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A Selection Process Based on Additive Consistency to Deal with Incomplete Fuzzy Linguistic Information

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Abstract: In group decision making situations, there may be cases in which experts do not have an in-depth knowledge of the problem to be solved and, as a result, they may present incomplete information. In this paper, we present a new selection process to deal with incomplete fuzzy linguistic information. As part of it, we use an iterative procedure to estimate the missing information. This procedure is guided by the additive consistency property and only uses the preference values provided by the experts. In addition, the additive consistency property is also used to measure the level of consistency of the information provided by the experts. The main novelties of this selection process are both the possibility to manage decision situations under incomplete fuzzy linguistic information and the importance of the experts' preferences in the aggregation processes is modeled by means of the experts' consistency.

Key Words: group decision making, incomplete information, fuzzy linguistic information, consistency, aggregation
Category: H.0, I.2, I.6, J.6

1 Introduction

In Group Decision Making (GDM) problems there are a set of alternatives to solve a problem and a group of experts, characterized by their own ideas, attitudes, motivations and knowledge, trying to achieve a common solution. To do this, experts have to express their preferences by means of a set of evaluations over the set of alternatives.

Preference relations are usually assumed to model experts' preferences in GDM problems [Orlovski 1978, Saaty 1980, Tanino 1984]. According to the nature of the information expressed for every pair of alternatives, there exist many different representation formats of preference relations. In this paper, we use fuzzy linguistic preference relations (FLPRs) because of most GDM problems present qualitative aspects that are complex to assess by means of precise and exact values and, in such cases, an ordinal fuzzy linguistic approach can be used to obtain a better solution [Herrera et al. 1997a, Herrera et al. 1998, Herrera-Viedma 2001, Herrera-Viedma et al. 2005, Herrera-Viedma et al. 2006, Zadeh 1975a, Zadeh 1975b, Zadeh 1975c]. FLPRs assessed on a 2-tuple fuzzy linguistic modelling [Herrera and Martínez 2000] are assumed because it provides some advantages with respect to the ordinal fuzzy linguistic modelling [Cabrerizo et al. 2009, Herrera and Martínez 2001]. The main advantage of pairwise comparison is that of focusing exclusively on two alternatives at a time, which facilitates experts when expressing their preferences. However, this way of providing preferences limits experts in their global perception of the alternatives and, as a consequence, the provided preferences could be not rational. Usually, rationality is related the consistency concept, which is associated with the *transitivity property*. Many properties have been suggested to model transitivity of a fuzzy preference relation [Herrera-Viedma et al. 2004]. One of these properties is the *additive consistency*, which, as it was shown in [Herrera-Viedma et al. 2004], can be seen as the parallel concept of Saaty's consistency property in the case of multiplicative preference relations [Saaty 1980].

It is obvious that consistent information, i.e., information which does not imply any kind of contradiction, is more relevant or important than information containing some contradictions. The general procedure for the inclusion of importance degrees in GDM problems involves the transformation of the preference values under the importance degrees to generate new values. This activity is carried out by means of a transformation function [Herrera and Herrera-Viedma 1997, Yager 1978, Yager 1994] or by using the importance degrees to induce the ordering of the preference values prior to their aggregation as in Induced Ordered Weighted Averaging (IOWA) operator [Yager and Filev 1999].

As aforementioned, each expert has his/her own knowledge concerning the problem being studied, which also may imply a major drawback, that of an expert not having a perfect knowledge of the problem to be solved. Indeed, experts could not be able to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. Experts would rather not guess those preference degrees in these situations and, as a consequence, they might provide incomplete information [Alonso et al. 2008, Kim et al. 1999, Herrera-Viedma et al. 2007a, Herrera-Viedma et al. 2007, Xu 2005]. In this way, a difficulty that has to be addressed is the lack of information in the experts' opinions and, therefore, it would be of great importance to provide experts some tools that allow them to express this lack of knowledge in their opinions.

The aim of this paper is to present a new selection process based on additive consistency property to deal with GDM problems with incomplete FLPRs. This new selection process is composed of three steps: (1) *estimation of missing preference values*, (2) *aggregation* and (3) *exploitation*. So, we define an *additive consistency* measure for FLPRs that is based on the additive transitivity property [Tanino 1984]. In the first step we use an iterative complete procedure to estimate missing information in the case of incomplete FLPRs. It is based on the linguistic extension of Tanino's consistency principle and it carries out the completion of a particular expert's incomplete FLPRs using only the information he/she provides. The, following the choice scheme proposed in [Fodor and Roubens 1994], *aggregation* following by *exploitation*, this new selection process is completed. Furthermore, we use the additive consistency measure to propose a new IOWA operator, which we call the additive-consistency 2-tuple linguistic IOWA operator. The aggregation step of a selection process consists in combining the experts' individual preferences into a collective one, in such a way, that it summarizes or reflects the properties contained in all the individual preferences. This aggregation is carried out by using that new linguistic IOWA operator. The exploitation phase transforms the global information about the alternatives into a global ranking of them. To do this, two quantifier guided choice degrees of alternatives are used: the dominance and non-dominance degrees. The main improvements of this new selection process is that it supports the management of incomplete fuzzy linguistic information and allows the aggregation of the experts' preferences, in such a way, that more importance is given to the most consistent ones.

The rest of the paper is set out as follows. Section 2 deals with the preliminaries necessary to develop the new selection process. Section 3 presents the new selection process based on additive consistency to deal with incomplete FLPR. Section 4 shows an example as to how to apply it. Finally, in Section 5, we draw our conclusions.

2 Preliminaries

In this section, we present those tools necessary to design the new selection process, that is, the concept of incomplete 2-tuple FLPR, consistency measures and the iterative procedure to estimate missing values.

2.1 Incomplete 2-tuple FLPRs

A preference relation is defined as $P^h \subset X \times X$, where the value $\mu_{P^h}(x_i, x_k) = p^h_{ik}$ is interpreted as the preference degree of the alternative x_i over x_k for the expert e_h. According to the nature of the information expressed for every pair of alternatives, there exist many different representation domains of preference relations. As aforementioned, we use the 2-tuple fuzzy linguistic model [Herrera and Martínez 2000] to represent experts' preferences.

Definition 1. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, being $g + 1$

the cardinality of S, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta\colon [0,g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{1}$$

where $round(\cdot)$ is the usual round operation, s_i has the closest index label to "β", and "α" is the value of the symbolic translation.

Proposition 2. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.*

$$\Delta^{-1}\colon S \times [-0.5, 0.5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{2}$$

A 2-tuple linguistic computational model to combine linguistic information is composed of following operators: h

1. *A 2-tuple comparison operator.* The comparison of linguistic information represented by linguistic 2-tuples is carried out according to an ordinary lexicographic order (see [Herrera and Martínez 2000] for more details).
2. *A 2-tuple negation operator.*
$$Neg(s_i, \alpha) = \Delta(g - (\Delta^{-1}(s_i, \alpha))). \tag{3}$$
3. *2-tuple aggregation operators.* Extending the classical aggregation operators, such as the Linguistic Ordered Weighted Averaging (LOWA) operator [Herrera et al. 1996], the weighted average operator, the Ordered Weighted Averaging (OWA) operator, etc., (see [Herrera and Martínez 2000]).

A linguistic term $s_i \in S$ can be seen as a linguistic 2-tuple by adding to it the value 0 as symbolic translation, i.e., $s_i \in S \equiv (s_i, 0)$.

Definition 3. A 2-tuple FLPR P^h on a set of alternatives $X = \{x_1, \ldots, s_n\}$ is a fuzzy set defined on the product set $X \times X$, whic is characterized by a 2-tuple linguistic membership function

$$\mu_{P^h} : X \times X \longrightarrow S \times [-0.5, 0.5). \tag{4}$$

When cardinality of X is small, the preference relation may be conveniently represented by a $n \times n$ matrix $P^h = (p^h_{ik})$, being $p^h_{ik} = \mu_{P^h}(x_i, x_k)$, $\forall i, k \in \{1, \ldots, n\}$ and $p^h_{ik} \in (S \times [-0.5, 0.5))$.

Usual models to solve GDM problems assume that experts are always able to provide all the preferences required. However, this situation is not always possible to achieve. Experts could have some difficulties in giving all their preferences due to lack of knowledge about part of the problem, or simply because they may not be able to quantify some of their degree of preference. It must be clear then that when an expert e_h is not able to express the particular value p^h_{ik}, this does not mean that he/she prefers both options with the same intensity.

In order to model these situations, in the following definitions we express the concept of an incomplete 2-tuple FLPR:

Definition 4. A function $f : X \times Y$ is partial when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a total function.

Definition 5. A 2-tuple FLPR P^h on a set of alternatives X with a partial 2-tuple linguistic membership function is an incomplete 2-tuple FLPR.

Obviously, a 2-tuple FLPR is complete when its membership function is totally defined. Clearly, definition (3) includes both definitions of complete and incomplete 2-tuple FLPRs.

2.2 Consistency measures

The previous definition of a 2-tuple FLPR does not imply any kind of consistency property. In fact, preference values of a preference relation can be contradictory. Obviously, an inconsistent source of information is not as useful as a consistent one and, thus, it would be quite important to be able to measure the consistency of the information provided by experts for a particular problem.

To make a rational choice, properties to be satisfied by such preference relations have been suggested. One of these properties is the *transitivity property*, which represents the idea that the preference value obtained by directly two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives. There are several possible characterizations for the transitivity property (see [Herrera-Viedma et al. 2004]). In this paper, we make use of the *additive transitivity property*, which can be seen for fuzzy preference relations as the parallel concept of Saaty's consistency property for multiplicative preference relations [Saaty 1980]. The mathematical formulation of the *additive transitivity* was given by Tanino in [Tanino 1984]:

$$(p_{ij}^h - 0.5) + (p_{jk}^h - 0.5) = (p_{ik}^h - 0.5),\ \forall i,j,k \in \{1,\dots,n\}. \tag{5}$$

Using the transformation functions Δ and Δ^{-1}, we define the linguistic additive transitivity property for 2-tuple FLPR as follows:

$$\begin{aligned}&\Delta[(\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(s_{g/2},0)) + (\Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0))] = \\ &\Delta[(\Delta^{-1}(p_{ik}^h) - \Delta^{-1}(s_{g/2},0)],\quad \forall i,j,k \in \{1,\dots,n\}.\end{aligned} \tag{6}$$

As in the case of additive transitivity, the linguistic additive transitivity implies linguistic additive reciprocity. Indeed, because $p_{ii}^h = (s_{g/2},0),\ \forall i$, if we make $k = i$ in (6), then we have: $\Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{ji}^h)) = (s_g,0),\ \forall i,j \in \{1,\dots,n\}$. Then, expression (6) could be rewritten as:

$$p_{ik}^h = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0)),\quad \forall i,j,k \in \{1,\dots,n\}. \tag{7}$$

A 2-tuple FLPR will be considered "additive consistent" when for every three options, $x_i, x_j, x_k \in X$, their associated 2-tuple fuzzy linguistic preference degrees, $p_{ij}^h, p_{jk}^h, p_{ik}^h$, fulfil (7). An additive consistent 2-tuple FLPR will be referred as consistent throughout the paper, as this is the only transitivity property we are considering.

Expression (7) could be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, the preference value p_{ik}^h $(i \neq k)$ can be estimated using an intermediate alternative x_j in three different ways:

1. From $p_{ik}^h = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate

$$(cp_{ik}^h)^{j1} = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0)). \tag{8}$$

2. From $p_{jk}^h = \Delta(\Delta^{-1}(p_{ji}^h) + \Delta^{-1}(p_{ik}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate

$$(cp_{ik}^h)^{j2} = \Delta(\Delta^{-1}(p_{jk}^h) - \Delta^{-1}(p_{ji}^h) + \Delta^{-1}(s_{g/2},0)). \tag{9}$$

3. From $p_{ij}^h = \Delta(\Delta^{-1}(p_{ik}^h) + \Delta^{-1}(p_{kj}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate

$$(cp_{ik}^h)^{j3} = \Delta(\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(p_{kj}^h) + \Delta^{-1}(s_{g/2},0)). \tag{10}$$

The overall estimated value cp_{ik}^h of p_{ik}^h is obtained as the average of all possible $(cp_{ik}^h)^{j1}$, $(cp_{ik}^h)^{j2}$ and $(cp_{ik}^h)^{j3}$ values:

$$cp_{ik}^h = \Delta\left(\frac{\sum_{j=1;i\neq k\neq j}^{n} (\Delta^{-1}((cp_{ik}^h)^{j1}) + \Delta^{-1}((cp_{ik}^h)^{j2}) + \Delta^{-1}((cp_{ik}^h)^{j3}))}{3(n-2)}\right). \tag{11}$$

We should point out that in expressions (8), (9) and (10), we could find that the value of argument of the function Δ could lie outside the interval $[0, g]$. In order to avoid this problem, the following function is used on the arguments of Δ:

$$f(y) = \begin{cases} 0, & \text{if } y < 0 \\ g, & \text{if } y > g \\ y, & \text{otherwise,} \end{cases} \tag{12}$$

When the information provided is completely consistent, then $(cp_{ik}^h)^{jl} = p_{ik}^h$, $\forall j, l$. The error between a preference value and its estimated one is defined as follows.

Definition 6. The error between a preference value and its estimated one in $[0, 1]$ is computed as:

$$\varepsilon p_{ik}^h = \frac{|\Delta^{-1}(cp_{ik}^h) - \Delta^{-1}(p_{ik}^h)|}{g}. \tag{13}$$

Thus, it can be used to define the consistency level of the preference degree p_{ik}^h.

Definition 7. The consistency level associated to p_{ik}^h is defined as:

$$cl_{ik}^h = 1 - \varepsilon p_{ik}^h. \tag{14}$$

When $cl_{ik}^h = 1$, then $\varepsilon p_{ik}^h = 0$ and there is no inconsistency at all. The lower the value of cl_{ik}^h, the higher the value of εp_{ik}^h and the more inconsistent is p_{ik}^h with respect to the rest of information.

In the following, we define the consistency levels associated with individual alternatives and the whole 2-tuple FLPR:

Definition 8. The consistency level, $cl_i^h \in [0, 1]$, associated to a particular alternative x_i of a 2-tuple FLPR, P^h, is defined as:

$$cl_i^h = \frac{\sum_{k=1; i \neq k}^{n} (cl_{ik}^h + cl_{ki}^h)}{2(n-1)}. \tag{15}$$

Definition 9. The consistency level, $cl^h \in [0, 1]$, of a 2-tuple FLPR, P^h, is defined as follows:

$$cl^h = \frac{\sum_{i=1}^{n} cl_i^h}{n}. \tag{16}$$

When working with an incomplete 2-tuple FLPR, expression (11) cannot be used to obtain the estimate of a known preference value. In these cases, the following sets can be defined [Herrera-Viedma et al. 2007]:

$$
\begin{array}{l}
A = \{(i,j) \mid i,j \in \{1,\dots,n\} \wedge i \neq j\} \\
MV^h = \{(i,j) \in A \mid p_{ij}^h \text{ is unknown}\} \\
EV^h = A \setminus MV^h \\
H_{ik}^{h1} = \{j \neq i,k \mid (i,j),(j,k) \in EV^h\} \\
H_{ik}^{h2} = \{j \neq i,k \mid (j,i),(j,k) \in EV^h\} \\
H_{ik}^{h3} = \{j \neq i,k \mid (i,j),(k,j) \in EV^h\} \\
EV_i^h = \{(a,b) \mid (a,b) \in EV^h \wedge (a = i \vee b = i)\},
\end{array}
\tag{17}
$$

where MV^h is the set of pairs of alternatives whose preference degrees are not given by expert e_h, EV^h is the set of pairs of alternatives whose preference degrees are given by the expert e_h; H_{ik}^{h1}, H_{ik}^{h2}, H_{ik}^{h3} are the sets of intermediate alternative x_j ($j \neq i,k$) that can be used to estimate the preference value p_{ik}^h ($i \neq k$) using (8)–(10), respectively; and EV_i^h is the set of pairs of alternatives whose preference degrees involving the alternative x_i are given by the expert e_h. Then, the estimated value of a particular preference degree p_{ik}^h ($(i,k) \in EV^h$) can be calculated as [Herrera-Viedma et al. 2007a, Herrera-Viedma et al. 2007]:

$$
\text{if } (\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3}) \neq 0 \Rightarrow
$$

$$
cp_{ik}^h = \Delta\left(\frac{\sum_{j \in H_{ik}^{h1}} \Delta^{-1}((cp_{ik}^h)^{j1}) + \sum_{j \in H_{ik}^{h2}} \Delta^{-1}((cp_{ik}^h)^{j2}) + \sum_{j \in H_{ik}^{h3}} \Delta^{-1}((cp_{ik}^h)^{j3})}{(\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3})}\right). \tag{18}
$$

An important factor to take into account when analyzing the consistency in decision making situations with incomplete information is the notion of completeness. Clearly, the higher the number of preference values provided by an expert the higher the chance of inconsistency [Herrera-Viedma et al. 2007]. Therefore, a degree of completeness associated with the number or preference values provided should also be taken into account to produce a fairer measure of consistency of an incomplete 2-tuple FLPR.

Given an incomplete 2-tuple FLPR, we can easily characterize two completeness levels, *the completeness level of a relation* and *the completeness level of an alternative*. For an incomplete 2-tuple FLPR P^h, its completeness level, C^h, can be defined as the ratio of the number of preference values known, $\#EV^h$, to the total possible number of preference values, $n^2 - n$:

$$
C^h = \frac{\#EV^h}{n^2 - n}. \tag{19}
$$

For an alternative x_i, we can define its completeness level according to the preferences provided by the expert e_h, C_i^h, as the ratio between the actual number of preference values known for x_i, $\#EV_i^h$, and the total number of possible preference values in which x_i is involved with a different alternative, $2(n-1)$:

$$C_i^h = \frac{\#EV_i^h}{2(n-1)}. \tag{20}$$

So, we can define the consistency level associated to a preference value in an incomplete 2-tuple FLPR as follows.

Definition 10. The consistency level cl_{ik}^h associated to p_{ik}^h $(i,k) \in EV^h$ is defined as a linear combination of its associated error and the average of the completeness values associated to the two alternatives involved in that preference degree

$$cl_{ik}^h = (1 - \alpha_{ik}^h) \cdot (1 - \varepsilon p_{ik}^h) + \alpha_{ik}^h \cdot \frac{C_i^h + C_k^h}{2};\ \alpha_{ik}^h \in [0,1], \tag{21}$$

where α_{ik}^h is a parameter to control the influence of completeness in the evaluation of the consistency levels for e_h defined as:

$$\alpha_{ik}^h = 1 - \frac{\#EV_i^h + \#EV_k^h - \#(EV_i^h \cap EV_k^h)}{4(n-1)-2}. \tag{22}$$

Clearly, expression (21) is an extension of expression (14), because when P^h is complete both EV^h and A coincide and $\alpha_{ik}^h = 0$, $\forall i, k$.

Definition 11. The consistency level of an incomplete 2-tuple FLPR is defined as follows:

$$cl^h = \frac{\sum_{(i,k)\in EV^h} cl_{ik}^h}{\#EV^h}. \tag{23}$$

2.3 Estimation procedure of missing values for incomplete 2-tuple FLPRs

We use an iterative complete procedure to estimate the missing values in an incomplete 2-tuple FLPR, which it is based on the linguistic additive consistency property. This procedure estimates missing values in an expert's incomplete 2-tuple FLPR using only the preference values provided by that particular expert. The procedure estimates missing values by means of two different tasks:

A) Choose those elements to be estimated in each iteration of the procedure

The subset of missing values MV^h that can be estimated in step t of our procedure is denoted by EMV_t^h and defined as follows:

$$EMV_t^h = \left\{ (i,k) \in MV^h \setminus \bigcup_{l=0}^{t-1} EMV_l^h \mid i \neq k \wedge \exists j \in \{H_{ik}^{h1} \cup H_{ik}^{h2} \cup H_{ik}^{h3}\} \right\}, \tag{24}$$

and $EMV_0^h = \emptyset$ (by definition). When $EMV_{maxIter}^h = \emptyset$, with $maxIter > 0$, the procedure will stop as there will not be any more missing values to be estimated. Furthermore, if $\bigcup_{l=0}^{maxIter} EMV_l^h = MV^h$, then all missing values are estimated, and, consequently, the procedure is said to be successful in the completion of the incomplete 2-tuple FLPR.

B) Estimate a particular missing value

In order to estimate a particular value p_{ik}^h with $(i,k) \in EMV_t^h$, the following function $estimate\_p(h,i,k)$ is proposed:

function estimate_p(h,i,k)
1) $(cp_{ik}^h)^1 = (s_0, 0),\ (cp_{ik}^h)^2 = (s_0, 0),\ (cp_{ik}^h)^3 = (s_0, 0),\ \mathcal{K} = 0.$
2) if $\#H_{ik}^{h1} \neq 0$, then $(cp_{ik}^h)^1 = \Delta((\sum_{j \in H_{ik}^{h1}} \Delta^{-1}((cp_{ik}^h)^{j1}))/\#H_{ik}^{h1}),\ \mathcal{K}{+}{+}.$
3) if $\#H_{ik}^{h2} \neq 0$, then $(cp_{ik}^h)^2 = \Delta((\sum_{j \in H_{ik}^{h2}} \Delta^{-1}((cp_{ik}^h)^{j2}))/\#H_{ik}^{h2}),\ \mathcal{K}{+}{+}.$
4) if $\#H_{ik}^{h3} \neq 0$, then $(cp_{ik}^h)^3 = \Delta((\sum_{j \in H_{ik}^{h3}} \Delta^{-1}((cp_{ik}^h)^{j3}))/\#H_{ik}^{h3}),\ \mathcal{K}{+}{+}.$
5) Calculate $cp_{ik}^h = \Delta\left(\frac{\Delta^{-1}(cp_{ik}^h)^1 + \Delta^{-1}(cp_{ik}^h)^2 + \Delta^{-1}(cp_{ik}^h)^3}{\mathcal{K}}\right).$
end function

Then, the complete iterative estimation procedure is the following:

0. $EMV_0^h = \emptyset$
1. $t = 1$
2. while $EMV_t^h \neq \emptyset$ {
3. for every $(i,k) \in EMV_t^h$ {
4. estimate_p(h,i,k)
5. }
6. $t{+}{+}$
7. }

3 A selection process based on additive consistency to deal with incomplete fuzzy linguistic information

In this section, we present a new selection process based on additive consistency to deal with incomplete fuzzy linguistic information. It consists of three phases: (1) *estimation of missing information*, (2) *aggregation* and (3) *exploitation*. The estimation of missing information completes the opinions provided by the experts. To do so, it uses the consistency based procedure to estimate missing information shown in Section 2.3. The aggregation phase defines a collective 2-tuple FLPR indicating the global preference between every ordered pair of alternatives, while the exploitation phase transforms the global information about the alternatives into a global ranking of them, from which a choice set of alternatives is derived.

3.1 Estimation of missing information

In this phase, each incomplete 2-tuple FLPR is completed following the consistency based procedure to estimate missing information shown in Section 2.3. In such a way, we allow to solve GDM situations with incomplete information because of if the missing information is not completed, we could find that some preference degrees of the collective preference relation cannot be computed in the aggregation phase and, consequently, the ordering of some alternatives cannot be computed in the exploitation phase. Therefore, for each incomplete 2-tuple FLPR, P^h, we obtain its corresponding complete 2-tuple FLPR, $\bar{P}^h$.

3.2 Aggregation

Once we have estimated all the missing values in every incomplete 2-tuple FLPR, we have a set of m individual 2-tuple FLPRs $\{\bar{P}^1, \ldots, \bar{P}^m\}$. From this set, a collective 2-tuple FLPR, $P^c = (p^c_{ik})$, must be obtained by means of an aggregation procedure. In our case, each value $p^c_{ik} \in S \times [-0.5, 0.5)$ will represent the preference of alternative x_i over alternative x_k according to the majority of the most consistent experts' opinions. To do that, we use a 2-tuple linguistic OWA operator to aggregate the experts' opinions.

Definition 12. A 2-tuple linguistic OWA operator of dimension n is a function $\phi : (S \times [-0.5, 0.5))^n \longrightarrow S \times [-0.5, 0.5)$, that has a weighting vector associated with it, $W = (w_1, \ldots, w_n)$, with $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$, and it is defined according to the following expression:

$$\phi_W(p_1, \ldots, p_n) = \Delta(\sum_{i=1}^{n} w_i \cdot \Delta^{-1}(p_{\sigma(i)})),\ p_i \in S \times [-0.5, 0.5), \qquad (25)$$

being $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation defined on 2-tuple linguistic values, such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $p_{\sigma(i)}$ is the i-highest 2-tuple linguistic value in the set $\{p_1, \ldots, p_n\}$.

A natural question in the definition of the OWA operator is how to obtain the associated weighting vector. In [Yager 1988], it was defined an expression to obtain W that allows to represent the concept of fuzzy majority [Kacprzyk 1986] by means of a fuzzy linguistic non-decreasing quantifier Q [Zadeh 1983]:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n. \qquad (26)$$

The 2-tuple linguistic OWA operator does not take into account the importance of the experts. However, a rational assumption in the resolution process of a GDM problem is that of associating more importance to the experts who provide the most "consistent" information. This assumption implies that GDM problems should be viewed as heterogeneous. Indeed, in any GDM problem with incomplete information, each expert e_h can have an importance degree associated with him/her, which, for example, can be his/her own consistency level of the relation cl^h or consistency levels of the preference values cl^h_{ik} in each preference value p^h_{ik}.

Usually, procedures for the inclusion of these importance values in the aggregation process involve the transformation of the preference values, p^h_{ik}, under the importance degree I^h, to generate a new value, $\tilde{p}^h_{ik}$ [Herrera et al. 1998, Herrera and Herrera-Viedma 1997]. Usually, this process is carried out by means of a transformation function g, $\tilde{p}^h_{ik} = g(p^h_{ik}, I^h)$ [Herrera et al. 1998, Yager 1978]. One alternative possibility could consist of using importance degrees or consistency levels as the order inducing values of the IOWA operator to be applied in the aggregation phase of the selection process. Yager and Filev defined the IOWA operator as an extension of the OWA operator [Yager 1988] to allow a different reordering of the values to be aggregated [Yager and Filev 1999].

Definition 13. A 2-tuple linguistic IOWA operator of dimension n is a function

$$\Phi_W(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle) = \Delta(\sum_{i=1}^{n} w_i \cdot \Delta^{-1}(p_{\sigma(i)})),\ p_i \in S \times [-0.5, 0.5), \quad (27)$$

being σ a permutation of $\{1, \ldots, n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $\langle u_{\sigma(1)}, p_{\sigma(1)}\rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$.

In the above definition, the reordering of the set of values to be aggregated, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated with them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order inducing variable $\{p_1, \ldots, p_n\}$ the values of the argument variable [Yager and Filev 1999].

In this case, to obtain the associated weighting vector, in [Yager 1996], Yager also proposed a procedure to evaluate the overall satisfaction of Q important (u_k) criteria (or experts) (e_k) by the alternative x_j. In this procedure, once the satisfaction values to be aggregated have been ordered, the weighting vector associated with an IOWA operator using a linguistic quantifier Q are calculated following the expression

$$w_i = Q\left(\frac{\sum_{k=1}^{i} u_{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{i-1} u_{\sigma(k)}}{T}\right), \quad (28)$$

being $T = \sum_{k=1}^{n} u_k$ the total sum of importance, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight to those experts with a zero importance degree. In our case, the consistency levels of the 2-tuple FLPRs are used to obtain the importance degrees associated with the experts.

Definition (13) allows the construction of many different operators. Indeed, the set of consistency levels of the preference values, $\{cl_{ik}^1, \ldots, cl_{ik}^m\}$, may be used to define an IOWA operator, i.e., and the ordering of the preference values to be aggregated $\{\bar{p}_{ik}^1, \ldots, \bar{p}_{ik}^m\}$ can be induced by ordering the experts from the most to the least consistent one. In such a way, we obtain an IOWA operator that we call the additive-consistency 2-tuple IOWA operator, which can be viewed as an extension of the AC-IOWA operator [Chiclana et al. 2007, Chiclana et al. 2004, Chiclana et al. 2004a, Herrera-Viedma et al. 2007a].

Definition 14. The additive-consistency 2-tuple linguistic IOWA operator of dimension m, Φ_W^{AC}, is a 2-tuple linguistic IOWA operator whose set of order inducing values is $\{cl_{ik}^1, \ldots, cl_{ik}^m\}$.

Then, the collective 2-tuple FLPR is obtained as follows:

$$p_{ik}^c = \Phi_Q^{AC}(\langle cl_{ik}^1, \bar{p}_{ik}^1 \rangle, \ldots, \langle cl_{ik}^m, \bar{p}_{ik}^m \rangle), \tag{29}$$

where Q is the fuzzy quantifier used to implement the fuzzy majority concept and, using (28), to compute the weighting vector of the additive-consistency 2-tuple IOWA operator, Φ_Q^{AC}.

3.3 Exploitation

In this phase, in order to select the "best" alternative(s) acceptable for the majority of the most consistent experts, we can use two different quantifier-guided choice degrees of alternatives [Herrera-Viedma et al. 2007a]:

- $QGDD_i$: This quantifier guided dominance degree quantifies the dominance that one alternative has over all the others in a fuzzy majority sense and is defined as follows:

$$QGDD_i = \phi_Q(p_{i1}^c, p_{i2}^c, \ldots, p_{i(i-1)}^c, p_{i(i+1)}^c, \ldots, p_{in}^c). \tag{30}$$

- $QGNDD_i$: This quantifier guided non-dominance degree gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives and is defined as follows:

$$\begin{aligned} QGNDD_i = \phi_Q(&Neg(p_{1i}^s), Neg(p_{2i}^s), \ldots, Neg(p_{(i-1)i}^s), \\ &Neg(p_{(i+1)i}^s), \ldots, Neg(p_{ni}^s)), \end{aligned} \tag{31}$$

where

$$p^s_{ki} = \begin{cases} (s_0, 0), & \text{if } p^c_{ki} < p^c_{ik} \\ \Delta(\Delta^{-1}(p^c_{ki}) - \Delta^{-1}(p^c_{ik})), & \text{if } p^c_{ki} \geq p^c_{ik}, \end{cases}$$

represents the degree in which x_i is strictly dominated by x_k.

The application of the above choice degrees of alternatives over X may be carried out according to two different policies: *sequential policy* and *conjunctive policy* [Herrera-Viedma et al. 2007a]. Thus, in a complete selection process, the choice degrees can be applied in three steps:

1. **Step 1.** The application of each choice degree of alternatives over X to obtain the following sets of alternatives:

$$X^{QGDD} = \{x_i \in X \mid QGDD_i = sup_{x_j \in X} QGDD_j\}, \tag{32}$$

$$X^{QGNDD} = \{x_i \in X \mid QGNDD_i = sup_{x_j \in X} QGNDD_j\}, \tag{33}$$

   whose elements are called maximum dominance elements on the fuzzy majority of X quantified by Q and maximal non-dominated elements by the fuzzy majority of X quantified by Q, respectively.

2. **Step 2.** The application of the conjunction selection policy, obtaining the following set of alternatives:

$$X^{QGCP} = X^{QGDD} \cap X^{QGNDD}. \tag{34}$$

   If $X^{QGCP} \neq \emptyset$, then End. Otherwise, continue.

3. **Step 3.** The application of the one of the two sequential selection policies, according to either a dominance or non-dominance criterion, i.e.:

   – *Dominance based sequential selection process QG-DD-NDD*. To apply the quantifier guided dominance degree over X, and obtain X^{QGDD}. If $\#(X^{QGDD}) = 1$, then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-DD-NDD} = \{x_i \in X^{QGDD} \mid QGNDD_i = sup_{x_j \in X^{QGDD}} QGNDD_j\}. \tag{35}$$

   This is the selection set of alternatives.

   – *Non-dominance based sequential selection process QG-NDD-DD*. To apply the quantifier guided non-dominance degree over X, and obtain X^{QGNDD}. If $\#(X^{QGNDD}) = 1$, then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-NDD-DD} = \{x_i \in X^{QGNDD} \mid QGDD_i = sup_{x_j \in X^{QGNDD}} QGDD_j\}. \tag{36}$$

   This is the selection set of alternatives.

4 Example of application

Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four alternatives and $S = \{N, MW, W, E, B, MB, T\}$ a set of seven linguistic labels with the following meaning:

$$N = \mathit{Null} \quad MW = \mathit{Much\ Worse} \quad W = \mathit{Worse} \quad E = \mathit{Equally\ Preferred}$$
$$B = \mathit{Better} \quad MB = \mathit{Much\ Better} \quad T = \mathit{Total}$$

Let us suppose that three different experts $E = \{e_1, e_2, e_3\}$ provide the following incomplete FLPRs using the linguistic expression domain S:

$$P^1 = \begin{pmatrix} - & x & \mathbf{W} & x \\ x & - & x & \mathbf{MW} \\ \mathbf{MB} & x & - & \mathbf{E} \\ x & \mathbf{B} & \mathbf{E} & - \end{pmatrix}; P^2 = \begin{pmatrix} - & \mathbf{W} & \mathbf{B} & \mathbf{B} \\ \mathbf{T} & - & \mathbf{B} & \mathbf{T} \\ \mathbf{W} & \mathbf{W} & - & \mathbf{MW} \\ \mathbf{N} & \mathbf{W} & \mathbf{MB} & - \end{pmatrix}; P^3 = \begin{pmatrix} - & \mathbf{MW} & x & x \\ \mathbf{B} & - & \mathbf{MB} & \mathbf{B} \\ \mathbf{W} & x & - & \mathbf{W} \\ \mathbf{W} & \mathbf{MW} & \mathbf{B} & - \end{pmatrix}.$$

Then, the respective 2-tuple FLPRs are the following:

$$P^1 = \begin{pmatrix} - & x & (\mathbf{W},\mathbf{0}) & x \\ x & - & x & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & x & - & (\mathbf{E},\mathbf{0}) \\ x & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix};$$

$$P^2 = \begin{pmatrix} - & (\mathbf{W},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{T},\mathbf{0}) & - & (\mathbf{B},\mathbf{0}) & (\mathbf{T},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{W},\mathbf{0}) & - & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{N},\mathbf{0}) & (\mathbf{W},\mathbf{0}) & (\mathbf{MB},\mathbf{0}) & - \end{pmatrix};$$

$$P^3 = \begin{pmatrix} - & (\mathbf{MW},\mathbf{0}) & x & x \\ (\mathbf{B},\mathbf{0}) & - & (\mathbf{MB},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & x & - & (\mathbf{W},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{MW},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & - \end{pmatrix}.$$

(A) Estimation of missing information

As we observe two 2-tuple FLPRs are incomplete $\{P^1, P^3\}$. As an example, we show how to complete P^1 using the consistency based procedure to estimate missing information shown in Section 2.3:

Step 1: The set of elements that can be estimated are:

$$EMV_1^1 = \{(1,4), (2,3), (3,2), (4,1)\}.$$

After these elements have been estimated, we have:

$$P^1 = \begin{pmatrix} - & x & (\mathbf{W},\mathbf{0}) & (W,-0.33) \\ x & - & (MW,0.33) & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & (B,0.33) & - & (\mathbf{E},\mathbf{0}) \\ (MB,-0.33) & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix}.$$

As an example, to estimate p_{14}^1 the procedure is as follows:
$H_{14}^{11} = \{3\} \;\Rightarrow (cp_{14}^1)^1 = \Delta(\Delta^{-1}(cp_{14}^1)^{31}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{13}^1) + \Delta^{-1}(p_{34}^1) - \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(2+3-3))) = \Delta(\Delta^{-1}(\Delta(2))) = (W, 0)$.

$H_{14}^{12} = \{3\} \;\Rightarrow (cp_{14}^1)^2 = \Delta(\Delta^{-1}(cp_{14}^1)^{32}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{34}^1) - \Delta^{-1}(p_{31}^1) + \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(3 - 5 + 3))) = \Delta(\Delta^{-1}(\Delta(1))) = (MW, 0).$
$H_{14}^{13} = \{3\} \;\Rightarrow (cp_{14}^1)^3 = \Delta(\Delta^{-1}(cp_{14}^1)^{33}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{13}^1) - \Delta^{-1}(p_{43}^1) + \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(2 - 3 + 3))) = \Delta(\Delta^{-1}(\Delta(2))) = (W, 0).$
$cp_{14}^1 = \Delta\left(\dfrac{\Delta^{-1}(cp_{14}^1)^1 + \Delta^{-1}(cp_{14}^1)^2 + \Delta^{-1}(cp_{14}^1)^3}{3}\right) = \Delta\left(\dfrac{2+1+2}{3}\right) = (W, -0.33).$

Step 2: The set of elements that can be estimated are:

$$EMV_2^1 = \{(1, 2), (2, 1)\}.$$

After these elements have been estimated, we have the following complete 2-tuple FLPR:

$$\bar{P}^1 = \begin{pmatrix} - & (E,0) & (\mathbf{W},\mathbf{0}) & (W,-0.33) \\ (E,0) & - & (MW,0.33) & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & (B,0.33) & - & (\mathbf{E},\mathbf{0}) \\ (MB,-0.33) & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix}.$$

As an example, to estimate p_{12}^1 the procedure is as follows:
$H_{12}^{11} = \{3, 4\} \;\Rightarrow (cp_{12}^1)^1 = \Delta\left(\dfrac{\Delta^{-1}(cp_{12}^1)^{31} + \Delta^{-1}(cp_{12}^1)^{41}}{2}\right) = (E, 0).$
$H_{12}^{12} = \{3, 4\} \;\Rightarrow (cp_{12}^1)^2 = \Delta\left(\dfrac{\Delta^{-1}(cp_{12}^1)^{32} + \Delta^{-1}(cp_{12}^1)^{42}}{2}\right) = (W, 0.33).$
$H_{12}^{13} = \{3, 4\} \;\Rightarrow (cp_{12}^1)^3 = \Delta\left(\dfrac{\Delta^{-1}(cp_{12}^1)^{33} + \Delta^{-1}(cp_{12}^1)^{43}}{2}\right) = (B, -0.33).$
$cp_{12}^1 = \Delta\left(\dfrac{\Delta^{-1}(cp_{12}^1)^1 + \Delta^{-1}(cp_{12}^1)^2 + \Delta^{-1}(cp_{12}^1)^3}{3}\right) = \Delta\left(\dfrac{3+2.33+3.67}{3}\right) = (E, 0).$

For P^3 we get:

$$\bar{P}^3 = \begin{pmatrix} - & (\mathbf{MW},\mathbf{0}) & (B,-0.25) & (E,-0.33) \\ (\mathbf{B},\mathbf{0}) & - & (\mathbf{MB},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (N,0.33) & - & (\mathbf{W},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{MW},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & - \end{pmatrix}.$$

The corresponding consistency level matrix associated with the incomplete 2-tuple FLPR P^1 is:

$$CL^1 = \begin{pmatrix} - & 0.80 & 0.70 & 0.76 \\ 0.80 & - & 0.78 & 0.70 \\ 0.70 & 0.80 & - & 0.90 \\ 0.80 & 0.70 & 0.90 & - \end{pmatrix}.$$

As an example, to compute cl_{13}^1, the following calculations are needed:

$EV_1^1 = \{(1, 3), (3, 1)\} \Rightarrow C_1^1 = 2/6.$
$EV_3^1 = \{(1, 3), (3, 1), (3, 4), (4, 3)\} \Rightarrow C_3^1 = 4/6.$
$EV_1^1 \cap EV_3^1 = \{(1, 3), (3, 1)\} \Rightarrow \alpha_{13}^1 = 1 - \frac{2+4-2}{10} = 0.6.$

For p^1_{13} we have that there is no intermediate alternative to calculate an estimated value and consequently we have:

$$\varepsilon p^1_{13} = 0 \Rightarrow cl^1_{13} = (1 - 0.6) \cdot (1 - 0) + 0.6 \cdot \frac{\frac{2}{6} + \frac{4}{2}}{2} = 0.7.$$

For P^2 and P^3 we get:

$$CL^2 = \begin{pmatrix} - & 0.83 & 0.92 & 0.80 \\ 0.50 & - & 0.53 & 0.75 \\ 0.92 & 0.70 & - & 0.50 \\ 0.36 & 0.92 & 0.50 & - \end{pmatrix}; CL^3 = \begin{pmatrix} - & 0.80 & 0.81 & 0.81 \\ 0.77 & - & 0.82 & 0.75 \\ 0.78 & 0.81 & - & 0.80 \\ 0.87 & 0.97 & 0.80 & - \end{pmatrix}.$$

(B) Aggregation

Once the incomplete 2-tuple FLPRs are completed, we aggregate them by means of the additive-consistency 2-tuple linguistic IOWA operator and using the consistency level of the preference values as the order inducing variable. We make use of the linguistic quantifier "most of", defined as $Q(r) = r^{1/2}$, which applying (28), generates a weighting vector of three values to obtain each collective preference value p^c_{ik}.

As example, the collective preference value p^c_{12} is obtained as follows:

$cl^1_{12} = 0.80,\ cl^2_{12} = 0.83,\ cl^3_{12} = 0.80.$
$\bar{p}^1_{12} = (E, 0),\ \bar{p}^2_{12} = (W, 0),\ \bar{p}^3_{12} = (MW, 0).$
$\sigma(1) = 2,\ \sigma(2) = 1,\ \sigma(3) = 3.$
$T = cl^1_{12} + cl^2_{12} + cl^3_{12}.$
$Q(0) = 0;\ Q\left(\frac{cl^3_{12}}{T}\right) = 0.33; Q\left(\frac{cl^3_{12}+cl^2_{12}}{T}\right) = 0.67;\ Q\left(\frac{cl^3_{12}+cl^2_{12}+cl^1_{12}}{T}\right) = 1.$
$w_1 = 0.33;\ w_2 = 0.34;\ w_3 = 0.33.$
$p^c_{12} = \Delta(w_1 \cdot \Delta^{-1}(\bar{p}^2_{12}) + w_2 \cdot \Delta^{-1}(\bar{p}^1_{12}) + w_3 \cdot \Delta^{-1}(\bar{p}^3_{12})) = (W, 0.01).$

Then, the collective 2-tuple FLPR is:

$$P^c = \begin{pmatrix} - & (W, 0.01) & (E, 0.32) & (E, -0.20) \\ (MB, -0.13) & - & (B, -0.29) & (B, -0.28) \\ (E, -0.10) & (W, 0.11) & - & (W, 0.36) \\ (W, -0.30) & (W, 0.09) & (B, 0.05) & - \end{pmatrix}.$$

(C) Exploitation

Using again the same linguistic quantifier "most of" and (26), we obtain the weighting vector $W = (w_1, w_2, w_3)$:

$$w_1 = Q(1/3) - Q(0) = 0.58 - 0 = 0.58.$$
$$w_2 = Q(2/3) - Q(1/3) = 0.82 - 0.58 = 0.24.$$
$$w_3 = Q(1) - Q(2/3) = 1 - 0.82 = 0.18.$$

and the following quantifier guided dominance and non-dominance degrees of all the alternatives:

	x_1	x_2	x_3	x_4
$QGDD_i$	(E,-0.04)	(B,0.38)	(W,0.21)	(E,0.16)
$QGNDD_i$	(MB,0.48)	(T,0.00)	(MB,0.01)	(MB,0.26)

To calculate the quantifier guided non-dominance degree the following matrix P^s is obtained:

$$P^c = \begin{pmatrix} - & (N,0.00) & (N,0.42) & (MW,0.10) \\ (E,-0.14) & - & (W,-0.40) & (E,-0.37) \\ (N,0.00) & (N,0.00) & - & (N,0.00) \\ (N,0.00) & (N,0.00) & (W,-0.31) & - \end{pmatrix}.$$

Clearly, the maximal sets are:

$$X^{QGDD} = \{x_2\} \text{ and } X^{QGNDD} = \{x_2\}.$$

Finally, applying the conjunction selection policy we obtain:

$$X^{QGCP} = X^{QGDD} \cap X^{QGNDD} = \{x_2\}.$$

which means that alternative x_2 is the best alternative according to "most of" the most consistent experts.

5 Conclusions

In this paper we have presented a new selection process based on additive consistency to deal with GDM problems under incomplete fuzzy linguistic information. This new selection process is composed of three phases: estimation of missing values, aggregation and exploitation. The main improvements of this selection process is that it supports the management of incomplete fuzzy linguistic information and it allows the aggregation of the experts' preferences in such a way that more importance is given to the most consistent ones.

In the future we think to research two new challenges: i) Study new strategies to compute the missing value, for example by using consensus criteria [Herrera et al. 1997a, Herrera-Viedma et al. 2005, Mata et al. 2009], and ii) design new selection process for GDM problems under unbalanced fuzzy linguistic information [Cabrerizo et al. 2009, Herrera-Viedma and López-Herrera 2007, Herrera et al. 2008].

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