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Mathematical Models of Endocrine Systems

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Abstract: A mathematical model is proposed to allow expressive representation of endocrine systems by graphic means. The differential equations exactly describing the system can be formulated easily and automatically by the graphic model. Different kinds of software are supposed to solve these equations easily. Chaotic operational range can be found by fitting the parameters of equations. The results can account for some endocrine diseases and would be able to help the therapy.

Keywords: endocrine, chaos, nonlinear differential equation, graphic
Categories: J.3, G.1.7

1 Introduction

The interactions of the parts of endocrine systems are well known ([1], [2], [3], [4]), their quantitative description is given by the specification of the hormone-concentration. The system shows typical actions, the most important of them are:

- Constant or slowly variable concentrations
- Periodically variable concentrations (fig.1)
- Random, irregular behaviour – this is not typical in general, often pathological (see Figure 2 which represents some events belonging to a given hormone-level)

We expect the model to reproduce the actions above. We hope that we can conclude on the mechanism of pathological changes by choosing the parameters of the fitted model and give the mathematical background for modelling the therapeutic possibilities.

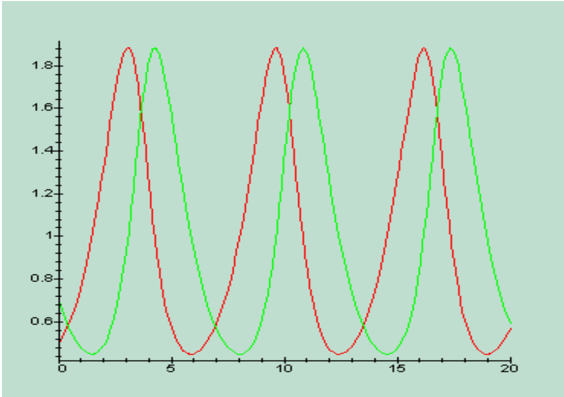

Figure 1: Periodic behavior of the endocrine system (concentration-time funcion in dimensionless scale)

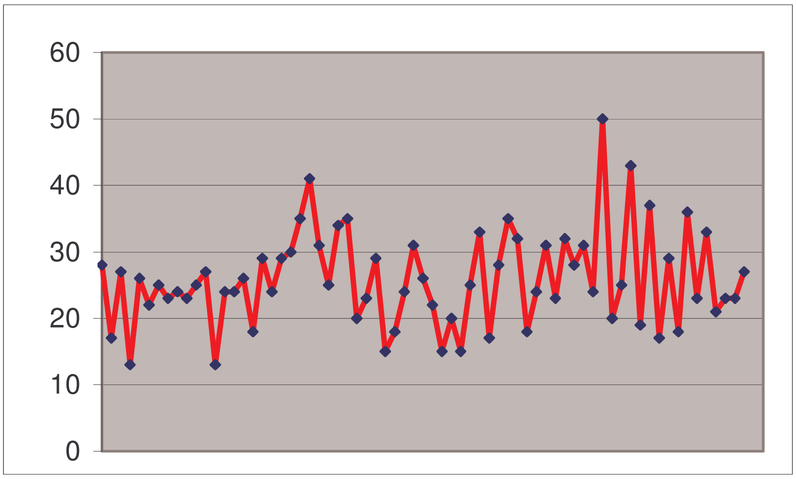

Figure 2: An example of aperiodic (perhaps chaotic) behaviour of the endocrine system (elapsed time between two items)

2 The model

A graphic notation was elaborated which is able to build up the model easily and to describe the endocrine system in numerous variations. There is mutually definite correspondence between the graphic model and a nonlinear ordinary differential-equation system. Solving the last problem we can discuss the behaviour of the model.

The model consists of *boxes* and *arrows* conjugatig the boxes (fig 3). The boxes symbolize the endocrine glands, hormone-concentrations respectively (some functions **x:R->R**), the arrows represent the interactions which are exciting (red arow) or inhibiting (blue arow). Among the interactions the *self-affect* of the glands is very important. The external constraints are described by the given function *F(t).* Our assumptions are:

- The differential of the hormone-concentration as time is going on is proportional to the sum of the excitations.
- The self-excitasion is constant, positive, negative resp.
- The effect of the other hormones raises or reduces the excitation proportionally to their concentration.
- The external excitations are constrained, they are independent from the concentrations.

The generic differential-equation describing the hormone-concentration is:

$$\dot{x} = (a + bx + \sum_{(other)} cy + F(t))x, \qquad (1)$$

where *a, b,* and *c* are constants, *F(t)* is a given function. One can prescribe such a type of differential-equation for all concentrations.

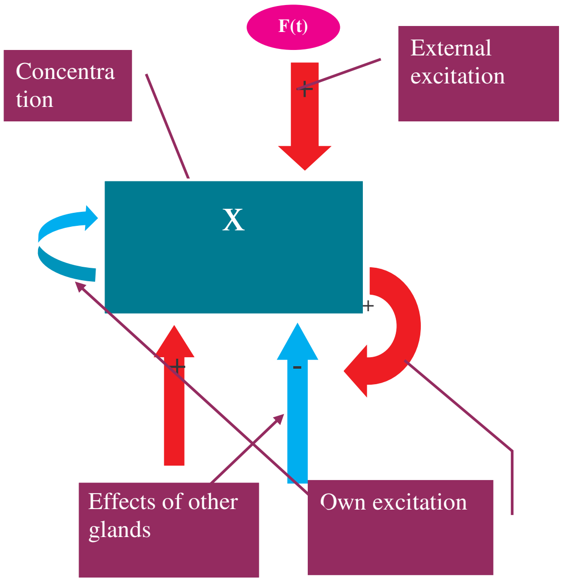

Figure 3: The graphical elements of the model

3 Some examples

3.1 A "single gland"

The simplest model consists of one gland, which is described by one interaction, shown in Figure 4 and the simple differential equation:

$$\dot{x} = ax, \qquad (2)$$

where a is a (positive or negative) constant. The solution is shown in Figure 5 (if $a>0$). This function has no finite limit, the solution is *unstable and* the model has no meaning itself.

Figure 4: The simplest model containing one gland only (+: activating effect, -: inhibiting effect)

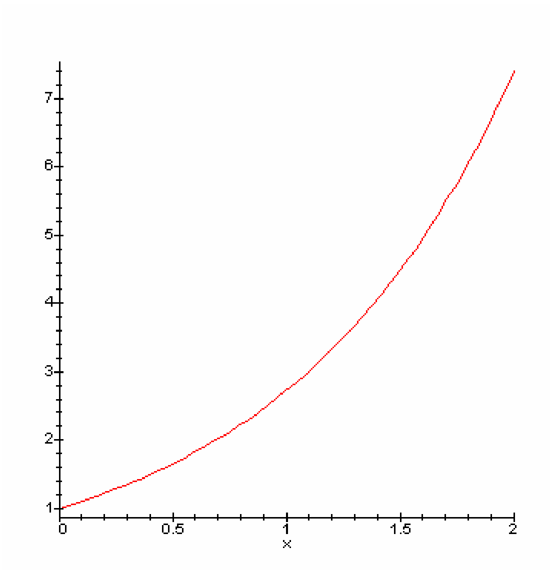

Figure 5: The solution of equation (2). The solution is a limitless function.

One can get stable finite solution if a self-action is allowed which is described by the equation:

$$\dot{x} = (a - bx)x, \qquad (3)$$

where *a, b* are positive constants. The graphic model is shown in Figure 6 and the solution of equation (3) in the Figure 7.

Figure 6: The graphic model belonging to the equation (3), +: activator, -: inhibitor)

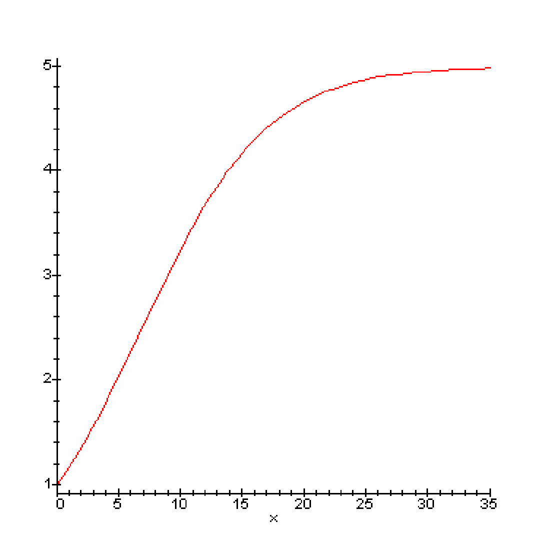

Figure 7: The solution of the equation (3)

3.2 The Lotka-Volterra-model

The well-known model from the population-dynamics is the *Lotka-Volterra-model (*[5], [6]*)*, which describes the states of two interacting elements (endocrine glands, see Figure 8). The glands exist and inhibit each other. The differential-equations describing the working of the system are:

$$\begin{aligned} \dot{x} &= (a - by)x \\ \dot{y} &= (-c + dx)y; a, b, c, d > 0 \end{aligned} \qquad (4)$$

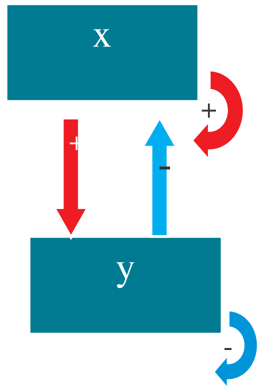

Figure 8: Graphic representation of Lotka-Volterra model

It seems that at the point *y0=a/b, x0=c/d* the system is in equilibrium. The equation system is not solvable in closed form, because it is a non-linear equation system. It is approximately possible (numerically) to solve them with the appropriate software, for example:

- *MAPLE* or other symbol-manipulator [7],
- *ODE* special software (**O**rdinary **D**ifferential **E**quations) [8].

The results are shown in Figure 1, Figure 9 resp. The hormone-concentrations are presented as the functions of time (Figure 1) and in the phase-space (Figure 9) resp. The interpretation of the model involves some problems, because the elements of the model are similar to the elements shown in Figure 7 and have no meaning themselves. One can explain their presence with the principle “the whole is more than the sum of the parts”.

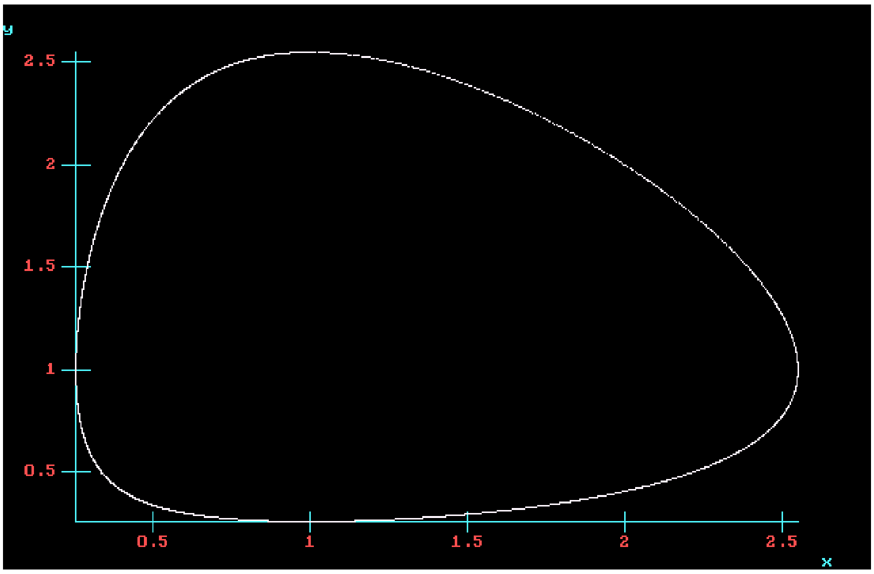

Figure 9: The solution of the equations (4), represented in the (x,y) phase-space

3.3 The external excitation

One can discuss another model which is a modification of the later. An external exciting constraint is allowed (as for example the light-fluctuation of day and night, which affects the hypophysis via the hypothalamus). The system is described in the Figure 10 and the appropriate differential-equations:

$$\begin{aligned} \dot{x} &= (a - by + p\sin(z))x \\ \dot{y} &= (-c + dx)y \\ \dot{z} &= \varpi \end{aligned} \qquad (5)$$

Note, that the initiation of a new variable z is indicated by the Poincare-Bendixson theorem [9]. Solving the equations we can get the results shown in Figure 11. One can observe the chaotic-like behavior of functions, which results in events as it is shown in Figure 2 (the events belong to fixed hormone-levels).

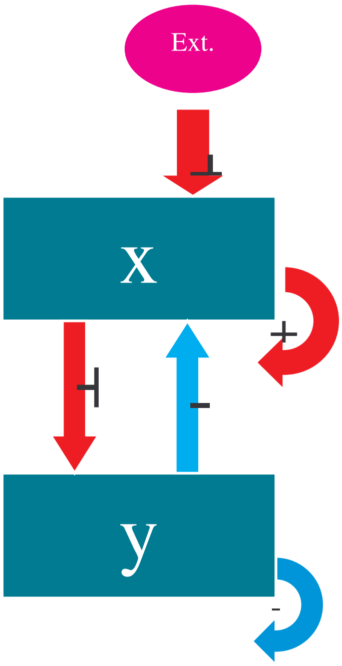

Figure 10: System with an external excitation

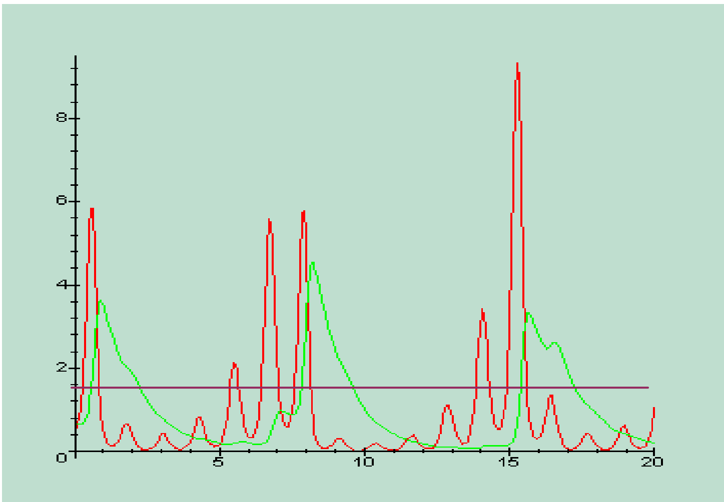

Figure 11: The chaotic-like solution of equation (5)

3.4 System with three glands

Finally we show a system consisting of tree elements and some possible interactions (Figure 12) The tree function is important when we want to search chaotic states, because the existence of chaotic states is required by the next conditions:

- The description of the system contains minimum tree independent functions.
- The differential-equation system may not be linear.

The differential equations in our case are:

$$\begin{aligned} \dot{x} &= (a - by - cz)x, \\ \dot{y} &= (-d + ex - fz)y, \qquad (6) \\ \dot{z} &= (-g + hy + ix)z, \end{aligned}$$

where parameters *a, b, c, d, e, f, g, h, i* are positive numbers. Figure 13 demonstrates the solution in the phase-space which appears as an attractor in the **(x,z)** plane.

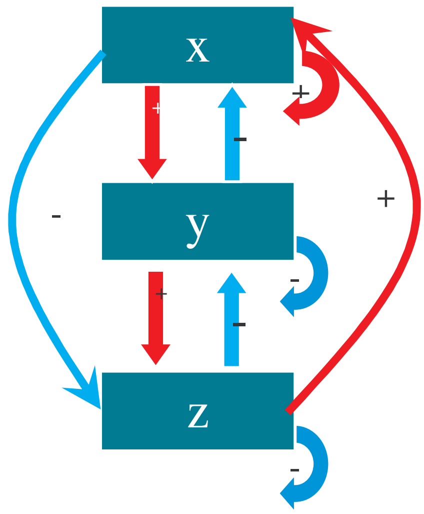

Figure 12: A model containing three glands

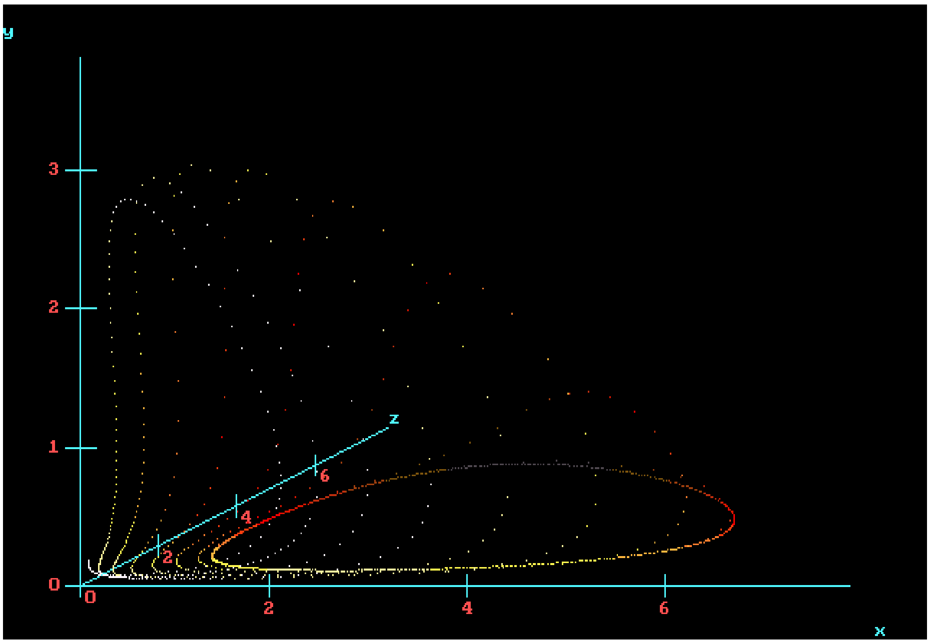

Figure 13: The solution of the equations (6) plotted in the three-dimensional phase-space. An ordinary attractor appears in the (x,z) plane

4 Future work

Our main effort is to find some chaotic states. It is very difficult to find strange attractors, because one must examine the points of minimum five dimensional spaces. We hope that far in the future the method will help us to medicate the pathologic mutations of endocrine systems. It is possible to dissolve the pathologic chaotic states in two ways:

- Modify the system parameters so that the system gets through to normal working.
- To affect the working of the system by external regulation. This is possible by an artificial neural network [10].

To find the answers to the questions raised requires further investigations.

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