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A Demand Forecasting Methodology for Fuzzy Environments

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Abstract: Several supply chain and production planning models in the literature assume the demands are fuzzy but most of them do not offer a specific technique to derive the fuzzy demands. In this study, we propose a methodology to obtain a fuzzy-demand forecast that is represented by a possibilistic distribution. The fuzzy-demand forecast is found by aggregating forecasts based on different sources; namely statistical forecasting methods and experts' judgments. In the methodology, initially, the forecast derived from the statistical forecasting techniques and experts' judgments are represented by triangular possibilistic distributions. Subsequently, those results are combined by using weights assigned to each of them. A new objective weighting approach is used to find the weights. The proposed methodology is illustrated by an example and a sensitivity analysis is provided.

Keywords: Fuzzy-demand forecast, objective weights, aggregation
Categories: I.2.3, I.2.1, F.2.1, G.3

1 Introduction

Scientists have studied forecasting since the 1930s. [Armstrong 01] provides summaries of important findings from the extensive forecasting literature. One of the most counterintuitive findings in forecasting is that judgmental forecasts by experts who ignore accepted forecasting principles have little value in complex and uncertain situations. This finding applies whether the opinions are expressed in words, spreadsheets, or mathematical models. Despite the lack of validity of expert unaided forecasts, many policy decisions are based on such forecasts. In complex and uncertain situations such as demand forecast, the expert forecasts should be integrated with statistical forecasting techniques [Armstrong et al. 08].

Demand forecast is one of the most important inputs of production planning and supply chain (SC) planning models. In the manufacturing process, demands are predicted to perform the basic planning activities such as capacity planning, resource planning, and raw material purchasing. Demand forecasts are of great importance for marketing activities and personnel management and its accuracy affects directly the profitability of the company. Underestimated demand causes raw-material and final product stock-outs that lead to possible profit losses while overestimated demand results with increases in inventories that cause an increase in the holding costs

[Shapiro 01, Petrovic et al. 06, Hsu and Wang 01]. Since demand forecast is related to several uncertainties, fuzzy sets can be used to represent demands in the models (i.e., fuzzy demands can be assumed).

In this study, we propose a methodology to generate fuzzy-demand forecasts by evaluating the historical data and the judgmental expressions. An objective weighting approach is proposed in order to combine forecasts coming from different sources. In the second section reasons of interest to the fuzzy-demand forecast in fuzzy SC and production planning models are explained. The details of the proposed methodology are then presented in the third section. Subsequently, a numerical example is used to illustrate the proposed methodology and a sensitivity analysis is conducted. Finally conclusions and suggestions for future work are provided.

2 Motivation of the study

2.1 Why fuzzy-demand forecasting?

In SCs or production systems, there are several uncertainties that should be taken into account. Especially in specific problems that necessitate future projections, such as new product design or strategic planning, there will be several parameters that cannot be estimated deterministically. Fuzzy logic is an important tool to model those types of uncertainties. There are several SC models that assume fuzzy parameters, such as demand [see Ryu et al. 04, Petrovic et al. 99, Wand and Shu 05, Wang and Fang 01, Chen and Lee 04, Torabi and Hassani 08, Xu et al. 08], operating time [see Wand and Shu 05], and price [see Torabi and Hassani 08]. As investigated from the literature, demand has been the most important and extensively studied source of uncertainty in SC problems [see Guillén et al. 05, Wang and Fang 01, Hsu and Wang 01, Liang 06, Petrovic 01, Das and Abdel-Malek 03, Leung et al. 06]. Given the fact that effectively meeting customer demand is what mainly drives most SC initiatives, it is appropriate to emphasize incorporating demand uncertainty into planning decisions. Furthermore, because demand fluctuations affect the production system and suppliers over time, demand is the main source of uncertainties.

Due to the fact that it implies judgments about the unknown future, demand forecasting is usually subject to uncertainties. Therefore in the SC and production planning literature, several studies assumed the demands to be fuzzy or represented them via possibilistic distributions [see Petrovic 01, Wang and Shu 05, Ouyang and Yao 02, Zhou and Liu 07, Wang and Fang 01, Chen and Lee 04, Torabi and Hassani 08]. However there is no detailed procedure to generate fuzzy demands from the historical and/or judgmental data. We present some studies that use fuzzy demands in the following.

[Petrovic 01] suggests that the uncertainties in parameters can be specified based on the managerial experience and subjective judgment in the situations where a lack of evidence or lack of confidence in evidence is available or simply evidence may not exist, as in the case of launching a new product. It is proposed to express these uncertainties using various imprecise linguistic expressions; for example, customer demand D is about d_m, but definitely not less than d_l and not greater than d_u. Fuzzy customer demands are represented by fuzzy membership functions that are derived from subjective manager belief. On the other hand, in [Petrovic et al. 08], demands

are described by imprecise linguistic expressions that are modeled by discrete fuzzy sets for coordinating a distribution SC. The paper mentions the arguments in a large body of recent literature that fuzzy sets theory could provide an appropriate framework for dealing with uncertainties in areas where intuition and subjective judgment play an important role. In such cases, uncertainty is caused by the imprecision of natural language description rather than the existence of statistical frequency of the occurrence of events. Similar to [Petrovic 01], [Petrovic et al. 08] described uncertain customer demand by imprecise linguistic expressions. However both papers do not propose any methodology that integrates different forecasts obtained from several subjective believes and statistical methods to obtain aggregated fuzzy demand.

[Wang and Shu 05] represents uncertain SC parameters, one of which is uncertain customer demand, by possibility distributions that can be further characterized by fuzzy numbers. The authors preferred six-point fuzzy numbers instead of most widely used fuzzy membership functions such as triangular or trapezoidal fuzzy numbers to avoid erroneous results for repeated uses of the multiplication and division operands and thus to improve approximation and computational efficiency. The estimation procedure of the six point fuzzy demands is not mentioned in the paper. It is assumed that fuzzy demands are known and given in the problem.

[Ouyang and Yao 02] applies a mini-max decision approach to solve a continuous review mixed inventory model in which the annual demand considered to be fuzzy. The paper first considers a case where the annual demand is treated as the triangular fuzzy number. Then, a statistical method is employed to construct a confidence interval for the annual demand, and through it to establish the corresponding fuzzy number. For the triangular fuzzy number case, the authors declared only that decision makers would determine fuzzy demands. If the actual annual demand is unknown or cannot be determined by the decision maker, it is attempted to use the random sample data of annual demand during past time to find the interval estimation of the real annual demand, and then to employ it to get the statistical based fuzzy annual demand.

[Zhou and Liu 07] models capacitated location–allocation problem assuming the demands are fuzzy. The paper considers that fuzzy demands are known. [Wang and Fang 01] presents a novel fuzzy linear programming method for solving the aggregate production planning problem with multiple objectives where several parameters including market demands are fuzzy in nature. Fuzzy parameters of the model are assumed to be fuzzy intervals in trapezoidal form. The membership functions of the fuzzy parameters are specified by an assistance of a decision maker. However the details of the specification procedure of membership functions are not given. [Chen and Lee 04] proposes a multi-product, multi-stage, and multi-period scheduling model to deal with multiple incommensurable goals for a multi-echelon SC network with uncertain market demands and product prices. The scenario-based representation for uncertain product demands is considered in the modeling. [Torabi and Hassani 08] considers a SC master planning model consisting of multiple suppliers, one manufacturer and multiple distribution centers. Due to incompleteness and/or unavailability of required data over the mid-term decision horizon, market demands are assumed to be fuzzy in nature. Furthermore, the pattern of triangular fuzzy number is adopted to represent fuzzy demand parameter. The most pessimistic value,

the most possible value, and the most optimistic value of the demand parameter are supposed to be estimated by a decision maker. [Kabak and Ülengin 07] proposes a possibilistic linear programming model to make long-term resource allocation and outsourcing decisions. Most of the parameters of the model, including demands, are assumed to be fuzzy in order to take into account environmental and system uncertainties of SCs.

The survey of the mentioned literature clearly reveals that the fuzzy demand is very crucial for the SC and production planning models in fuzzy environment. Fuzzy demands are generally used in the models where statistical data is not available or reliable and expert judgments are provided. In fact, when the statistical data is available if uncertainty is very high, expert judgments are still very important in order to improve the accuracy of the forecast. However above mentioned studies do not offer an integrated fuzzy-demand generating algorithm that considers both statistical and judgmental expressions.

2.2 Forecasting fuzzy-demands by combining statistical and judgmental inputs

In the literature, there are several studies using fuzzy approaches to demand forecasting [see Efendigil et al 09, Chang and Wang 06, Lee and Chen 01, Kuo, 01]. [Kahraman et al. 06] provides a detailed revision of fuzzy sets applications in forecasting. However, in this paper, the topic of interest is related to generating fuzzy demands rather than fuzzy demand forecasting. As far as our investigation, there is no study that is directly devoted to generating fuzzy demands.

On the other hand, [Petrovic et al. 06] proposes a demand forecasting model taking into account both statistical forecasting techniques based on historical data as well as expert judgments. It presents a new decision support system for demand forecasting by combining four forecast values. Two of them are obtained independently by a customer and a market expert. Subjective judgments of the experts on future demand are represented as linguistic values. Other two forecasts are crisp values that are obtained by using predetermined conventional statistical methods, namely a time-series analysis based on decomposition, and an auto regressive integrated moving average model. Fuzzy IF-THEN rules are applied to combine these four forecast values into an improved crisp forecast. Practically the model is restricted to its predetermined types of statistical and judgmental inputs and the crisp output of the model cannot be used in the fuzzy SC and / or production planning models.

[Sayed et al. 09] proposes a hybrid forecasting model in which cause and effect based forecasts and time series related forecasts are combined to generate better demand forecast accuracy. Modified genetic algorithm is used to choose the best combination of methods. Literature also includes other studies that propose combination of statistical forecasts [see Sánchez 08, Colloby and Armstrong 92]. These papers mostly do not consider expert judgments and the final results are crisp.

It is concluded from the literature survey that a fuzzy-demand forecast methodology that integrates the statistical and judgmental forecast and gives a fuzzy result can be a fine contribution.

3 Proposed Fuzzy-Demand Forecast Methodology

A range of forecasting models and statistical techniques are available for demand forecasting. In fact the source of the forecasts can be grouped into two categories. The first one is the historical data about the corresponding product, and other related industrial and macroeconomic variables. Statistical forecasting methods are used to analyze the related data. The second source is judgmental expressions of product and/or market experts. This is especially preferred when the historical data is either not available or not very relevant [Shapiro 01].

The proposed methodology combines results of the statistical and judgmental forecasts using a fuzzy logic-based aggregation procedure. For this purpose, initially, the forecasts generated by statistical forecasting methods are represented by triangular possibility distributions (TPDs). Subsequently, all the forecast results initially obtained as well as the historical data are presented to experts in order to provide a basis for their judgments. The experts express their judgments as optimistic, expected, and pessimistic values. If no historical data is available, the procedure starts with judgmental forecast based solely on the opinions of experts. The judgmental forecasts are also represented by TPDs. Then weights of the forecasts indicating the importance of each forecast are determined by an objective weighting approach. Finally, all the forecast results obtained through multiple sources are aggregated using objective weights. [Fig. 1] shows the steps of the methodology.

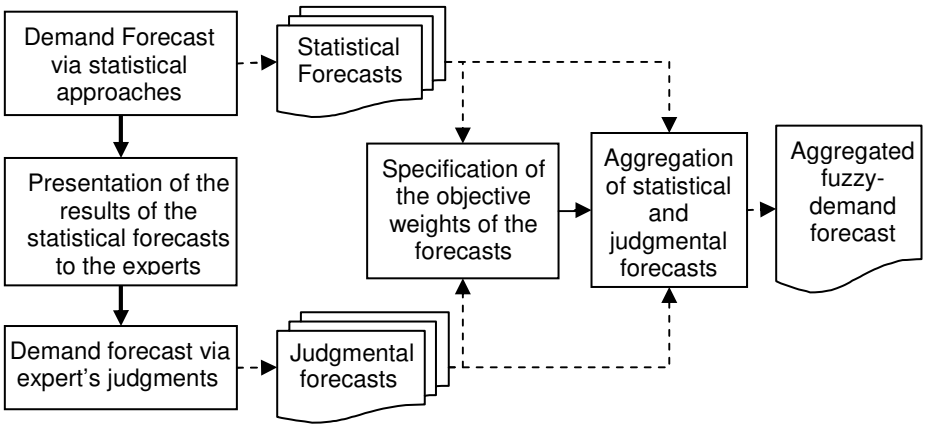

Figure 1: Steps of the proposed methodology

3.1 Demand forecast via statistical approaches

Statistical approaches for demand forecast includes classical methods such as moving average, exponential smoothing, trend estimation as well as time series models such as Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), and causal models such as regression models [Hair et al. 95].

[Ouyang and Yao 02] uses sample data during past time to find the interval estimation of the real demand, and then employed it to get the statistical-based fuzzy

number. The estimation procedure uses the mean of the sample data and the confidence interval to form the critical points (i.e., right support, the most possible, and left support) of the triangular fuzzy number.

We use a procedure to form TDPs from the results of statistical approaches that is inspired from [Ouyang and Yao 02]'s procedure of generating triangular fuzzy number from sample data. In general, when a forecast is made with a statistical approach, the result is given with expected value (f^e) and a confidence interval on the bases of α significance level ($\left[f_\alpha^l, f_\alpha^r\right]$). These results can be considered as the most possible, pessimistic and optimistic outcomes. In other words, the expected value gathered from the statistical approach can be stated as the most possible value; left and right boundaries of the confidence interval can be assigned as pessimistic and optimistic values, respectively.

Therefore, the demand forecast with s^{th} statistical approach (where s an index for statistical approaches and S is the total number of statistical approaches employed, s=1,…,S) can be represented by a TPD, $\tilde{D}_s = \left(D_s^p, D_s^m, D_s^o\right)$, as follows:

$$D_s^o = f_{s\alpha}^r,\ D_s^p = f_{s\alpha}^l,\ D_s^m = f_s^e \qquad [1]$$

where p, m, and o stands for pessimistic, most possible, and optimistic demands, respectively and α is the significance level that is determined by the decision analyzer. For instance, suppose that moving average is applied to a sample data of 24 months and the forecast related to 25th month is generated as 889 (i.e., f^e=889) with %95 confidence interval of [720;1057] (i.e., $f_\alpha^l = 720, f_\alpha^r = 1057$). Then this forecast is represented with the following TPD: D_1(720, 889, 1057).

In the proposed methodology, the number and type of statistical approach to generate demand forecast is selected by the decision analyzer.

3.2 Demand forecast via expert judgments

To generate judgmental demand forecasts from experts, firstly, the results of the statistical methods as well as the crude historical data are presented in order to inform them about the data output. Experts are then requested to indicate their opinions about future demands from pessimistic, optimistic and most possible perspectives. These values are used to form related TPDs. If judgments are made in a group meeting, experts may indicate their opinions individually or they may reach a consensus result. The resulting demand forecasts are represented by TPDs as $\tilde{D}_j = \left(D_j^p, D_j^m, D_j^o\right)$, where j is an index for different experts (j =1,…,J).

3.3 Specification of the objective weights of the forecasts

In order to conclude a final demand forecast, the forecasts coming from different sources are aggregated. We utilize weights that show the relative importance of the forecasts for the aggregation.

3.3.1 Weight assessment issue

The issue of the relative importance to be assigned to each conflicting attribute necessitates the specification of the weight and is fundamental to any analysis of multi criteria decision making (MCDM) problems.

The assessment and interpretation of importance weights has often been a matter of controversy. The meaning of the numerical weight parameter will differ according to the particular preference model being used, and often also according to the range of alternatives under consideration. In MCDM, weights of criteria or attributes, representing their relative importance or value trade-offs, are normally determined based on subjective preferences or perceptions of the decision makers (DMs) and/or stakeholders. Several methods for determining criteria weights have been proposed. However, no single method can guarantee a more accurate result which implies that there is no easy way for determining criteria weights and there are no criteria for determining what the true weight set is.

There are many techniques to elicit attribute weights, for review see [Steward 92] and [Weber and Borcherding 93]. Literature surveys also indicate that there are two main categories of determining the weights [Ma et al. 99]. Criteria importance is a reflection of the DM's subjective preference as well as the objective characteristics of the criteria themselves.

A good review of various subjective weighting methods commonly used in MCDM is given by a number of authors based on various performance measures, such as [Barron and Barrett 96], [Hobbs 78], and [Schoemaker and Waid 82]. The results of these studies show that the same DM may obtain different weights using different methods. This may mainly be due to the fact that the DM cannot always give consistent judgment under different weighting schemes and the weighting process itself is essentially context dependent [Deng et al. 00].

The inconsistency problem in subjective weighting has been well addressed by some papers, reflecting the inherent difficulty of assigning reliable subjective weights. [Fisher 95] and [Mareschal 88] uses a sensitivity analysis approach to give DMs flexibility in assigning criteria weights and help them understand how criteria weights affect the decision outcome. Although these approaches can reduce DMs' cognitive burden in determining precise weights, it may become tedious and difficult to manage as the number of the criteria increases.

Objective weights of criteria importance, measured by average intrinsic information generated by a given set of alternatives through each criterion, reflect the nature of conflicting criteria and enable the incorporation of inter-dependent criteria [Diakoulaki et al. 95]. Main drawback of objective approaches for weight assessing is that such approaches neglect the subjective judgment information of the DM [Ma et al. 99].

In the objective approach category, the entropy method can be cited [Hwang and Yoon 81]. In this method weights are determined without a direct involvement of the decision maker, in the terms of the values of the alternatives for each criterion. The essential idea is that the importance relative to a criterion j, measured by the weight w_j, is a direct function of the information conveyed by the criterion relative to the whole set of alternatives. The degree of divergence of the average intrinsic information contained by criterion j is calculated as entropy value of criterion j. In

other words, the most important criteria are those which have the greatest discriminating power between alternatives.

In addition to the entropy measure, any other method of measuring the divergence in performance ratings could be used to determine the objective weights. [Diakoulaki et al. 1995] proposes the CRITIC (CRiteria Importance Through Intercriteria Correlation) method. It is based on how great the correlation is between the columns of the decision matrix.

The weights can also be objectively obtained by solving a mathematical programming model. The aim here is to minimize the sum of the linear equal weighted summation of the separation between each alternative and the ideal one, measured by the squared distance. [Ma et al. 99] proposes an integrated approach where the pair-wise comparison matrix on attributes is given by a DM, and all attributes are objective. [Horowitz and Zappe 95] presented a linear programming model for the development of attribute weights. A number of mathematical approaches have been used by researchers including regression analysis, multidimensional scaling, logit, etc. [Shirland et al. 03].

A subjective weighting approach is not preferred in this study because firstly, as mentioned above, it may give inconsistent results. In the particular methodology, this problem can be seen even more dramatically since experts (of weight assessment) are asked to evaluate forecasts coming from two different kinds of source; namely judgments and the statistical forecasting methods.

Additionally, in relation to the inconsistency problem, the true method of gathering evaluations may be hard to decide. Finally, the subjective weighting methods need people to evaluate the given data. However in the addressed problem it is not clear who will make the evaluation. Are the weights determined by the problem owner? Who is the problem owner in a demand forecasting problem? Will the weights be evaluated by the experts? Could the experts manipulate their forecast and weight evaluation? These are the questions that cannot be answered properly if a subjective weighting method is selected.

3.3.2 Proposed objective weight assessment approach

Based on the above-mentioned reasons, we prefer an objective method to decide on the values of weights independently from any people's judgments. For this purpose an approach that is similar to the entropy method is proposed.

In information theory, entropy, or mostly referred as Shannon entropy, is a measure of the uncertainty associated with a random variable. Shannon entropy quantifies the information contained in a message. In fuzzy theory, much attention has being paid to the measure of fuzziness of a fuzzy subset, while entropy, as a measure of uncertainty, plays a significant role in the field of information theory [Xiaoshu and Fanlun, 00]. Since this study aims to give more importance, thus higher weight, to the forecast that has more information, measure of fuzziness is used to generate the weights.

The measure of fuzziness for a discrete fuzzy set A is defined as a mapping $H : P_n(X) \to R^+$ that quantifies the degree of fuzziness present in A where $P_n(X)$ is the set of all fuzzy subsets of *X*. Based on the De Luca and Termini axioms [De Luca

and Termini 72], [Ebanks 83] suggests that a measure of fuzziness should satisfy at least the following five well-known properties:

Sharpness $P1: H(A)=0 \Leftrightarrow \mu_A(x)=0 \text{ or } 1 \quad \forall x \in X;$

Maximality $P2: H(A)$ is maximum $\Leftrightarrow \mu_A(x)=0.5 \quad \forall x \in X;$

Re *solution* $P3: H(A) \geq H(A^*)$, where A^* is a sharpened version of A;

Symmetry $P4: H(A)=H(\overline{A})$, where $\mu_{\overline{A}}(x)=1-\mu_A(x) \quad \forall x \in X;$

Valuation $P5: H(A \cup B)+H(A \cap B)=H(A)+H(B)$.

Among the methods that satisfy all the above-mentioned axioms, a distance-based Minkowski index of fuzziness is used in this study due to its easy applicability. Index of fuzziness, which is a measure of fuzziness, is defined in terms of metric distance of a fuzzy set A, from any of the nearest crisp set, say crisp set C, for which defined as follows [Klir and Folger 88].

$$\begin{aligned} &\mu_C(x)=0 \quad \text{if } \mu_A(x) \leq 1/2 \\ &\mu_C(x)=1 \quad \text{if } \mu_A(x) > 1/2 \end{aligned} \qquad [2]$$

The Minkowski class of distance yields a class of fuzzy measures as given in [3] [Klir and Folger 88].

$$\begin{aligned} H_k(A) &= \left(\sum_{x \in X} \left| \mu_A(x) - \mu_C(x) \right|^k \right)^{1/k} \quad \textit{for discrete membership functions} \\ H_k(A) &= \left(\int_{x \in X} \left| \mu_A(x) - \mu_C(x) \right|^k \right)^{1/k} \quad \textit{for continuous membership functions} \end{aligned} \qquad [3]$$

where $k \in [1,+\infty]$. Special cases of $k = 1$ and $k = 2$ are Hamming and Euclidean distances respectively. For a TPD B(*p,m,o*), the fuzziness index can be measures as follows.

$$H_k(B) = \left(\frac{2^{k+1}-1}{2^k(k+1)} (p-o) \right)^{1/k} \qquad [4]$$

Equation [4] indicates that fuzziness of a TPD is proportional to the difference between optimistic and pessimistic values (i.e., the length of its support). When entropy of each forecast is calculated, their normalized values are used as their respective weights. Therefore the weight of the statistical forecast (w_s) and the judgmental forecast (w_j) are calculated as follows.

$$w_j = \frac{H_k(\tilde{D}_j)}{\sum_{k \in S,J} \left(H_k(\tilde{D}_j) + H_k(\tilde{D}_s) \right)} \quad \textit{and} \quad w_s = \frac{H_k(\tilde{D}_s)}{\sum_{k \in S,J} \left(H_k(\tilde{D}_j) + H_k(\tilde{D}_s) \right)} \qquad [5]$$

where the entropy of the s^{th} statistical forecast (or j^{th} judgemental forecast) is the numerator of the equation and the sum of the entropies of all forecast, which is used as a normalization factor, is the denominator.

For instance, if k is specified as 1 (i.e., hamming distance), then entropy will be equal to $H(B) = 3/4 (p-o)$ and the corresponding weights are calculated as follows:

$$w_j = \frac{\left(D_j^o - D_j^p\right)}{\sum_{k \in S,J}\left(D_k^o - D_k^p\right)} \quad and \quad w_s = \frac{\left(D_s^o - D_s^p\right)}{\sum_{k \in S,J}\left(D_k^o - D_k^p\right)} \qquad [6]$$

By the changing values of $k \in [1,+\infty]$, the weights also change. When k increases, differences between weights decrease. For instance, suppose that four forecasts are available with supports (i.e., (p-o) values) of 3, 7, 10, and 15. Corresponding weights for different k values are given in [Tab. 1].

K	**D_1**	**D_2**	**D_3**	**D_4**
Support →	3	7	10	15
1	0.086	0.200	0.286	0.429
2	0.152	0.232	0.277	0.339
3	0.181	0.240	0.270	0.309
4	0.197	0.243	0.266	0.294
5	0.207	0.245	0.263	0.285
10	0.228	0.248	0.257	0.268
30	0.242	0.249	0.252	0.256
100	0.248	0.250	0.251	0.252
+∞	0.250	0.250	0.250	0.250

Table 1: Weights for different k values

It is clearly seen in [Tab. 1] that when k =1, D_4, which has the highest support, gets the highest weight while D_1, which has the lowest support, has the lowest weight. When k increases the weights corresponding to the demands approaches each other. When k=+∞, all weights are equal to each other.

3.4 Aggregation of statistical and judgmental forecasts

In this step, all the forecast values coming from different sources are combined into an aggregated fuzzy forecast. The aim of the aggregation procedure is to obtain a TPD by combining all the TPDs coming from different sources. In that perspective, the problem is converted to find an optimum TPD that has the minimum weighed distance from several TPDs. If $\tilde{D}$ shows the aggregated demand, the problem can be formulated as given in [7].

$$Find\ \tilde{D} = (D^p, D^m, D^o)$$

$$Minimizing\ TotDist = \left(\sum_s w_s * Dist(\tilde{D}_s, \tilde{D}) + \sum_j w_j * Dist(\tilde{D}_j, \tilde{D}) \right) \qquad [7]$$

where $Dist()$ is a function that indicates distance between two possibility distributions, w_s is the weight of s^{th} statistical method (s=1,…S), w_j, is the weight of j^{th} expert (j=1,…,J).

Distance measures are generally studied in ranking of the fuzzy sets or numbers. Review and comparison of related literature can be seen in [Chen and Hwang 92]. Hamming distance and λ-cut based methods are most popular distance measures. In study, we select a λ-cut based distance measure because it can be applied in the

optimization problems like the one given in [7]. The distance between two arbitrary possibility distributions $\tilde{A}$ and $\tilde{B}$ with λ-cut sets, $[A^-(\lambda), A^+(\lambda)]$ and $[B^-(\lambda), B^+(\lambda)]$, is defined as follows [Zeng and Li 07]:

$$Dist(\tilde{A},\tilde{B}) = \int_0^1 \left[A^-(\lambda) - B^-(\lambda)\right]^2 d\lambda + \int_0^1 \left[A^+(\lambda) - B^+(\lambda)\right]^2 d\lambda \qquad [8]$$

For a TPD, A = $\left(A^p, A^m, A^o\right)$ λ-cut levels can be found as follows:

$$\begin{aligned} A^-(\lambda) &= A^m - (1-\lambda)(A^m - A^p) = A^p + \lambda(A^m - A^P) \\ A^+(\lambda) &= A^m + (1-\lambda)(A^o - A^m) = A^o + \lambda(A^m - A^o) \end{aligned} \qquad [9]$$

By using the equations presented in [9], the distance measure can be updated as follows:

$$\begin{aligned} Dist(\tilde{A},\tilde{B}) = &\int_0^1 \left[A^p + \lambda(A^m - A^P) - B^p - \lambda(B^m - B^P)\right]^2 d\lambda \\ &+ \int_0^1 \left[A^o + \lambda(A^m - A^o) - B^o - \lambda(B^m - B^o)\right]^2 d\lambda \end{aligned} \qquad [10]$$

When the integral given in [10] is calculated following result is obtained:

$$Dist(\tilde{A},\tilde{B}) = \left(\begin{aligned} &\left(A^m - B^m\right)^2 + \left(A^m - B^m\right)\left(A^p - B^p\right) + \left(A^p - B^p\right)^2 \\ &\quad + \left(A^m - B^m\right)^2 + \left(A^m - B^m\right)\left(A^o - B^o\right) + \left(A^o - B^o\right)^2 \end{aligned} \right) \Big/ 3 \qquad [11]$$

To solve the problem given in [7] with the distance function in [11], partial derivatives of the function with respect to three parameters of the aggregated demand ($\tilde{D} = (D^p, D^m, D^o)$) is calculated and equated to zero [Zeng and Li 07].

$$\frac{\partial TotDist}{\partial D^i} = \left(\sum_s w_s * \frac{\partial Dist(\tilde{D}_s, \tilde{D})}{\partial D^i} + \sum_j w_j * \frac{\partial Dist(\tilde{D}_j, \tilde{D})}{\partial D^i} \right) = 0 \quad (i=p,m,o) \qquad [12]$$

Partial derivatives in [12] are calculated as follows:

$$\frac{\partial TotDist}{\partial D^p} = \sum_s w_s * \left(-\left(D_s^m - D^m\right) - 2\left(D_s^p - D^p\right)\right) + \sum_j w_j * \left(-\left(D_j^m - D^m\right) - 2\left(D_j^p - D^p\right)\right) = 0$$

$$\begin{aligned} \frac{\partial TotDist}{\partial D^m} = &\sum_s w_s * \left(-4\left(D_s^m - D^m\right) - \left(D_s^p - D^p\right) - \left(D_s^o - D^o\right)\right) \\ &+ \sum_j w_j * \left(-\left(D_j^m - D^m\right) - 2\left(D_j^p - D^p\right) - \left(D_j^o - D^o\right)\right) = 0 \end{aligned}$$

$$\frac{\partial TotDist}{\partial D^o} = \sum_s w_s * \left(-\left(D_s^m - D^m\right) - 2\left(D_s^o - D^o\right)\right) + \sum_j w_j * \left(-\left(D_j^m - D^m\right) - 2\left(D_j^o - D^o\right)\right) = 0$$

The final result of the above mentioned equalities are found as follows:

$$D^i = \left(\sum_s D_s^i * w_s + \sum_j D_j^i * w_j \right) \Big/ \left(\sum_s w_s + \sum_j w_j \right) \qquad (i=p,m,o) \qquad [13]$$

The aggregated demand function is the weighted average of the input demand forecasts.

4 Numerical Example

To illustrate the proposed methodology an example referred in [Petrovic et al. 06] is used. A time series that records monthly demands for a product for a period of two years is given in [Fig. 2].

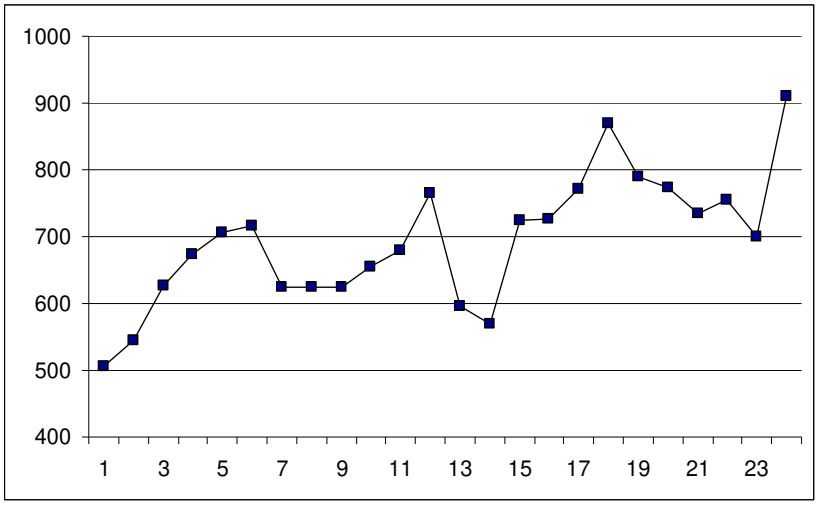

Figure 2: Historical data used in the example

Using STATGRAPHICS Centurion software package, three different time series analysis techniques, namely ARIMA(1,1,0), linear trend, and simple exponential smoothing, are applied to given data. These methods are chosen because the first one is refereed in [Petrovic et al. 06], and other two gives best results with respect to root mean square error (RMSE). The results are shown in [Tab. 2].

According to the results in [Tab. 2], the corresponding TPDs are as follows:

$$\tilde{D}_1 = (720,889,1057);\ \tilde{D}_2 = (662,818,973);\ \tilde{D}_3 = (692,844,997) \qquad [14]$$

According to the results in [14], the estimated demand range is between 622 and 1057. This is a very large range for the decision maker to give accurate decisions concerning the activities based on this forecast. These results also show a high level of variation. Hence expert judgments are crucial to get a narrower range estimate in this problem. As a result; the judgmental forecasts are employed in the problem.

s	Source of the Forecast	Forecast for the 25th month	%95 confidence interval for forecast	RMSE
1	ARIMA (1,1,0)	889	[720;1057]	81.12
2	Linear trend	818	[662, 973]	67.67
3	Simple Exponential smoothing	844	[692, 997]	79.32

Table 2: Results of the statistical methods

The expert judgments are determined based on [Petrovic et al. 06]. As a customer forecast, the expected demand is 750 while the pessimistic and optimistic forecasts are 700 and 800 respectively. The same values for the expert forecast are 730, 680, and 780, respectively. Therefore the related TPDs are as follows:

$$\tilde{D}_1 = (700,750,800);\ \tilde{D}_2 = (680,730,780) \qquad [15]$$

Finally, the results of five different forecasts are aggregated to get one fuzzy-demand forecast. For this purpose initially, the weights are determined by using the entropy approach given in [Section 3.3]. In [Tab. 3] the entropies of the forecasts and the weights are given (*k* is considered to be 1). According to these results weights of the forecasts obtained from statistical approaches are higher than weight of experts' forecasts. These results seem logical because the statistical forecasts give more information about the possible future values of the demand.

When [13] is applied to the data given in [Tab. 3], the aggregated demand is found to be $\tilde{D}(691.75,\ 832.10,\ 972.16)$. The crisp result of [Petrovic et al. 06] is 774 and the actual demand is 747. Because the result of the proposed methodology covers ones of [Petrovic et al. 06] and the actual demand, it can be considered as satisfactory.

As can be seen in [Fig. 3], the aggregate forecast value is close to the statistical forecasts due to the fact that weights of the statistical forecasts are greater than expert forecasts. In fact, the range estimated by the proposed model is narrower than the ones found by the classical statistical forecasting method. However even further refinement can be realized by a Delphi type of analysis [Rowe and Wright 99] that will be conducted with the experts.

	Source of forecast	**Forecast**	**Entropy**	**Weight**
s=1	ARIMA (1,1,0)	(720,889,1057)	252.75	0.29228
s=2	Linear trend	(662,818,973)	233.25	0.26973
s=3	Simple Exponential smoothing	(692,844,997)	228.75	0.26453
j=1	Customer forecast	(700,750,800)	75.00	0.08673
j=2	Expert forecast	(680,730,780)	75.00	0.08673

Table 3: Aggregation of the forecasts

4.1 Sensitivity analysis on the numerical example

In order to analyze the robustness of the result of the proposed methodology, sensitivity analyses are conducted. For this purpose, initially, the impact of the number of the input forecasts is analyzed through testing the possibility of dominance of the methods on the aggregated result. The number of experts and the statistical methods to be included in the analysis is not pre-determined in the methodology. One may think that if one of the statistical methods is omitted from the evaluation or other statistical methods are employed, the result of the proposed methodology may change dramatically. This sensitivity analysis is important to show the robustness of the methodology to the selection of the statistical methods or decision makers.

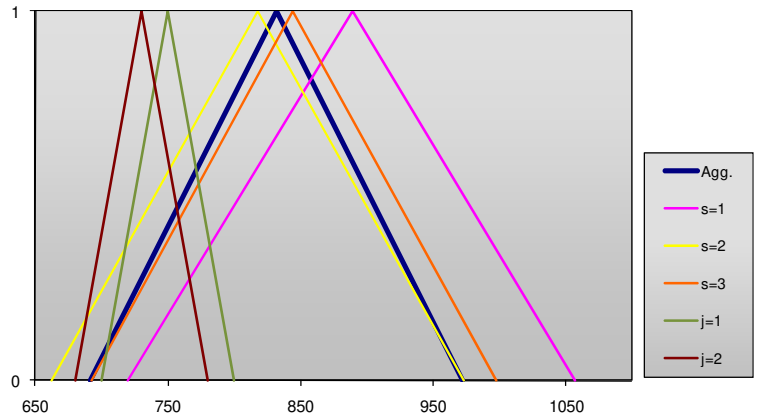

Figure 3: Results of the numerical example

The possibility of dominance of the methods on the aggregated result is tested by designing five experiments where one forecast is eliminated from the evaluation each time. According to the results given in [Fig. 4], the aggregated demands of the five experiments do not change dramatically compared to the original solution. Only the result of the experiment "s=1 excluded" seems a part from the others. This may be because of the significant difference of ARIMA's forecast from the other forecasts. Since ARIMA is included in the analysis as it is refereed in [Petrovic et al. 06] despite its high RMSE value, this result is understandable. It can be concluded from this analysis that small differences on the selection of sources do not affect the aggregated forecast noticeably. Moreover, if the RMSE of the statistical forecast is relatively high, then it is better not include that forecast in the evaluation.

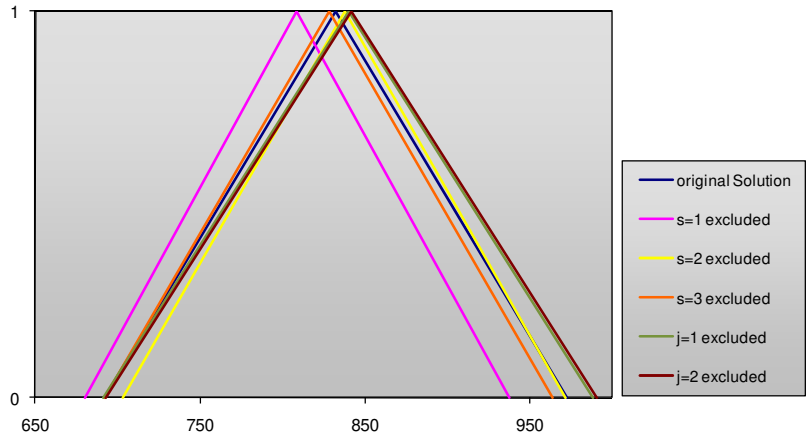

Figure 4: Results of experiments on the number of sources

Secondly, the effect of the confidence level of the statistical forecasts on the aggregated result is investigated. This analyse show the extent of effect of the confidence level on the results. For this purpose three experiments are designed where confidence levels are considered as 90%, 92%, and 98%. Results of these experiments are compared with the originally determined 95% level.

According to the results given in [Fig. 5], the increase in the confidence interval results with an increase in the supports of TPD of the statistical methods which in turn increase the support of the aggregated forecast. The most possible value of the aggregated forecast does not change dramatically based on the change of the confidence interval.

The sensitivity analysis shows that the model result is robust when the number of source and the confidence interval level are changed.

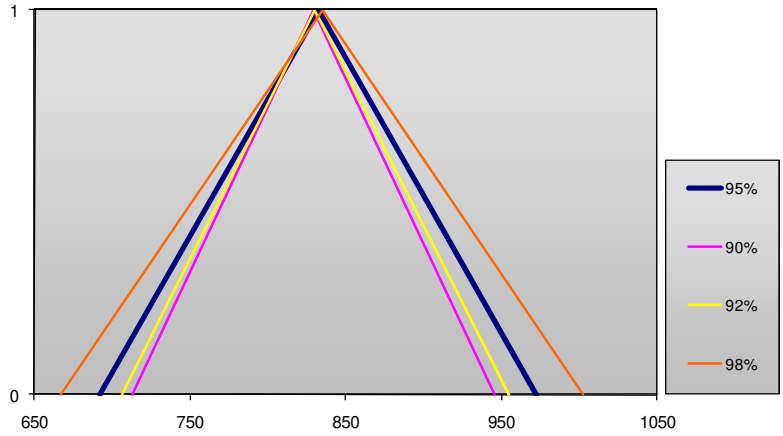

Figure 5: Results of the experiments on the confidence level

5 Conclusions and Suggestions for Future Work

In this study, a methodology is proposed for aggregating forecasts of different sources to reach a combined forecast that is represented by a TPD. The results of the methodology can be used in SC and production planning models where fuzzy demands are input. It is also beneficial to have a range estimation rather than point estimation in such uncertain phenomena.

When compared to [Petrovic et al. 06], the proposed model is easier to apply. Additionally, it satisfies fuzzy output rather than a crisp one. While the number of statistical and judgmental forecasts are limited in [Petrovic et al. 06], the number of input forecasts is not restricted in the proposed method.

In order to improve the accuracy of the proposed methodology it is necessary to obtain forecasts from heterogeneous experts. Experts vary in their knowledge and in how they approach the problems. A willingness to bring additional information and

different approaches to bear on the demand forecasting problem improves accuracy. In addition, in situations in which experts might be biased, it is important to obtain forecasts from experts with different biases [Armstrong et al. 08].

This approach is particularly applicable for situations where reliable subjective weights cannot be obtained. In fact, taking into account the fact that even with objective weighting methods there will be different weights, in order to improve the accuracy of the objective weighting it may be worthwhile to use several objective weighting approach and then take the minimum [Wang and Lee 09].

On the other hand, if it is possible to find appropriate experts who may provide reliable subjective weights, a further suggestion may be to use both subjective and objective weighting methods as suggested by [Singh and Singh 09].

Related future studies may be about getting more accurate information from the experts. Instead of asking the optimistic/most possible/pessimistic demands to the decision maker, standard linguistic expressions such as "between *a* and *b*", "more than *a*", "at least *b*" etc. can be proposed to get the information from decision makers.

Rather than relying on untested statistical procedures to demand forecast, the proposed methodology could rely upon prior evidence of which statistical forecasting methods works best under which conditions. Thus, one could turn to empirical evidence drawn from a wide variety of forecasting problems. This evidence is summarized in the Forecasting Method Selection Tree at http://forecastingprinciples.com.

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