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Equivalent Transformations of Automata by Using Behavioural Automata [1]

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Abstract: This paper uses category theory to emphasize the relationships between Mealy, Moore and Rabin-Scott automata, and the behavioural automata are used as a unifying framework. Some of the known links between Mealy, Moore and Rabin-Scott automata are translated into isomorphisms of categories, and we also show how behavioural automata connect to these automata. Considering the distinction between final and sequential behaviours of an automaton, we define a sequential version of Mealy automata and study its relationship to behavioural automata.

Key Words: Mealy, Moore and Rabin-Scott automata, semiautomata, behavioural automata, final and sequential behaviours of automata, category theory
Category: F.1, F.1.1.

1 Introduction

Several kinds of automata and behaviours of automata have been studied in the literature. Well-known classes of automata are Mealy, Moore and Rabin-Scott automata. Their behaviour can be classified as total vs external, and sequential vs final (see [Rudeanu 1989a, Rudeanu 1989b]). In this paper we continue the previous investigation on sequential and final behaviour [Ciobanu and Rudeanu 2006], especially on the sequential behaviour. Informally speaking, a sequential machine has a finite set S of states, receives inputs from a given set I, and produces outputs from a given set O. If the input signals $i_1, \ldots, i_n$ are successively applied, a sequence $o_1, \ldots, o_n$ of outputs is produced; it depends on the state s of the machine at the moment when the first input is received. We identify the above sequences with the words $i_1 \ldots i_n \in I^+$ and $o_1 \ldots o_n \in O^+$. We refer to $o_1 \ldots o_n$ as the *sequential behaviour* of the machine.

[1] C.S. Calude, G. Stefanescu, and M. Zimand (eds.). *Combinatorics and Related Areas. A Collection of Papers in Honour of the 65th Birthday of Ioan Tomescu.*

The sequential functioning has led us to an equivalent "sequential" definition of Mealy automata, the equivalence meaning in fact an isomorphism of categories (Theorem 5). It has been noted in the literature that

– a Rabin-Scott automaton can be viewed as a Moore automaton with output set $\{0,1\}$.
  The behavioural automata were introduced and studied in [Rudeanu 1989a] and [Rudeanu 1989b], based on the remarks that

– both a Rabin-Scott automaton and a Mealy automaton can be viewed as a behavioural automaton.
  In [Ciobanu and Rudeanu 2006] it has been noted that

– a Moore automaton can be also viewed as a behavioural automaton.

The first remarks are formally presented in this paper as isomorphisms of categories (Theorems 8, 11 and 16), while for the last remark we construct a functor from the category of Moore automata to a category of behavioural automata, which is not even a monofunctor, although it is the identity on morphisms (Proposition 18).

Unless otherwise stated, in the identities below we understand the quantifiers $\forall\, s \in S$; $\forall\, i \in I; \forall\, w, w_1, w_2 \in I^*$.

2 Mealy Automata

Recall that a *Mealy automaton* is an algebra (S, I, O, δ, μ), where (S, I, δ) is a semiautomaton, and $\mu : S \times I \longrightarrow O$; the former condition means that $\delta : S \times I^* \longrightarrow S$ is a function which satisfies

$$(1) \qquad \delta(s,\varepsilon) = s \ \ \& \ \ \delta(\delta(s,w_1),w_2) = \delta(s,w_1w_2)\ .$$

Definition 1. Let **Mealy** be the category of Mealy automata as prescribed by multi-sorted universal algebra. Let **Mealy**IO be the subcategory of **Mealy** whose objects are the Mealy automata with fixed input set I and output set O, while the morphisms are the morphisms in **Mealy** of the form $(h, 1_I, 1_O)$.

In other words, the objects of **Mealy**IO are the Mealy automata of the form (S, I, O, δ, μ) with fixed I, O, while the morphisms $h : (S, I, O, \delta, \mu) \longrightarrow (S', I, O, \delta', \mu')$ are defined by

$$(2) \qquad h(\delta(s,w)) = \delta'(h(s),w)\ ,$$

$$(3) \qquad \mu(s,i) = \mu'(h(s),i)\ .$$

Condition (2) defines the *semiautomaton morphisms* $(h, 1_I)$, namely $(h, 1_I)$: $(S, I, \delta) \to (S', I, \delta')$. We will identify the morphism $(h, 1_I, 1_O)$ with h.

The distinction between final and sequential behaviours yields a "sequential" definition of Mealy automata.

Definition 2. Let $\mathbf{Mealy}^*IO$ be the category whose objects are the algebras $(S, I, O, \delta, \alpha)$, where (S, I, δ) is a semiautomaton and $\alpha : S \times I^* \longrightarrow O^*$ satisfies

$$\alpha(s, i) \in O \;, \tag{4}$$

$$\alpha(s, wi) = \alpha(s, w)\alpha(\delta(s, w), i) \;, \tag{5}$$

while the morphisms $h : (S, I, O, \delta, \alpha) \longrightarrow (S', I, O, \delta', \alpha')$ are the semiautomaton morphisms h which satisfy

$$\alpha(s, i) = \alpha'(h(s), i) \;. \tag{6}$$

Proposition 3. *In* $\mathbf{Mealy}^*IO$ *the components* α *of the objects and the morphisms* h *satisfy the identities*

$$\alpha(s, \varepsilon) = \varepsilon \;, \tag{7}$$

$$\alpha(s, w_1w_2) = \alpha(s, w_1)\alpha(\delta(s, w_1), w_2) \;, \tag{8}$$

$$\alpha(s, w) = \alpha'(h(s), w) \;. \tag{9}$$

Proof. Taking $w := \varepsilon$ in (5) and using (1), we obtain $\alpha(s, i) = \alpha(s, \varepsilon)\alpha(s, i)$, which implies (7).

We prove (8) by induction on w_2. For $w_2 := \varepsilon$, (8) is verified via (1). For $w_2 := i$, (8) reduces to (5). Supposing that (8) holds for w_2, we prove it for w_2i by using in turn (5), then the inductive hypothesis and (1), and finally (5):

$$\alpha(s, w_1)\alpha(\delta(s, w_1), w_2i) = \alpha(s, w_1)\alpha(\delta(s, w_1), w_2)\alpha(\delta(\delta(s, w_1), w_2), i)$$

$$= \alpha(s, w_1w_2)\alpha(\delta(s, w_1w_2), i) = \alpha(s, w_1w_2i) \;.$$

Similarly, property (9) is verified for $w := \varepsilon$ in view of (7), while for $w := i$ it reduces to (6). The passage from w to wi follows by using in turn (5), the inductive hypothesis, (2) and again (5):

$$\alpha(s, wi) = \alpha(s, w)\alpha(\delta(s, w), i) = \alpha'(h(s), w)\alpha'(h(\delta(s, w)), i)$$

$$= \alpha'(h(s), w)\alpha'(\delta'(h(s), w), i) = \alpha'(h(s), wi) \;.$$

Corollary 4. *Conditions* (4), (5), (6) *are equivalent to* (4), (8), (9).

Theorem 5. *The categories* $\mathbf{Mealy}IO$ *and* $\mathbf{Mealy}^*IO$ *are isomorphic.*

Proof. Define

$$F_1 : \mathbf{Mealy}IO \longrightarrow \mathbf{Mealy}^*IO$$

by $F_1(S, I, O, \delta, \mu) = (S, I, O, \delta, \alpha)$, where $\alpha : S \times I^* \longrightarrow O^*$ is defined by (7),

$$\alpha(s, i) = \mu(s, i) \ , \tag{10}$$

and (5), while $F_1(h) = h$ on morphisms.

Since (10) implies (4), F_1 is well defined on objects. If h is a morphism in $\mathbf{Mealy}IO$, then h is a semiautomaton morphism which satisfies (3), hence we obtain (6) via (10):

$$\alpha(s, i) = \mu(s, i) = \mu'(h(s), i) = \alpha'(h(s), i) \ .$$

So h is a morphism in $\mathbf{Mealy}^*IO$ too, showing that F_1 is well defined on morphisms.

Define

$$G_1 : \mathbf{Mealy}^*IO \longrightarrow \mathbf{Mealy}IO$$

by $G_1(S, I, O, \delta, \alpha) = (S, I, O, \delta, \mu)$, where $\mu : S \times I \longrightarrow O$ is defined by

$$\mu(s, i) = \alpha(s, i) \ , \tag{10'}$$

while $G_1(h) = h$ on morphisms.

It follows by (10′) and (4) that $\mu(s, i) \in O$, therefore G_1 is well defined on objects. If h is a morphism in $\mathbf{Mealy}^*IO$, then h is a semiautomaton morphism which satisfies (6). According to (10′), identity (6) becomes (3), so that h is a morphism in $\mathbf{Mealy}IO$ too, showing that G_1 is well defined on morphisms.

It remains to prove that G_1F_1 and F_1G_1 are identity maps on objects. Indeed,

$$G_1F_1(S, I, O, \delta, \mu) = G_1(S, I, O, \delta, \alpha) = (S, I, O, \delta, \mu') \ ,$$

where $\mu'(s, i) = \alpha(s, i) = \mu(s, i)$ by (10′) and (10). Besides,

$$F_1G_1(S, I, O, \delta, \alpha) = F_1(S, I, O, \delta, \mu) = (S, I, O, \delta, \alpha') \ ,$$

where we have to prove that $\alpha' = \alpha$. Indeed, $\alpha'(s, \varepsilon) = \varepsilon = \alpha(s, \varepsilon)$ by (7), while $\alpha'(s, i) = \mu(s, i) = \alpha(s, i)$ by (10) and (10′). Finally if $\alpha'(s, w) = \alpha(s, w)$, then by using (5), the inductive hypothesis and $\alpha'(s, i) = \alpha(s, i)$, we obtain

$$\alpha'(s, wi) = \alpha'(s, w)\alpha'(\delta(s, w), i) = \alpha(s, w)\alpha(\delta(s, w), i) = \alpha(s, wi) \ .$$

3 Moore and Rabin-Scott Automata

Recall that a *Moore automaton* is an algebra (S, I, O, δ, μ) where (S, I, δ) is a semiautomaton and $\mu : S \longrightarrow O$. The category **Moore** of Moore automata is defined as prescribed by multi-sorted universal algebra.

Definition 6. The subcategory $\mathbf{Moore}IO$ of $\mathbf{Moore}$ has as objects the Moore automata with fixed input set I and output set O, while the morphisms are the morphisms in $\mathbf{Moore}$ of the form $(h, 1_I, 1_O)$. So $h : S \longrightarrow S'$ and we will identify the morphism with its component h.

In view of the above definition, the morphism conditions are (2) and

$$\mu(s) = \mu'(h(s)) \; . \tag{11}$$

Recall also that a *Rabin-Scott automaton* is an algebra (S, I, δ, F) where (S, I, δ) is a semiautomaton and F is a subset of S (the set of *final states*).

Definition 7. Let $\mathbf{RS}I$ be the category of Rabin-Scott automata with a fixed input set I, the morphisms being the semiautomaton morphisms h which satisfy

$$s \in F \Longleftrightarrow h(s) \in F' \; . \tag{12}$$

Theorem 8. *The categories* $\mathbf{Moore}I\{0,1\}$ *and* $\mathbf{RS}I$ *are isomorphic.*

Proof. Define

$$F_2 : \mathbf{Moore}I\{0,1\} \longrightarrow \mathbf{RS}I$$

by $F_2(S, I, \{0,1\}, \delta, \mu) = (S, I, \delta, F)$ with $s \in F \Longleftrightarrow \mu(s) = 1$, and $F_2(h) = h$. But h is a semiautomaton morphism and (11) implies (12) because

$$s \in F \Longleftrightarrow \mu(s) = 1 \Longleftrightarrow \mu'(h(s)) = 1 \Longleftrightarrow h(s) \in F' \; ,$$

so that h is a morphism in $\mathbf{RS}I$ and the definition $F_2(h) = h$ is correct.

Define

$$G_2 : \mathbf{RS}I \longrightarrow \mathbf{Moore}I\{0,1\}$$

by $G_2(S, I, \delta, F) = (S, I, \{0,1\}, \delta, \mu)$ where $\mu : S \longrightarrow \{0,1\}$ is defined by $\mu(s) = 1 \Longleftrightarrow s \in F$, while $G_2(h) = h$. But h is a semiautomaton morphism and (12) implies (11) because

$$\mu(s) = 1 \Longleftrightarrow s \in F \Longleftrightarrow h(s) \in F' \Longleftrightarrow \mu'(h(s)) = 1 \; ,$$

so that h is a morphism in $\mathbf{Moore}I\{0,1\}$ and the definition $G_2(h) = h$ is correct.

The functoriality of F_2 and G_2 and the fact that G_2F_2 and F_2G_2 are identity functors, are immediate.

4 Behavioural Automata and Connections

Definition 9. Consider two fixed sets I and Ω and a fixed partial function $\Phi : \Omega^2 \xrightarrow{\circ} \Omega$. A *behavioural automaton* or *b-automaton* for short, is an algebra $(S, I, \Omega, \delta, \alpha)$, where (S, I, δ) is a semiautomaton and $\alpha : S \times I^* \longrightarrow \Omega$ is a function which satisfies

$$\alpha(s,w) = \Phi(\alpha(s,\varepsilon), \alpha(s,w)) \;, \tag{13}$$

$$\alpha(\delta(s,i),w) = \Phi(\alpha(s,i), \alpha(s,iw)) \;, \tag{14}$$

to the effect that the right-hand sides exist and the equalities hold.

In [Rudeanu 1989a] and [Rudeanu 1989b], the behavioural automata were called *behaviouristic*[2] automata.

Definition 10. The category $\mathbf{Beh}I\Omega\Phi$, or simply $\mathbf{Beh}$, of b-automata has these algebras as objects, while the morphisms are the triples $(h, 1_I, 1_\Omega)$, where $h : S \longrightarrow S'$ is a semiautomaton morphism and

$$\alpha(s,w) = \alpha'(h(s),w) \;. \tag{9}$$

Theorem 11. *The category* $\mathbf{Beh}I\{0,1\}\Phi_2$*, where* $\Phi_2 : \{0,1\}^2 \to \{0,1\}$*,* $\Phi_2(x,y) = y$*, is isomorphic to the category* $\mathbf{RS}I$*.*

Proof. Define

$$F_3 : \mathbf{Beh}I\{0,1\}\Phi_2 \longrightarrow \mathbf{RS}I$$

by $F_3(S,I,\{0,1\},\delta,\alpha) = (S,I,\delta,F)$ with $F = \{\delta(s,w) \mid \alpha(s,w) = 1\}$, and $F_3(h) = h$. Then $F_3(h) = h$ is a morphism of Rabin-Scott automata because it is a semiautomaton morphism and (9) implies (12):

$$s \in F \Longleftrightarrow \delta(s,\varepsilon) \in F \Longleftrightarrow \alpha(s,\varepsilon) = 1$$

$$\Longleftrightarrow \alpha'(h(s),\varepsilon) = 1 \Longleftrightarrow \delta'(h(s),\varepsilon) \in F' \Longleftrightarrow h(s) \in F' \;.$$

Define

$$G_3 : \mathbf{RS}I \longrightarrow \mathbf{Beh}I\{0,1\}\Phi_2$$

by $G_3(S,I,\delta,F) = (S,I,\{0,1\},\delta,\alpha)$ with $\alpha(s,w) = 1 \Longleftrightarrow \delta(s,w) \in F$, and $G_3(h) = h$. To prove that G_3 is correctly defined on objects, we note that condition (13) is trivially satisfied in $\mathbf{Beh}I\{0,1\}\Phi_2$, while condition (14) reduces to $\alpha(\delta(s,i),w) = \alpha(s,iw)$, which is fulfilled because

$$\alpha(\delta(s,i),w) = 1 \Longleftrightarrow \delta(\delta(s,i),w) \in F \Longleftrightarrow \delta(s,iw) \in F \Longleftrightarrow \alpha(s,iw) = 1 \;.$$

Besides, $G_3(h) = h$ is indeed a morphism in $\mathbf{Beh}I\{0,1\}\Phi_2$, because h is a semiautomaton morphism and (12) implies (9):

$$\alpha(s,w) = 1 \Longleftrightarrow \delta(s,w) \in F \Longleftrightarrow h(\delta(s,w)) \in F'$$

[2] Behaviourism : theory that man's actions are automatic responses to stimuli and not dictated by consciousness; behaviouristic, a.; cf. The New National Dictionary, Collins, London and Glasgow, 1966.

$$\iff \delta'(h(s), w) \in F' \iff \alpha'(h(s), w) = 1 \ .$$

The functoriality of F_3 and G_3 is immediate. To prove the isomorphism, we compute

$$G_3F_3(S, I, \{0,1\}, \delta, \alpha) = G_3(S, I, \delta, F) = (S, I, \{0,1\}, \delta, \alpha') \ ,$$

where $\alpha'(s, w) = 1 \iff \delta(s, w) \in F \iff \alpha(s, w) = 1$, showing that $\alpha' = \alpha$ and hence G_3F_3 is the identity, and

$$F_3G_3(S, I, \delta) = F_3(S, I, \{0,1\}, \delta, \alpha) = (S, I, \delta, F') \ ,$$

where

$$s \in F' \iff \delta(s, \varepsilon) \in F' \iff \alpha(s, \varepsilon) = 1 \iff \delta(s, e) \in F \iff s \in F \ ,$$

showing that $F' = F$ and hence F_1G_1 is the identity.

Corollary 12. *The categories* $\mathbf{Moore}I\{0,1\}$ *and* $\mathbf{Beh}I\{0,1\}\Phi_2$ *are isomorphic.*

Definition 13. Consider the category $\mathbf{Beh}IO^*\Phi_{2-1}$, where $\Phi_{2-1} : O^* \times O^* \overset{\circ}{\longrightarrow} O^*$ is defined by $\Phi_{2-1}(\omega_1, \omega_1\omega) = \omega$, else nil; in other words, $\Phi_{2-1}(\omega_1, \omega_2) = \omega \iff \omega_1\omega = \omega_2$.

It follows that condition (13) becomes $\alpha(s, w) = \alpha(s, \varepsilon)\alpha(s, w)$, which is equivalent to $\alpha(s, \varepsilon) = \varepsilon$. Therefore the objects of $\mathbf{Beh}IO^*\Phi_{2-1}$ are the algebras $(S, I, O^*, \delta, \alpha)$ where (S, I, δ) is a semiautomaton and the map $\alpha : S \times I^* \longrightarrow O^*$ satisfies

$$\alpha(s, e) = \varepsilon \ , \tag{7}$$

$$\alpha(s, wi) = \alpha(s, w)\alpha(\delta(s, w), i) \ , \tag{5}$$

(which are the translations of (13), (14)), while the morphisms are the semiautomaton morphisms $h : S \longrightarrow S'$ which satisfy (9).

Lemma 14. *The map* $\alpha \in (S, I, O^*, \delta, \alpha) \in \mathbf{Beh}IO^*\Phi_{2-1}$ *satisfies*

$$\alpha(s, w_1w_2) = \alpha(s, w_1)\alpha(\delta(s, w_1), w_2) \ . \tag{8}$$

Proof. For $w_1 := \varepsilon$, condition (8) reduces to an identity via (1), while for $w_2 := i$ it reduces to (5). Now if (8) holds, then by using in turn (8) with $w_2 := iw_2$, then (5) with $s := \delta(s, w_1)$ and finally (8) with $w_2 := i$, we obtain

$$\alpha(s, w_1iw_2) = \alpha(s, w_1)\alpha(\delta(s, w_1), iw_2)$$

$$= \alpha(s, w_1)\alpha(\delta(s, w_1), i)\alpha(\delta(\delta(s, w_1), i), w_2) = \alpha(s, w_1i)\alpha(\delta(s, w_1i), w_2) \ .$$

Definition 15. Let $\mathbf{Beh}_{MealyIO}$ be the full subcategory of $\mathbf{Beh}IO^*\Phi_{2-1}$ whose objects are the b-automata which satisfy

$$\alpha(s,i) \in O \ . \tag{4}$$

Theorem 16. *The categories* $\mathbf{Mealy}^*IO$ *and* $\mathbf{Beh}_{MealyIO}$ *coincide.*

Proof. According to Definition 2, the category $\mathbf{Mealy}^*IO$ is characterized by conditions (4),(5),(6), while in view of Definitions 13 and 15 $\mathbf{Beh}_{MealyIO}$ is characterized by conditions (4),(5),(7),(9). But clearly (9)$\Longrightarrow$(6), hence, using also Proposition 3, we get

$$\{(4),(5),(7),(9)\} \Longrightarrow \{(4),(5),(6)\} \Longrightarrow \{4),(5),(7),(9)\} \ .$$

Corollary 17. *The categories* $\mathbf{Beh}_{\mathrm{Mealy}IO}$ *and* $\mathbf{Mealy}IO$ *are isomorphic.*

Proof. By Theorem 5 and Theorem 16.

Remark A: There is a fully faithful monofunctor

$$F_4 : \mathbf{Mealy}IO \longrightarrow \mathbf{Moore}IO$$

defined by $F_4(S,I,O,\delta,\mu) = (S',I,O,\delta',\mu')$, where $S' = S \times O$, $\delta'((s,o),i) = (\delta(s,i),\mu(s,i))$, $\mu'(s,o) = o$ and $F_4(h) = h$.

It is worth to note that the transformation F_4 over objects has been used in the literature to prove that Mealy automata and Moore automata have the same generative power ([Creanga et al. 1973], Theorem II.6.7).

Remark B: There is a functor

$$G_4 : \mathbf{Moore}IO \longrightarrow \mathbf{Mealy}IO$$

defined as follows: $G_4(S,I,O,\delta,\mu) = (S,I,O,\delta,\mu')$ with $\mu'(s,i) = \mu(\delta(s,i))$, and $G_4(h) = h$.

Proposition 18. *There is a functor*

$$G'_4 : \mathbf{Moore}IO \longrightarrow \mathbf{Beh}_{Mealy}$$

as follows: $G'_4(S,I,O,\delta,\mu) = (S,I,O^*,\delta,\alpha)$, *where*

$$\alpha(s,\varepsilon) = \varepsilon \ , \tag{7}$$

$$\alpha(s,wi) = \alpha(s,w)\mu(\delta(s,wi)) \ , \tag{15}$$

and $G'_4(h) = h$ *on morphisms.*

Proof. In view of Theorem 5 and Remark A we obtain the functor

$$G'_4 = F_1 \circ G_4 : \mathbf{Moore}IO \longrightarrow \mathbf{Beh}_{Mealy}$$

such that $G'_4(S, I, O, \delta, \mu) = (S, I, O^*, \delta, \alpha)$ satisfies (7),

$$\alpha(s, i) = \mu'(s, i) = \mu(\delta(s, i)) \tag{16}$$

and $G'_4(h) = h$. It remains to prove (15), which follows from

$$\alpha(s, i_1 \dots i_n) = \mu(\delta(s, i_1))\mu(\delta(s, i_1 i_2)) \dots \mu(\delta(s, i_1 \dots i_n)) \ . \tag{17}$$

And (17) can be proved without difficulty, by induction.

To see that G_4 is not a monofunctor, consider the following example. Suppose $\mathcal{A}_1 = (S, I, O, \delta, \mu_1)$ and $\mathcal{A}_2 = (S, I, O, \delta, \mu_2)$ are Moore automata for which there is $\sigma \in S$ such that $\sigma \neq \delta(s, w)\ \forall\, s \in S\ \forall\, w \in I^+$, while μ_1 and μ_2 coincide outside σ. Then $G_4(\mathcal{A}_1) = G_4(\mathcal{A}_2)$.
In particular this might happen even if $\mathcal{A}_1$ and $\mathcal{A}_2$ are reachable automata with common initial state s_0 and $\sigma = s_0$.

Remark C: We can associate with each category $\mathbf{K}$ considered above, a category $\mathbf{K}^0$ whose objects are the objects of $\mathbf{K}$ endowed with an *initial state* s_0, while the morphisms are those morphisms h in $\mathbf{K}$ which satisfy an extra condition $h(s_0) = s'_0$. Then the results we have established above are duplicated by practically identical results for the category $\mathbf{K}^0$.

The functors between categories are summarized in the following diagrams:

$$\begin{tikzcd}
\mathbf{Beh}_{Mealy}IO \arrow[r, equal] & \mathbf{Mealy}^*IO \arrow[d, shift right, "G_1"'] \arrow[d, phantom, "\simeq"] \\
\mathbf{Moore}IO \arrow[u, "G'_4"] \arrow[r, shift right, "G_4"'] & \mathbf{Mealy}IO \arrow[l, shift right, "F_4"'] \arrow[u, shift right, "F_1"']
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{Moore}I\{0,1\} \arrow[r, shift left, "F_2"] \arrow[r, phantom, "\simeq"] & \mathbf{RS}I \arrow[l, shift left, "G_2"] \arrow[r, shift right, "G_3"'] \arrow[r, phantom, "\simeq"] & \mathbf{Beh}I\{0,1\}\varPhi_2 \arrow[l, shift right, "F_3"']
\end{tikzcd}$$

5 Conclusion and Related Work

In this paper we define a sequential version of Mealy automata, and study its relationship to behavioural automata. Category theory is used to emphasize the relationships between Mealy, Moore and Rabin-Scott automata, and show how behavioural automata connect to these automata. The categorical approach has enabled us to evaluate the degree of connections of the form "A is a particular case of B": sometimes they can be refined to an isomorphism of categories, but other times they cannot.

We conjecture that functors F_4 and G_4 form an adjunction. Further work includes an algebraic investigation of the functions which can appear as sequential and final behaviour of some automata.

Automata have a long and respectable history in computing. Attempts to express automata theory in terms of category theory have appeared many years ago. An approach to automata theory by using categories and functors was given in the papers written by Arbib and Manes, see, e.g., [Arbib and Manes 1974], [Arbib and Manes 1975]. They studied automata in the category of sets for the linear sequential automata and the tree automata, or the category of modules for the linear sequential automata. Their main idea is to express the type of studied automata by a suitable functor on the corresponding category. We also mention the books [Ehrig and Pfender 1972] and [Eilenberg 1974], and emphasize the contribution of [Cazanescu 1967] where the monomorphisms and epimorphisms in the category of Mealy automata are proven to be the injective and surjective morphisms, respectively.

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