A \Diamond R\ B \arrow[dl, shift left] \\
B \Diamond R^{-1}\ A \arrow[ur, shift left] & B \boxplus R^{-1}\ A \arrow[l] \arrow[u]
\end{tikzcd}$$

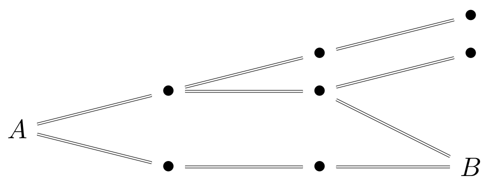
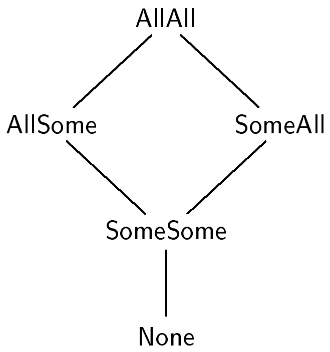


Figure 8: Routes examples

There are five possible subsets of these statements that may hold together, and we thus obtain a five-valued relation from R which we denote by $\star R$. The five values which may occur for $a \star R\ b$ will be called AllAll, SomeAll, AllSome, SomeSome and None. They are arranged in a lattice $\mathscr{L}$ as follows:



We define the relation $\star R$ using these five values, by means of the following table of conditions specifying when $a \star R\ b$ holds.

AllAll	$(a \boxplus R\ b) \wedge (b \boxplus R^{-1}\ a)$
AllSome	$(a \boxplus R\ b) \wedge \neg(b \boxplus R^{-1}\ a)$
SomeAll	$\neg(a \boxplus R\ b) \wedge (b \boxplus R^{-1}\ a)$
SomeSome	$(a \Diamond R\ b) \wedge \neg(a \boxplus R\ b) \wedge \neg(b \boxplus R^{-1}\ a)$
None	$\neg(a \Diamond R\ b)$

An interesting feature of the lattice $\mathscr{L}$, is that if we write $\sqcap$ for the meet or greatest lower bound operation then we have the result that for all $A, B, C \in X$ and all $\ell_1, \ell_2, \ell_3 \in \mathscr{L}$,

$$\text{If } (A\ \ell_1\ B), (B\ \ell_2\ C) \text{ and } (A\ \ell_3\ C) \text{ then } \ell_1 \sqcap \ell_2 \leqslant \ell_3.$$

This compositional result indicates an important distinction between the spatial and the spatio-temporal relations. It shows that if we know how A is related to B in $\star R$ and if we know how B is related to C in $\star R$ then we have some information about the relation between A and C. This is different from the spatial situation, where, for example, if we know A overlaps B and B overlaps C then we have no useful information about the spatial relationship of A to C.

3.6 Modelling filiation

An entity may continue in existence from one time to the next, in this case the identity of the entity is perpetuated. However, identities may come into existence and disappear. When someone eats an apple it is usual to consider that the apple ceases to exist although it can be discussed whether the apple's identity is subsumed by the person's or whether it vanishes completely. Such points are important in philosophy [Williams 1989, Gallois 1998], but for our purposes such issues are a matter of convention that can be made to suit the application. Entities may also derive their existence from other entities, as for example in the case of children from parents. In our model an entity can be related to another at a later time by a relationship of filiation, but the semantics of the relation is not specified. Thus in this paper, if a is related to b by filiation then this means some kind of descent can be discerned from a to b, but we leave open the ability to interpret this in different ways to suit different applications.

A filiation is semantically related to a given application. For example, in a genealogy filiation, a person z who exists at time u is in temporal filiation with two parents x and y which exist at time $t < u$. In this case, the filiation between x, y and z is purely biological. This filiation is written symbolically as $x \; \rho_f \; z$ and $y \; \rho_f \; z$ and in general we adopt the dotted line to indicate this as in Figure 9.

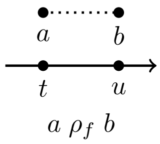
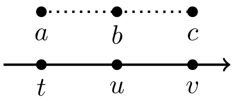


Figure 9: Notation for the filiation relation

The notions of ancestry and descent introduced for earlier relations can be used here too, and a simple example is provided in Figure 10. We have the 1-step ancestor, or parent, function ${\rho_f}^{-1} : \sigma \to \mathcal{P}(\sigma)$ which maps an entity e to the set of entities involved at the preceding timepoint in the formation of e. Similarly, the 1-step descent, or child, function ρ_f maps an entity to the set of entities at the next time step to which it contributes. Each of these can be iterated d steps, and where we leave d unspecified, we have ${\rho_f}^{+}$ taking an entity to all those later entities it engenders, and similarly the converse relation ${\rho_f}^{-}$ takes an entity e to all earlier entities giving rise to e.

We have seen that the relation of spatial connection between entities may be refined into seven kinds of connection and one of disconnection in the RCC8 classification. There are many other ways of refining spatial connection, and this



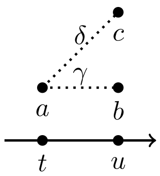
$$\rho_f{}^{-1}(b) = \{a\} \qquad \rho_f{}^{-}(c) = \{a, b\}$$
$$\rho_f(b) = \{c\} \qquad \rho_f{}^{+}(a) = \{b, c\}$$

Figure 10: Temporal ancestry and descent functions

topic has been widely studied in the literature. It would be possible similarly to discuss many different types of filiation, but in the present paper the emphasis is on the inter-rationship between the spatial, the spatio-temporal and filiation relations rather than on the technicalities of any one of these in isolation. We are thus concerned to present a model which is sufficiently flexible to allow for different kinds of spatial, spatio-temporal or filation relation yet which is capable of showing how these three kinds of relation would be related among themselves. For this reason we distinguish here just two types of filiation (Figure 11):

continuation the first entity is the same as second entity. (e.g. one person at two times). We denote this relationship by γ.

derivation the first entity creates (possibly with others) the second entity. (e.g. a parent of a child). We denote this relationship by δ.



$a \; \delta \; c$

$a \; \gamma \; b$

Figure 11: Filiation between two entities: continuation between a and b, derivation between a and c

By having two kinds of arrow we have a three-valued relation which leads to a more complex analysis than the previous spatio-temporal case, but we can nevertheless make the route extension to the filiation relation. Before doing this we need to make clear the constraints imposed by the distinction between continuation and derivation. There are two key issues:

– One entity may not continue as two distinct entities.

– If one entity continues as a second, then no entity other than the first has a filiation relation to the second.

If we use the notation of figure 11 to add labels for the different kinds of filiation, then conditions prohibit these situations Figure 12.

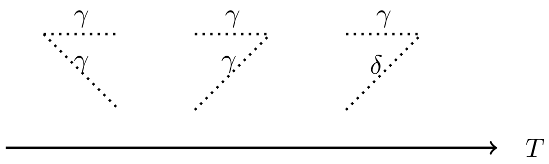
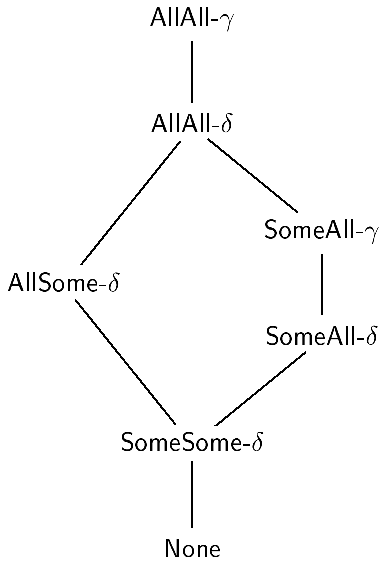


Figure 12: Unauthorised continuation and derivation

As in the case of the relation ρ_{st} we can consider routes in the relation. In the current case a route carries labels which can be composed. That is, for example, is a is continued by b and b is continued by c then a is continued by c, but if a is continued by b and from b c is derived then from a is c derived. By composing labels in this way we can assign a label to each route. This is because in the case of filiation, unlike spatial connection, we do have a transitive relation.

When we carry out the extension to the route relation $\star\rho_f$ we obtain a relation which takes values in a lattice below of not five but seven logical values. This is because we can distinguish cases according as the route in question is labelled by γ or by δ.



3.7 Spatio-temporal graph G_{ST}

The three relations introduced so far can be combined into a single graph. which we call the spatio-temporal graph, G_{ST}. An example is provided in Figure 13. This shows three times, t_1, t_2 and t_3 with four entities A, B, C and D present at the first time. These four are spatially related as indicated by the solid lines, and develop to the three entities shown at stage t_2. In the move from the second to third stages one entity (CD) continues while two others (A and B) combine to a new entity AB which derives from both of them.

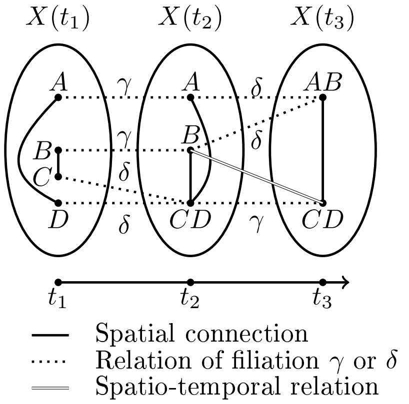


Figure 13: Spatio-temporal graph G_{ST}. Spatial relations are denoted in plain lines; filiations in dotted line with labels γ or δ to denote continuations and derivations, respectively; spatio-temporal relations in double-line

4 Application to the propagation of bramble plants

The modelling approach can be applied to a large range of domains oriented to the study of evolving entities such as in epidemiology, transportation planning and biology. This can be the case for applications that combine different categories of relations in space and time. The example chosen to illustrate the potential of our approach is the propagation of brambles in a particular area of land. This propagation encompasses several categories of processes and properties with four modes of proliferation: layering, basal shoot, diffusion of seeds, and grafting.

In order to give more background to this application context, we now describe some basic principles that describe the propagation of brambles [Wehrlen 1985].

Four main propagation modes have been identified. They can be spatially qualified as three being based on division and one on fusion. These four modes are illustrated in Figure 14.

Let us present a more formal description. We assume $t_j, t_{j+1} \in T$ are two successive times. We use x and y to denote plants at time t_j, and there may be several plants x_i, where $i \in \mathbb{N}$, arising from x at time t_{j+1}. The four types of propagation are then as follows.

- Basal Shoot (Figure 14a): a sucker x_i is a plant genetically identical[1] to its parent plant x. Basal shoot propagation happens when a part of a parent plant x becomes detached, and creates a new bramble x_i. We have $x\ \delta\ x_i$ and $x\ \rho_{st}\ x_i$. This method of propagation is illustrated in Figure 15a.

- Layering is shown in Figure 14b. In this case a plant x_i that results from a layering is genetically identical to its parent plant x. It comes from a branch of the parent plant which is buried into the ground, and that generates a new plant x_i. Generally, the branch involved in this process dies. We can see that $x\ \delta\ x_i$, and $x\ \rho_{st}\ x_i$.

- Seed dispersal is shown in Figure 14c. The fruit of brambles consist of an agglomeration of a variable number of small seeds. When they are eaten by animals (e.g. deer, birds), the animals droppings can give rise to new brambles. One daughter plant x_i is derived from its parent x, that is, $x\ \delta\ x_i$. In general it need not be the case that $x\ \rho_{st}\ x_i$ as shown in Figure 15b.

- Grafting (Figure 14d) takes place between two brambles of distinct identities, which we will denote x and y. This process happens when a part of a bramble y is grafted onto a branch of another bramble x. This generates a fusion of two brambles at time t_{j+1} and this fused bramble can be denoted xy. In the case of grafting, we have $x\ \delta\ xy$ and that $y\ \delta\ xy$. In general $x\ \rho_{st}\ xy$ and also $y\ \rho_{st}\ xy$ will hold as illustrated in Figure 15c.

We now present further formal details of the propagation model. We do not make a distinction between a bramble e and its materialisation as a spatial entity, so e will also denote the space occupied by the bramble. Using the filiations described above we consider propagations over a given time period $T = \{t_1, t_2, t_3, t_4\}$. A specific example is presented in Figures 16, and 17.

We consider the case of the transmission of an attribute of brambles by filiation. The case studied is a form of disease of the plants, and the affected plants are said to be contaminated and the transmission process is one of contamination. We refer to the graph shown in Figure 17. Two specific approaches are

[1] As defined in [Wehrlen 1985], child plants are genetically identical to their parent and reveal preservation of a lineage within a same filiation.

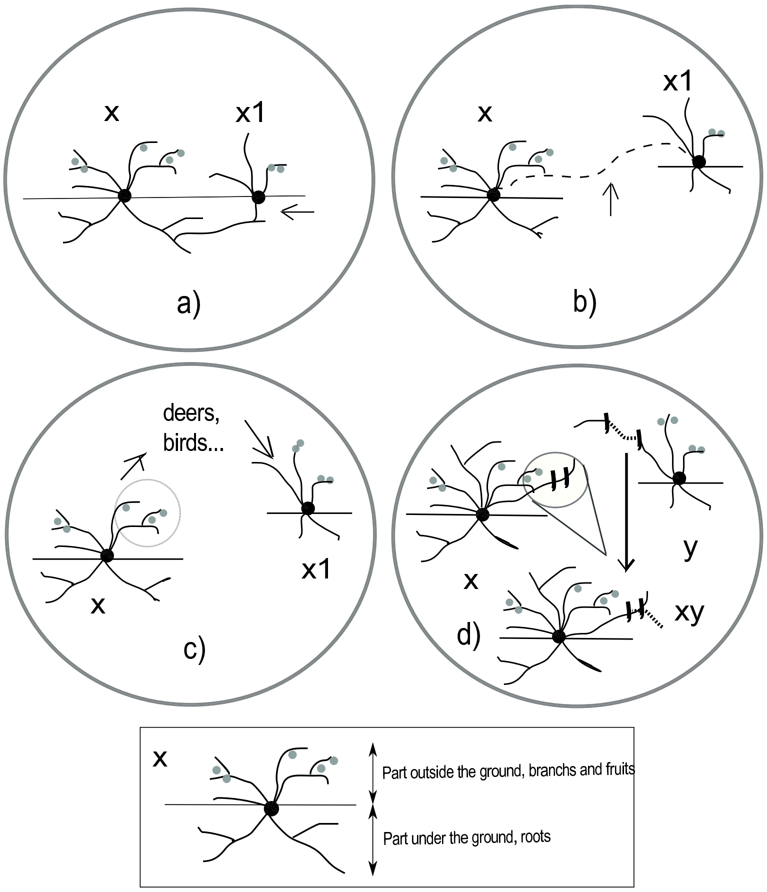


Figure 14: Schematic view of three mechanisms of division and one fusion of a parent entity: (a) Basal shoot (b) Layering (c) Seed dispersal (d) Grafting

used to explore this graph model. The first one is based on the application of functions, the second employs the route concept.

Approach 1: Contamination modelled by the application of functions

Two differents cases are considered. In the first one, the objective is to model the process of contamination. In the second one, we search for the contaminated entities.

Case 1 Let us assume that the entities that have been contaminated are known. The contamination process is unknown, it could be the result of one or several processes illustrated in Figure 15, and potentially several differ-

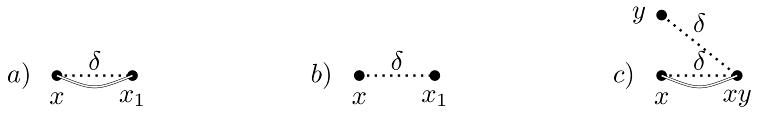


Figure 15: Three propagation cases: a) layering or basal shoot, b) seed dispersal and c) grafting

ent relations might be involved: spatial connection, filiation or spatio-temporal. Suppose the contaminated brambles are as follows:

At time t_1: B

At time t_2: A, B

At time t_3: H, B, C_1, D_1

At time t_4: H_1, C_1, D_{11}, E_{11}

We can express that an entity x is contaminated only if there is an entity y contaminated such that

$$(y \in {\rho_{st}}^{-}(x) \wedge y \in {\rho_f}^{-}(x)) \vee y \in \rho_s(x).$$

This corresponds to the case a) depicted on Figure 15 that is, by layering or by basal shoot or by the spatial connection. This example shows how the functions provided allow identification of the process responsible for the contamination.

Case 2 In this case we assume that the contamination process is known. Suppose that this contamination results from one of the processes previously identified (i.e. layering, basal shoot mode, or spatial connection) and that we are searching for all contaminated entities at earlier times given those contaminated at time t_4. In order to find all contaminated entities, the following algorithm is applied:

Let E_i the set of contaminated entities at time t_i

(1) $i := 4$.

(2) Find all contaminated entities y such that for each entity $x \in E_i$, $(y \in {\rho_{st}}^{-1}(x) \wedge y \in {\rho_f}^{-1}(x))$. This set of contaminated entities is called $CSET$.

(3) For each entity $y \in CSET$, find all entities $z \in X(t_{i-1})$ such that $z \in {\rho_s}^n(y)$, for all $n \in \mathbb{N}$ such that ${\rho_s}^n(y) \neq \{\}$. Then, let $CSET$ be the set of these contaminated entities.

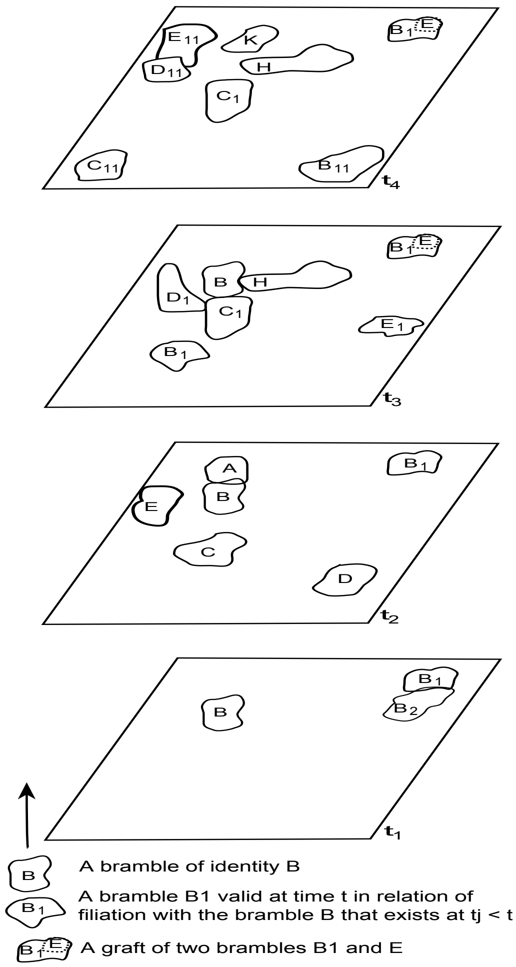


Figure 16: An illustrative view of the propagation of brambles on a period of time $T = \{t_1, t_2, t_3, t_4\}$

(4) If $i \neq 1$ then $i := i - 1$, $E_i := CSET$ and go back to (2), otherwise the result is $\cup_i(E_i)$.

This example shows how functions can identify all earlier contaminated entities from knowledge of the process and the set of contaminated entities identified at one time.

Approach 2: Contamination shown by routes

Let us revisit the second case of the first approach, but now with contami-

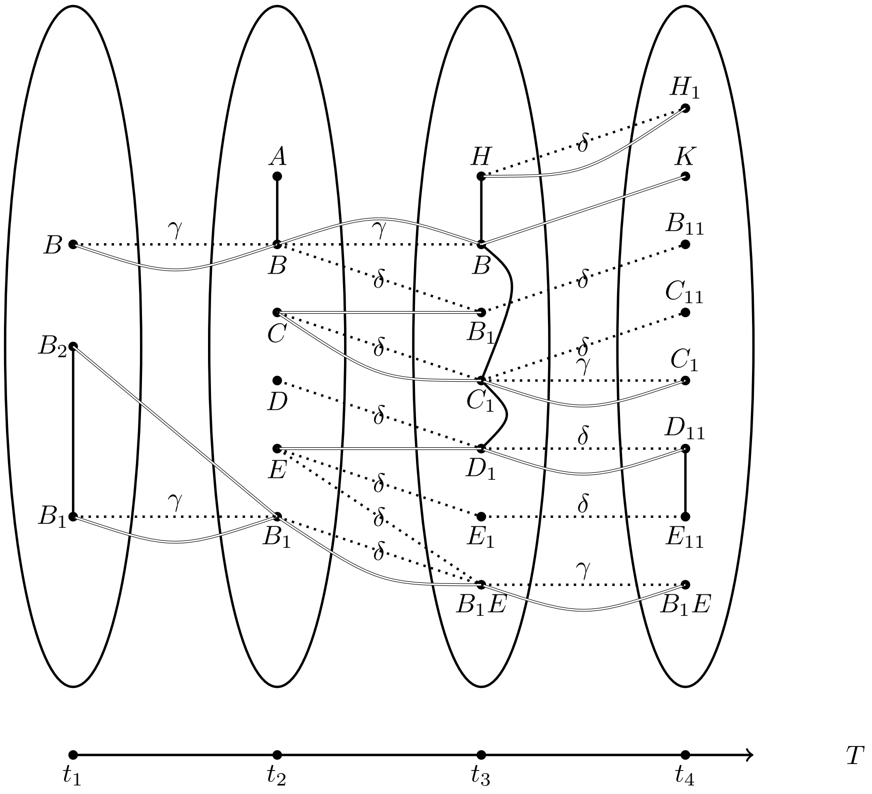


Figure 17: Modelisation of the brambles propagation

nation only transmitted by the spatio-temporal relation[2]. To illustrate how properties of this situation can be derived, let us consider two given brambles x and y at times t_i and t_j respectively, with $i < j$.

(a) Suppose that the entity x is contaminated. If $(x \star R\ y = \mathsf{AllAll}) \vee (x \star R\ y = \mathsf{SomeAll})$, then it follows that all the entities on this route are contaminated.

Conversely, it is possible to infer the origin of the contamination. This is can be seen in the example where $C \star R\ C_1 = \mathsf{SomeAll}$, between times t_2 and t_4. It is also evident between times t_1 and t_4 and for $B_1 \star R\ B_1E = \mathsf{AllAll}$.

(b) On the other hand, if there is $x \star R\ y = \mathsf{AllSome}$ and y at time t_j is contaminated, then it is not possible to conclude that x at t_i is the

[2] Note that the a similar approach can be applied with the contamination process of Case 1.

origin of the contamination. This is the case between times t_1 and t_2 for $B_2 \star R\, B_1 = \mathsf{AllSome}$.

Routes within a given filiation or spatio-temporal relation relate brambles that may not be at consecutive times. Overall, routes of filiation support deduction of additional information on the processes and inter-relations between brambles in space and time. The information that emerges can be refined according to the sort of filiation involved (whether continuation or derivation). This could be of interest when one is searching for a process of contamination as in the following cases:

For entity B at times t_1 and t_3 we see composition of two continuations so that B at t_1 continues to B at t_3.

Between B at t_1 and B_{11} at t_4 there is a composition of one continuation and two derivations, thus the second of these two entities is derived from the first.

These examples illustrate of the potential of the modelling approach when combined with the application of functions, and routes. Their potential can be enlarged by the integration of additional semantics.

5 Conclusion

The World is made of entities that evolve and generate complex interaction networks in space and time. Modelling such interaction networks require the design of appropriate spatio-temporal representations, this is of major interest for many scientific studies directed to the analysis and understanding of the patterns that emerge. The research presented in this paper introduces a graph-based modelling approach for the propagation of spatio-temporal entities. We make a distinction between the different ways entities are related in space and time according to the locations they share at some time, and how entities at different time are related by a filiation relation. In a specific model, several properties can be inferred from the different dimensions represented: spatial relations in space, spatio-temporal relations and temporal filiations. These relations allow for a representation of spatial and temporal connections, and the development of several manipulation operations: entities form networks in space and time whose structural properties can be studied at the local or global levels. We use the local level when analysing the neighbourhood of a given entity in space and time, and the global when studying the routes or path that relate one entity to another. The notion of route allows for a generation of a 5-valued relation when considering spatio-temporal relations, and a 8-valued relation when considering filiations, that is, continuation and derivation. The approach is exemplified by a case study in modelling concepts for the analysis of bramble propagation. The model can be enriched still further by additional concepts and semantics. For example, entities may continue their existence in different ways, can be absorbed by others, or even entities generate replicas in some sense. These few examples give some directions to explore regarding the semantics attached to the relations and networks that support the modelling approach we have introduced.

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