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Trading Links and Paths on a Communication Bandwidth Market

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Abstract: This paper presents a novel market model for balancing communication bandwidth trade. The distinguishing characteristic of the model is that it assumes that market players can place buy offers not only for isolated network resources (inter-node links), but also for end-to-end network paths of predefined capacity. It also enables effective balancing of sell and buy offers for network resources in such a way which maximizes the global economic welfare. From a formal point of view, the model produces a linear programming problem for clearing a multi-commodity market. Three simple examples are used to discuss and illustrate the proposed model.
Key Words: bandwidth market, resource allocation, network design
Category: C.2.1, K.6.0

1 Introduction

The still dominating form of communication bandwidth trading consists of bilateral agreements between telecommunications companies, usually involving complex and nontransparent negotiations. With such trading patterns it is difficult to balance the demand and supply sides in an optimal fashion, especially in short and medium time horizons. Moreover, with the rapidly increasing number of market players and the growing complexity of network resources and services, value-based pricing, rather than cost-based pricing, becomes the only realistic and reasonable approach. Thus the need for more flexible, fair and optimized trading patterns, in the form of exchanges/auctions, becomes quite obvious. Potential advantages of bandwidth exchanges and auctions are discussed in [Chiu 1999]. Although bandwidth exchanges and auctions are being introduced (Arbinet [Arbinet 2006] and Merkato [Merkato 2006]), there are still many theoretical and practical challenges to address in order to make such trading patterns really efficient, reliable and scalable. This paper concerns the theoretical aspect of the issue.

The idea of a “smart bandwidth market” [MacKie-Mason and Varian 1994] was introduced in the context of problems involved in managing congestion and pricing problems in the Internet. This seminal work inspired many pricing ideas and schemes [DaSilva 2000], [Falkner 2000]. A common feature of these schemes is that competition between bandwidth providers (sellers) is not considered. Different types of auctions are widely used as a trading mechanism for bandwidth allocation. In [Courcoubetis et al. 1999] a separate auction for each network link is organized; a special, iterative mechanism is introduced to coordinate the individual link-auctions. Also, in [Lazar and Semret 1999] an iterative auction scheme for allocating bandwidth on single communication links is presented. The scheme is based on a second-price auction known as a Vickrey auction [Sandholm 2000]. The advantage of this type of auction is that players are encouraged to bid truthfully. A further modification of that trading mechanism is presented in [Bitsaki et al. 2005]. Algorithmic issues regarding bandwidth trading are studied in [Bhatia et al. 2003] and [Jain and Varaiya 2004].

The idea of a global bandwidth broker has been described in [Cheliotis 2000]. It is assumed that buy and sell offers may concern different kind of network resources. In particular, buy offers may concern communication paths, while sell offers concern network links, i.e. building blocks of end-to-end network paths. This differentiation plays an important role in the approach presented in our paper. Pricing issues in modern communications networks are discussed comprehensively in [Courcoubetis et al. 2003]. General considerations concerning auction principles and mechanism are comprehensively analysed in [Klemperer 2000].

We consider bandwidth trading from the point of view of network operators and service providers who are active market players. They buy and sell bandwidth in the process of network resources provision and management. They can also treat bandwidth buying and reselling as an investment and a potential source of profit.

The proposed model assumes that participants of the trade can place buy offers concerning not only such resources as links between network node pairs, but also concerning end-to-end transmission paths consisting of some fraction of bandwidth on a sequence of network links. The model can be applied to trading resources of any layer of a communication network architecture. The description of traded objects – links and paths, can obtain different interpretation depending on the considered type of network resources (physical media, optical links, SDH containers, ATM cell streams, IP packet streams, etc.).

Efficient market balancing requires joint optimization of trade of different kind of network resources (elementary commodities). For this purpose multi-commodity exchange models should be used, in addition to single-commodity exchanges and bilateral trading. The basic multi-commodity market clearing model developed in [Toczyłowski 2002] is in the linear programming form and enables maximizing global economic welfare and effective balancing of sell and buy offers for bundles of elementary commodities. It has all positive features of the classical single-commodity market model, yet enabling handling many real-world requirements. Since the proposed model is linear and does not involve integer-variables computations, it can be applied to large networks. It should be noted however, that the model is directed toward welfare optimization, which of course is not the only possible criteria in constructing trading mechanisms.

The paper is organized as follows. The formulation of the model is presented in Section 2. Section 3 contains a discussion of its general features. Simple examples illustrating basic properties of the model are presented in Section 4. Section 5 summaries main results.

2 The model

We will start the description of the proposed model by defining its variables and parameters: let D be the set of buy offers and E the set of sell offers.

A given buy offer d can be described as a vector $[h_d, E_d]$, where h_d is the maximal bandwidth capacity the buyer wishes to purchase, and E_d is the maximal acceptable unit price of bandwidth. In a similar way, a given sell offer e can be described as a vector $[y_e, S_e]$, where y_e is the maximal capacity the seller is willing to sell and S_e is the minimal acceptable unit price of bandwidth. It is assumed that offers can be realized partially; x_d is the realization volume of buy offer d, x_e is the realization volume of sell offer e. Variable x_{ed} denotes the bandwidth capacity allocated to sell offer e to serve buy offer d.

Every offer concerns a point-to-point bandwidth connection between a pair of specified locations in a communication network. The locations form a set of network nodes V. It is assumed that the connections are unidirectional, i.e. they have source and sink nodes. The source node for a buy offer d is denoted by s_d and the sink node by t_d. The assignment between sell offers and network nodes is expressed by the incidence matrix $[a_{ve}]$, where $a_{ve} = 1$ if offer e originates in node v, -1 if e terminates in node v, and 0 otherwise.

When offers e and d are matched, the trade is realized at contract prices p_{ed}. The contract price p_{ed} is limited: $S_e \leq p_{ed} \leq E_d$.

Trade surplus for each player is described by a relation between the individual evaluation of the offer and the value set by transaction price p_{ed}. For a buy offer d this surplus is defined as:

$$E_d x_d - \sum_{e \in \gamma(d)} p_{ed} x_{ed}$$

where $\gamma(d)$ is a set of paths allocated for buy offer d. This expression is the value of the bandwidth "seen" by the buyer, decreased by the amount spent on contracted bandwidth of related sell offers.

The surplus for a sell offer e is defined as:

$$\sum_{d:e\in\gamma(d)} (p_{ed} - S_e)x_{ed}$$

This expression is the difference between the transaction price p_{ed} and the offer price multiplied by the transaction bandwidth volume, summed over all buy offers matched to sell offer e. Thus the transaction price is a point of division of transaction surplus between the buyer and the seller.

The sum of surplus for all market participants defines the market welfare:

$$Q = \sum_d \left(E_d x_d - \sum_{e\in\gamma(d)} p_{ed}x_{ed} \right) + \sum_e \left(\sum_{d:e\in\gamma(d)} (p_{ed} - S_e)x_{ed} \right)$$

This expression can be simplified to:

$$Q = \sum_d E_d x_d - \sum_e S_e x_e$$

The aim of the trading mechanism described by the presented model is to find optimal bandwidth allocation which maximizes total individual satisfaction of market players. Thus the maximization of overall market welfare is the objective of the model.

The model is formulated as a mathematical linear program presented below.

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indices:
$d = 1, 2, 3, \ldots, D$ buy offers
$v = 1, 2, 3, \ldots, V$ network nodes
$e = 1, 2, 3, \ldots, E$ sell offers
parameters:
a_{ve} = 1 if offer e originates in node v, -1 if e terminates in node v, 0 otherwise
s_d source node for offer d
t_d sink node for offer d
h_d capacity of offer d
E_d offered unit price of buy offer d
y_e capacity of offer e
S_e offered unit price of sell offer e
variables:
x_{ed} bandwidth flow serving buy offer d allocated to sell offer e
x_d contracted bandwidth capacity for offer d
x_e contracted bandwidth capacity for offer e

constraints:

$$\sum_{e \in E} a_{ve} x_{ed} = \begin{cases} x_d & v = s_d \\ 0 & v \neq s_d, t_d \\ -x_d & v = t_d \end{cases} \quad \forall v \in V, \forall d \in D \tag{1}$$

$$\sum_{d \in D} x_{ed} \leq x_e \quad \forall e \in E \tag{2}$$

$$0 \leq x_d \leq h_d \quad \forall d \in D \tag{3}$$

$$0 \leq x_e \leq y_e \quad \forall e \in E \tag{4}$$

$$0 \leq x_{ed} \quad \forall e \in E, \forall d \in D \tag{5}$$

objective:

$$\hat{Q} = \max \left(\sum_{d \in D} E_d x_d - \sum_{e \in E} S_e x_e \right) \tag{6}$$

Both the objective function and the model constraints are linear. The flow constraints (1) are expressed by demand realization variables x_d (not by the nominal volumes of the offered demands h_d). Constraints (3) and (4) set upper bounds on bandwidth realization of demand and supply offers. The constraint (2) sets the values of contracted bandwidth volume on each link according to flows allocated to network paths.

Note that in the above formulation the market welfare is dependent on offer prices but not on transaction prices (contract price variables do no appear in the model), i.e. the transaction prices are not of direct interest; what is of interest is the allocation of bandwidth resources to buyers. This feature is a direct consequence of the adopted definition of welfare.

3 General features of the model

In the proposed model efficient market balance is obtained in the effect of joint optimization of many elementary buy and sell offers. The formulated multi-commodity exchange problem preserves all positive features of the classical single-commodity market model. The benefit of the multi-commodity exchange is in marshalling competition among the greatest number of potential buyers and sellers. From a global perspective, the multi-commodity exchange is effective in the sense that it maximizes the global economic surplus. Thus, for given offers, no better allocation of bandwidth resources is possible.

From an individual player point of view an important feature of the exchange is the "transparency" and fairness conditions of clearing, which encourage players to place sincere offers and to use truthful bidding strategies reflecting their underlying values. The presented multi-commodity exchange model provides transparent and fair conditions of clearing, since the dual prices in the optimal solution enables setting the competitive market prices for all bandwidths resources on individual links [Toczyłowski 2002]. In particular, any competitive offer (that provides a positive surplus over it's market value) is always selected for realization. On the other hand, any offer that provides a negative surplus over its market value is rejected.

In an auction analysis it is usually assumed that players have independent private values of the goods, which means that each player knows its own valuation that is unaffected by the value other players place on the good. An auction is incentive compatible if the players optimize their expected utilities by bidding their true values of the goods. This is a desirable feature because a player's decision depends only on local information, and he gains no advantage from "modeling" other agents. An allocation is efficient if no further gains from trade is possible, i.e. it maximizes the total welfare. This implies that resources are allocated to the players who value them most highly.

In the multi-commodity exchange model some resources may be allocated to buyers who temporarily offer the highest bid prices, which is not necessarily is the most efficient. However, the exchange model guarantees truthful bidding in a "weak" sense: the buyers and sellers that are price-takers, i.e. that have negligible chance of being pivotal and setting the common market clearing prices, have incentives to place their bids very close to true values [Toczyłowski 2002]. On the other hand, if on the market there is a high concentration of bids and only few strong players account for the winning bids, there still may remain some incentives for bidding strategies that could exploit the market power for revenue gains.

Our formulation of the bandwidth trading model resembles a multi-commodity flow problem which appears in designing (dimensioning) communication networks [Pióro and Medhi 2004]. The resemblance is apparent if the bandwidth buy and sell offers are interpreted as traffic demands and network links, respectively. There are however two important differences. Firstly, the buy offers in the presented model may be realized only partially, whereas in a usual network design problem the objective is to satisfy fully all traffic demands; if this is not the case, the network design objective is not achieved. Secondly, while for the bandwidth market case the objective is to maximize global economic welfare, in the network design problem the objective is to minimize total cost of network resources, i.e. to minimize the total cost of investment for each particular network being designed and for an assumed resources cost structure. In other words, the differences are both in the objective function and the constraints.

4 Basic properties with examples

In the following some basic properties of the proposed model are illustrated with the examples. The examples are very simple in order to enable inspection of the properties without tedious calculations.

Example 1

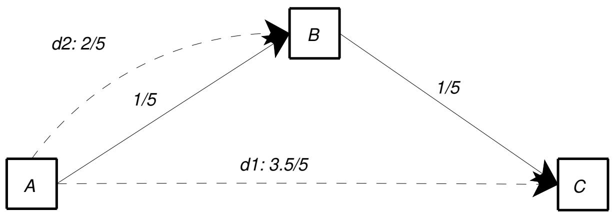

Figure 1: *Example 1*

The goal of this example is to show the difference between two cases: in case (1) buy offers concern end-to-end network path (Fig. 1), whereas in case (2) buy offers concern separate network links (i.e. resources between adjacent network nodes). In both cases sell offers concern network links.

In Fig. 1 solid lines denote sell offers; in the "x/y" notation, x is the sell offer price and y is the offered capacity. Broken lines represent buy offers described similarly by "d: x/y", where x is buy offer price, y is the required capacity and d is a label of the buy offer.

(1) Welfare maximization ($\hat{Q} = 7.5$) results in realizing demand $d1$ on path (A, B, C); demand $d2$ is not realized.

(2) Buy offer $d1$ is modified comparing to case (1); it has the form of two separate buy offers: for link (A, B) and link (B, C). Suppose that the offered price is 1.75 for each link, i.e. 3.5 for path (A, C), as in case (1). Sell offers – as in case (1). Buy offer $d2$ – as in case (1). Since $d2$ offers a higher price for link (A, B) than $d1$, so $d2$ is realized and $d1$ is not. The welfare in this case is $\hat{Q} = 5$, i.e. is not maximized. In effect case (1) is superior.

Example 2

Consider the example from Fig. 2. The notation convention is the same as in the previous example. As can be readily seen, there are (at least) two solutions which maximize welfare ($\hat{Q} = 40$):

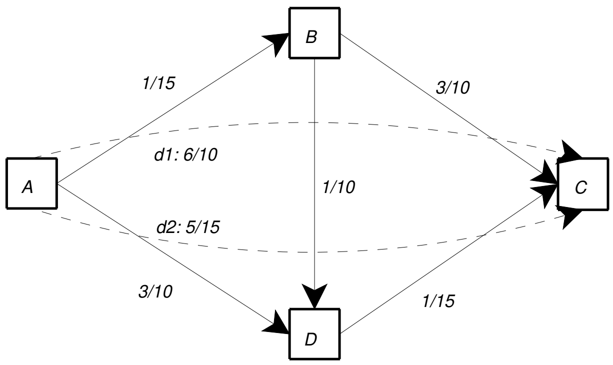

Figure 2: *Example 2*

(1) $d1$ is fully realized with path (A, B, D, C) of capacity 10; $d2$ is realized only partially with two paths: (A, B, C) of capacity 5 and (A, D, C) of capacity 5

(2) $d1$ is fully realized with path (A, D, C) of capacity 10, $d2$ is also fully realized with path (A, B, D, C) of capacity 5 and path (A, B, C) of capacity 10.

Although both solution are optimal in the sense of maximal market welfare, the solution (2) may be considered superior, because all demands are realized.

In general however, the question arises: which of the equivalent solutions in terms of welfare should be chosen. The model presented in the previous section does not give the answer, so it should be enhanced with some criterion in addition to welfare maximization. The objective of the additional criterion might be, for example, to maximize the volume of realized buy offers, defined as $\xi = \sum_{d \in D} x_d$. Applying this to the cases above yields $\xi = 20$ for case (1) and $\xi = 25$ for case (2), so solution (2) is chosen.

An other criterion for choosing from equivalent solutions could be e.g.: choose the solution that fully satisfies the greatest number of buy offers. With this criterion also solution (2) would be chosen.

The problem indicated in the example above may be also solved in a different way, notably by appending an additional criterion directly to the original model. For example, trade volume maximization could be the criterion. This however would lead to a multi-criteria optimization problem. To stay within the single-criterion problem one can introduce some additional criterion into the objective function in the form of a penalty factor, e.g.:

$$\hat{Q} = \max\left(\sum_{d\in D} E_d x_d - \sum_{e\in E} S_e x_e + \delta \sum_{d\in D} x_d\right) \tag{7}$$

where δ is a coefficient indicating the weight of the penalty.

Example 3

The example shown in Fig. 3 describes bandwidth allocation on a bottleneck resource – interpreted as a bandwidth sell offer between nodes E and F. There are three buy offers and five sell offers, the notation convention is the same as in the previous examples.

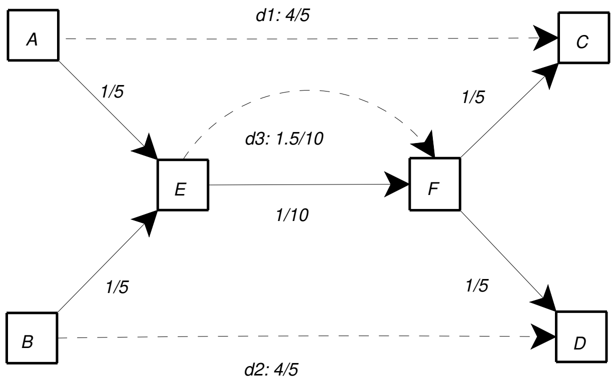

Figure 3: *Example 3*

Welfare maximization yields $\hat{Q} = 10$. Whereas offer $d1$ is realized on path (A, E, F, C) and $d2$ is realized on path (B, E, F, D), buy offer $d3$ is not realized.

Note that if the offer price of buy offer $d3$ is increased from 1.5 to over 2, then welfare maximization results in realizing $d3$ and rejecting $d1$ and $d2$. In such a case, all links except (E, F) are not traded in the considered market clearing instance, i.e. may become objects of trade in the future – in some future clearing instance, and thus present some potential source of future market welfare. This kind of circumstance is not taken into account in the clearing algorithm considered in the paper as it does not refer to time-dependant (multi-instance) welfare optimization issues. This, however, is an important, real-world issue requiring extensions of the basic model.

5 Conclusions

In this paper we have addressed the problem of balancing communication bandwidth trade. The proposed model provides effective allocation of network resources aiming at maximization of the global economic welfare. It follows a centralized approach towards bandwidth allocation: a single market operator matches market offers. Market players place buy offers not only for isolated network resources, i.e., inter-node links, but also for end-to-end network paths of predefined capacity. The model is formulated as a linear programming optimization problem. Some of its characteristic features and properties have been presented and discussed.

The model may be extended in several ways in order to take account of specific, real-world trade circumstances. For example, taking into account capacity modularity is usually required. This however leads to integer variable problems, i.e. increases computational complexity. It might be also useful to assume that players may formulate their offers in a more complex way, e.g. by expressing the buy/sell capacity in some range, rather than just specifying the maximal capacity. Extensions are also necessary to take account of selling/buying of whole subnetworks, rather than just individual links/paths. Real world exchanges may also need to take into consideration the hierarchically layered structure of network resources.

These and other extensions of the basic model proposed in this paper will be presented in the authors' future papers.

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