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P Systems with Active Membranes and Separation Rules [1]

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Abstract: The P systems are a class of distributed parallel computing devices of a biochemical type. In this paper, a new definition of separation rules in P systems with active membranes is given. Under the new definition, the efficiency and universality of P systems with active membranes and separation rules instead of division rules are investigated.

Key Words: Natural computing, Membrane computing, Distributed computing

Category: C.1.4, F.1.1

1 Introduction

The P systems are a class of distributed parallel computing devices of a biochemical type, introduced in [Păun 2000], which can be seen as a general computing architecture where various types of objects can be processed by various operations. The area starts from the observation that certain processes which take place in the complex structure of living cells can be considered as computations. For details we refer to [Păun 2000], [Păun 2002].

Informally speaking, in P systems with active membranes one uses six types of rules: (a) multiset rewriting rules, (b) rules for introducing objects into membranes, (c) rules for sending objects out of membranes, (d) rules for dissolving membranes, (e) rules for dividing elementary membranes, and (f) rules for dividing non-elementary membranes.

Membrane division – inspired from cell division well-known in biology – is the most investigated way for obtaining an exponential working space in a linear time, and solving on this basis hard problems, typically **NP**-complete problems, in polynomial (often, linear) time. Details can be found in [Păun 2001, Păun 2002, Pérez-Jiménez et al. 2002]. Recently, also **PSPACE**-complete problems were attacked in this way (see [Sosík 2003, Alhazov et al. 2003]).

[1] Dedicated to Professor Zhang Kemin for his 70th birthday

Separation is also a well known phenomenon of cell biology. Many macromolecules are too large to be transported through membranes by means of protein channels, and then they are transported through vesicle formation. This process can transport packages of chemicals out of the cell.

By division, the two new membranes have exactly the same objects except for at most a pair of different objects; it is also possible that the two new membranes have different charges. However, in the biological phenomenon of separation, the contents of the two new membranes evolved from a membrane can be significantly different. In [Alhazov and Ishdorj 2004], separation rules are introduced to P systems with active membranes, where for each separation rule, a different subset U of objects is considered to activate the membrane to separate. In this paper, we give a new definition of separation rules. In the new definition, for each separation rule one object is used to activate separation; for all separation rules a uniform subset O_1 of objects is used to denote which membrane objects should go. From this point of view, the new definition is an improvement of the definition in [Alhazov and Ishdorj 2004]. Under the new definition, the efficiency and universality of P systems with active membranes and separation rules instead of division are investigated.

2 P Systems with Active Membranes

We assume the reader to be familiar with basic elements of complexity theory and formal language theory, for instance, from [Papadimitriou 1994, Salomaa 1973, Rozenberg and Salomaa 1997], as well as with the basic knowledge of membrane computing, for instance, from [Păun 2002] (details and recent results from membrane computing can be found at the web address `http://psystems.disco.unimib.it`). We only mention that RE denote the family of recursively enumerable languages, and that for a family of languages FL, by $PsFL$ we denote the family of Parikh sets of languages in FL; as usual, the Parikh mapping associated with an alphabet V is denoted by Ψ_V.

A P system *with active membranes* (and electrical charges) is a construct

$$\Pi = (O, H, \mu, w_1, \ldots, w_m, R),$$

where:

1. $m \geq 1$ (the initial degree of the system);
2. O is the alphabet of *objects*, where $O = O_1 \cup O_2$, $O_1, O_2 \neq \emptyset$, $O_1 \cap O_2 = \emptyset$;
3. H is a finite set of *labels* for membranes;
4. μ is a *membrane structure*, consisting of m membranes, labeled (not necessarily in a one-to-one manner) with elements of H;

5. $w_1, \ldots, w_m$ are strings over O, describing the *multisets of objects* placed in the m regions of μ;

6. R is a finite set of *developmental rules*, of the following forms:

   (a) $[\ a \to v]_h^e$,
   for $h \in H, e \in \{+, -, 0\}, a \in O, v \in O^*$
   (object evolution rules, associated with membranes and depending on the label and the charge of the membranes, but not directly involving the membranes, in the sense that the membranes are neither taking part in the application of these rules nor are they modified by them);

   (b) $a[\ \ ]_h^{e_1} \to [\ b]_h^{e_2}$,
   for $h \in H, e_1, e_2 \in \{+, -, 0\}, a, b \in O$
   (communication rules; an object is introduced in the membrane, possibly modified during this process; also the polarization of the membrane can be modified, but not its label);

   (c) $[\ a\ ]_h^{e_1} \to [\ \ ]_h^{e_2} b$,
   for $h \in H, e_1, e_2 \in \{+, -, 0\}, a, b \in O$
   (communication rules; an object is sent out of the membrane, possibly modified during this process; also the polarization of the membrane can be modified, but not its label);

   (d) $[\ a\ ]_h^e \to b$,
   for $h \in H, e \in \{+, -, 0\}, a, b \in O$
   (dissolving rules; in reaction with an object, a membrane can be dissolved, while the object specified in the rule can be modified);

   (e) $[\ a\ ]_h^{e_1} \to [\ b\ ]_h^{e_2} [\ c\ ]_h^{e_3}$,
   for $h \in H, e_1, e_2, e_3 \in \{+, -, 0\}, a, b, c \in O$
   (division rules for elementary membranes; in reaction with an object, the membrane is divided into two membranes with the same label, possibly of different polarizations; the object specified in the rule is replaced in the two new membranes by possibly new objects);

   (f) $[\ [\ \ ]_{h_1}^{\alpha_1} \cdots [\ \ ]_{h_k}^{\alpha_1} [\ \ ]_{h_{k+1}}^{\alpha_2} \cdots [\ \ ]_{h_n}^{\alpha_2}]_{h_0}^{\alpha_0}$
   $\to [\ [\ \ ]_{h_1}^{\alpha_3} \cdots [\ \ ]_{h_k}^{\alpha_3}]_{h_0}^{\alpha_5} [\ [\ \ ]_{h_{k+1}}^{\alpha_4} \cdots [\ \ ]_{h_n}^{\alpha_4}]_{h_0}^{\alpha_6}$,
   for $k \geq 1, n > k, h_i \in H, 0 \leq i \leq n$, and $\alpha_0, \ldots, \alpha_6 \in \{+, -, 0\}$ with $\{\alpha_1, \alpha_2\} = \{+, -\}$; if the membrane with the label h_0 contains other membranes than those with the labels $h_1, \ldots, h_n$ specified above, then they must have neutral charges in order to make this rule applicable; these membranes are duplicated and then are part of the contents of both new copies of the membrane h_0

(division of non-elementary membranes; this is possible only if a membrane contains two immediately lower membranes of opposite polarization, $+$ and $-$; the membranes of opposite polarizations are separated in the two new membranes, but their polarization can change; always, all membranes of opposite polarizations are separated by applying this rule).

(Note that, in order to simplify the writing, in contrast to the style customary in the literature, we have omitted the label of the left parenthesis from a pair of parentheses which identifies a membrane.) The rules of type (a) are applied in the parallel way (all objects which can evolve by such a rule should do it), while the rules of types $(b), (c), (d), (e), (f)$ are used sequentially, in the sense that one membrane can be used by at most one rule of these types at a time. In total, the rules are used in the non-deterministic maximally parallel manner: all objects and all membranes which can evolve, should evolve. Only halting computations give a result, non-halting computations give no output.

The separation rules introduced in [Alhazov and Ishdorj 2004] are of the following form (without polarizations):

(h_0) $[\ O]_h \to [\ U]_h[\ O-U]_h$, for $h \in\ H, U \subset O$.

If polarization is considered, they will be of the form:

(h) $[\ O]_h^{e_1} \to [\ U]_h^{e_2}[\ O-U]_h^{e_3}$, for $h \in\ H, U \subset O$.

Here, we introduce a new definition of separation rules.

(g) $[\ a]_h^{e_1} \to [\ O_1]_h^{e_2}[\ O_2]_h^{e_3}$,
for $h \in H$, $e_1, e_2, e_3 \in \{+, -, 0\}$, $a \in O$
(separation rules for elementary membranes; in reaction with an object, the membrane is separated into two membranes with the same label, possibly different polarization; at the same time, the object a can evolve; the objects from O_1 are placed in the first membrane, those from O_2 are placed in the second membrane).

As the rules of types $(b), (c), (d), (e)$, and (f), the rules of type (g) are used sequentially, in the sense that one membrane can be used by at most one rule of these types at a time. If at the same time a membrane h is separated, and there are objects in this membrane which evolve by means of rules of type (a), then in the two new membranes we introduce the result of evolution; that is, we suppose that first the evolution rules of type (a) are used, changing the objects, then separation is produced. Of course, this process takes one step.

Note the difference between division rules and separation rules. In a division rule, besides the fact that the object specified in the rule is replaced by two possibly different new objects, the two new membranes have the same copies of objects. In contrast, in a separation rule, the new objects are placed into the two new membranes according to O_1 and O_2; these two new membranes can be much different, for instance, in the number of objects.

For the difference between the definitions of (g) and (h), see the introduction.

To understand what it means solving a problem in a uniform and confluent way, here we briefly recall some related notions. Given a decision problem X, we say that it can be solved in polynomial (linear) time by recognizing P systems in a uniform way, if, informally speaking, we can construct in polynomial time a family of recognizing P systems Π_n, $n \in \mathbb{N}$, associated with the sizes n of instances $X(n)$ of the problem, such that the system will always stop in a polynomial (linear, respectively) number of steps, sending out the object `yes` if the instance $X(n)$ has a positive answer and the object `no` if the instance $X(n)$ has a negative answer. If the computation of a P system is nondeterministic, but all branches of a computation eventually reach a unique configuration, then we say that the system is confluent.

In the next section, we will show that P systems with separation rules instead of division rules can solve the `SAT` problem in linear time in a uniform and deterministic way.

3 Solving `SAT` by P Systems with Separation Rules

Theorem 1. *P systems with rules of types* $(a), (b), (c)$*, and* (g) *can solve* `SAT` *in linear time in a uniform and deterministic way.*

Proof. Let us consider a propositional formula in the conjunctive normal form:

$$\begin{aligned} &\beta = C_1 \wedge \cdots \wedge C_m, \\ &C_i = y_{i,1} \vee \cdots \vee y_{i,l_i},\ 1 \le i \le m, \text{ where} \\ &y_{i,k} \in \{x_j, \neg x_j \mid 1 \le j \le n\},\ 1 \le i \le m, 1 \le k \le l_i. \end{aligned}$$

The instance β (to which the size (m,n) is associated) is encoded as a multiset over

$$V(\langle n,m\rangle) = \{x_{i,j}, \bar{x}_{i,j} \mid 1 \le i \le m, 1 \le j \le n\}.$$

The object $x_{i,j}$ represents the variable x_j appearing in the clause C_i without negation, and object $\bar{x}_{i,j}$ represents the variable x_j appearing in the clause C_i with negation. Thus, the input multiset is

$$\begin{aligned} w = &\{x_{i,j} \mid x_j \in \{y_{i,k} \mid 1 \le k \le l_i\}, 1 \le i \le m, 1 \le j \le n\} \\ &\cup \{\bar{x}_{i,j} \mid \neg x_j \in \{y_{i,k} \mid 1 \le k \le l_i\}, 1 \le i \le m, 1 \le j \le n\}. \end{aligned}$$

For given $(n,m) \in \mathbb{N}^2$, we construct a recognizing P system $(\Pi(\langle n,m\rangle), V(\langle n,m\rangle), 2)$ with:

$$\Pi(\langle n,m\rangle) = (O(\langle n,m\rangle), H, \mu, w_1, w_2, R),$$

$$
\begin{aligned}
O(\langle n,m\rangle) &= O_1 \cup O_2,\\
O_1 &= \{x_{i,j}, \bar{x}_{i,j}, \mid 0 \le i \le m, 1 \le j \le n\} \cup \{c_i \mid 1 \le i \le m+2\}\\
&\quad \cup \{d_i \mid 1 \le i \le 2n+2m+2\} \cup \{r_{i,j} \mid 0 \le i \le m, 1 \le j \le n\}\\
&\quad \cup \{e, t, \lambda, \mathtt{yes}, \mathtt{no}\},\\
O_2 &= \{x'_{i,j}, \bar{x}'_{i,j}, \mid 0 \le i \le m, 1 \le j \le n\} \cup \{d'_i \mid 1 \le i \le n\}\\
&\quad \cup \{r_{i,j} \mid 0 \le i \le m, 1 \le j \le n\} \cup \{e'\},\\
\mu &= [\,[\,]_2]_1,\\
w_1 &= \lambda,\ w_2 = d_1,\\
H &= \{1,2\},
\end{aligned}
$$

and the following rules (we also give explanations about the use of these rules):

1. $[\,d_i]_2^0 \rightarrow [\,O_1]_2^+[\,O_2]_2^-$, $1 \le i \le n$.
In membrane with label 2, when it is "electrically neutral", object d_i causes the membrane to separate and to choose for a variable x_i, $1 \le i \le n$, both values $true$ and $false$, in form of charges $+$ and $-$ of the two created membranes with the label 2. These rules allow us to have 2^n internal membranes, in n steps.

2. $[\,x_{i,j} \rightarrow x_{i,j}x'_{i,j}]_2^0$, $1 \le i \le m$, $1 \le j \le n$.
$[\,\bar{x}_{i,j} \rightarrow \bar{x}_{i,j}\bar{x}'_{i,j}]_2^0$, $1 \le i \le m$, $1 \le j \le n$.
$[\,d_i \rightarrow d_{i+1}ed'_{i+1}e']_2^0$, $1 \le i \le n-1$.
At the same time with using the rule of type (1), the objects evolve by rules of type (2).

3. $[\,x_{i,1} \rightarrow r_{i,1}]_2^+$, $1 \le i \le m$.
$[\,\bar{x}_{i,1} \rightarrow \lambda]_2^+$, $1 \le i \le m$.
$[\,x'_{i,1} \rightarrow \lambda]_2^-$, $1 \le i \le m$.
$[\,\bar{x}'_{i,1} \rightarrow r'_{i,1}]_2^-$, $1 \le i \le m$.
The rules of type (3) try to implement a process allowing the internal membranes to encode the assignment of a variable and, simultaneously, to check the value of all clauses by this assignment, in such a way that, if the clause is true, then an object $r_{i,1}$ or $r'_{i,1}$ will appear in the membrane. In the other case, the object encoding the variable will disappear.

4. $[\,x_{i,j} \rightarrow x_{i,j-1}]_2^+$, $1 \le i \le m$, $2 \le j \le n$.
$[\,\bar{x}_{i,j} \rightarrow \bar{x}_{i,j-1}]_2^+$, $1 \le i \le m$, $2 \le j \le n$.
$[\,x'_{i,j} \rightarrow x_{i,j-1}]_2^-$, $1 \le i \le m$, $2 \le j \le n$.
$[\,\bar{x}'_{i,j} \rightarrow \bar{x}_{i,j-1}]_2^-$, $1 \le i \le m$, $2 \le j \le n$.
The check process described in the rules of type (3) is always made with respect to the first variable appearing in the internal membrane. Hence, the

rules of type (4) take charge of making a cyclic path through all the variables to get that, initially, the first variable is x_1, then x_2, and so on.

5. $[\ e]_1^+ \rightarrow [\ ]_1^0 e.$
$[\ e']_1^- \rightarrow [\ ]_1^0 e.$
$[\ d'_i \rightarrow d_i]_2^-$, $1 \leq i \leq n$.
The auxiliary objects e and e' exit the membrane changing the polarizations to neutral and the object d'_i in the membrane with negative charge evolves to the object d_i (for the use of the rules of type (1) and the third rules of type (2)), so that the above described generating process of the assignments and the encoding of the satisfied clauses can cycle.

6. $[\ r_{i,k} \rightarrow r_{i,k+1} r'_{i,k+1}]_2^0$, for $1 \leq i \leq m$, $1 \leq k \leq n-1$.
$[\ r'_{i,k} \rightarrow r_{i,k+1} r'_{i,k+1}]_2^0$, for $1 \leq i \leq m$, $1 \leq k \leq n-1$.
These rules are designed to denote the fact that if clause C_i is satisfied by the assignment encoded by a membrane, then the new membranes obtained from it by separation also satisfy the clause C_i. The second subscript of $r_{i,k}$ or $r'_{i,k}$ is used for synchronization, which is necessary, because of the use of objects $r_{i,n}$ and $r'_{i,n}$ in the rules of types (11), (12), and (13).

7. $[\ d_i \rightarrow d_{i+1}]_2^0$, for $n \leq i \leq 2n-2$.
$[\ d_{2n-2} \rightarrow d_{2n-1} c_1]_2^0.$
Through the counter objects d_i, the rules of type (7) control the process of synchronization of the objects $r_{i,k}$ and $r'_{i,k}$ in the internal membranes.

8. $[\ d_{2n-1}]_2^0 \rightarrow [\ ]_2^+ d_{2n-1}.$
The application of the rules of type (8) will show that the system is ready to check which clauses are true by the assignment encoded by an internal membrane.

9. $[\ d_i \rightarrow d_{i+1}]_1^0$, $2n-1 \leq i \leq 2n+2m+1$.
The rules of type (9) supply counter objects d_i in the skin, in such a way that, if objects $d_{2n+2m-1}$ appear, then they show the end of the checking of the clauses. The objects d_i, with $2n+2m \leq i \leq 2n+2m+2$, will control the final stage of the computation.

10. $[\ r_{1,n}]_2^+ \rightarrow [\ ]_2^- r_{1,n}.$
$[\ r'_{1,n}]_2^+ \rightarrow [\ ]_2^- r_{1,n}.$
For all 2^n internal membranes, we check whether $r_{1,n}$ or $r'_{1,n}$ is present in each membrane. If this is the case, then $r_{1,n}$ or $r'_{1,n}$ is sent out of the membrane where it is present (one copy of $r_{1,n}$ or $r'_{1,n}$ exits the membrane where it is present, the other copies will evolve to $r_{0,n}$ or $r'_{0,n}$ by the rules of type (10), which will never evolve again), changing in this way the polarization of that membrane, to negative. The membranes which do not contain the object $r_{1,n}$

or $r'_{1,n}$ remain positive and they will no longer evolve, as no further rule can be applied to them.

11. $[\ r_{i,n} \to r_{i-1,n}]_2^-$, for $1 \leq i \leq n$.
$[\ r'_{i,n} \to r'_{i-1,n}]_2^-$, for $1 \leq i \leq n$.
The rules of type (10) always work with respect to the objects $r_{1,n}$ or $r'_{1,n}$, The relabel in the rules of type (11) takes charge of making a cyclic path through all $r_{i,n}$ and $r'_{i,n}$, and evolves the objects $r_{1,n}$ or $r'_{1,n}$ to $r_{0,n}$ or $r'_{0,n}$ (which will never evolve again).

12. $r_{1,n}[\ ]_2^- \to [\ r_{0,n}]_2^+$.
The objects $r_{1,n}$ from the skin membrane return to internal membranes with negative charge, changing to $r_{0,n}$, and returning the polarization of the membrane to positive. This makes possible the use of rules of type (10).

    Note that in the skin membrane the number of copies of $r_{1,n}$ is equal to the number of membranes with negative charge; thus, because of parallelism, each membrane which previously contained objects $r_{1,n}$ or $r'_{1,n}$ will now contain an object $r_{0,n}$.

13. $[\ c_i \to c_{i+1}]_2^-$, $1 \leq i \leq m$.
The presence of object c_i (with $2 \leq i \leq m+1$) in the internal membrane shows that the assignment makes the first $i-1$ clauses true.

14. $[\ c_{m+1}]_2^+ \to [\ ]_2^+ c_{m+1}$.
The presence of the object c_{m+1} shows that all clauses are satisfied by the assignment encoded by an internal membrane. The rule of type (14) sends to the skin the objects c_{m+1} appearing in the internal membranes.

15. $[\ c_{m+1} \to c_{m+2}t]_1^0$.
The objects c_{m+1} in the skin evolve to objects $c_{m+2}t$. The objects t in the skin are produced simultaneously with the appearance of the objects $d_{2n+2m+1}$ in the skin, and they will be used to output the computing result.

16. $[\ t]_1^0 \to [\ ]_1^+ t$.
The rule of type (16) sends out of the system an object t changing the polarization of the skin to positive, then objects t remaining in the skin are not able to evolve. Then by rule of type (17), the object c_{m+2} can exit the skin producing an object `yes`, telling us that the formula is satisfiable, and the computation halts.

17. $[\ c_{m+2}]_1^+ \to [\ ]_1^- \mathtt{yes}$.
The application of the rule of type (17) changes the polarization in the skin membrane to negative in order that the objects c_{m+2} remaining in it are not able to continue evolving.

18. $[\ d_{2n+2m+2}]_1^0 \to [\ ]_1^+ \mathtt{no}$.
    By the rule (18) the object $d_{2n+2m+2}$ only evolves when the skin has neutral charge (this is the case when the formula is not satisfiable). Then the system will evolve sending out to the environment an object no and changing the polarization of the skin to positive, in order that objects $d_{2n+2m+2}$ remaining in the skin, do not evolve.

From the previous explanation of the use of rules, one can easily see how this P system works. It is clear that the object yes is sent to the environment if and only if the formula β is satisfiable. This is achieved in $3n + 2m + 4$ steps: in $2n$ steps we create 2^n internal membranes (as well as the 2^n different truth-assignments), then n steps for synchronization; it takes $2m$ steps to check whether all clauses are satisfied by an assignment; further 4 steps are necessary to output the computing result yes. If formula β is not satisfiable, then at step $3n + 2m + 4$ the system sends the object no to the environment. Therefore, the family of membrane systems we have constructed is sound, confluent, and linearly efficient.

To prove that the family is uniform, we have to show that for a given size, the construction of P systems described in the proof can be done in polynomial time by a Turing machine. We omit the detailed construction due to the fact that it is straightforward but cumbersome as explained in the proof of Theorem 7.2.3 in [Păun 2002] (although P systems in [Păun 2002] are semi-uniform). So SAT problem was decided in linear time $(3n + 2m + 2)$ by recognizing active P systems with separation rule in a uniform way, and this concludes the proof. □

4 Removing Polarizations

Following the idea in [Alhazov and Pan 2004, Alhazov et al. 2003], let us consider now rules of types $(a) - (e)$ and (g) without polarizations. They are of the following forms (because "no polarization" means "neutral polarization", we add the subscript 0 to the previous letters identifying the six types of rules; as above, $O = O_1 \cup O_2$ is the alphabet of objects and H is the set of labels of membranes):

(a_0) $[\ a \to v]_h$, where $a \in O, v \in O^*$, and $h \in H$,

(b_0) $a[\ ]_h \to [\ b]_h$, where $a, b \in O$ and $h \in H$,

(c_0) $[\ a]_h \to [\ ]_h b$, where $a, b \in O$ and $h \in H$,

(d_0) $[\ a]_h \to b$, where $a, b \in O$ and $h \in H$,

(e_0) $[\ a]_h \to [\ b]_h [\ c]_h$, where $a, b, c \in O$ and $h \in H$,

(g_0) $[\ a]_h \to [\ O_1]_h [\ O_2]_h$, for $h \in H$, $a \in O$.

Rules of types $(b), (c), (e), (g)$ in Section 2 were introduced without the capability of changing the label of membranes they involve (this makes no sense for dissolving rules), but in [Păun 2002] one already considers rules of type (e) which can change both the label and the polarization of membranes. Such rules are of the form

$[\ a]_{h_1}^{e_1} \to [\ b]_{h_2}^{e_2}[\ c]_{h_3}^{e_3}$, with $a, b, c \in O$, $e_1, e_2, e_3 \in \{+, -, 0\}$, and $h_1, h_2, h_3 \in H$,

and they have been called of type (e'). We extend this idea and this notation to rules of types (b_0), (c_0), (e_0), (g_0): their primed versions indicate the fact that the labels can be changed. Specifically, these rules are of the following forms:

(b_0') $a[\ ]_{h_1} \to [\ b]_{h_2}$, where $a, b \in O$ and $h_1, h_2 \in H$,

(c_0') $[\ a]_{h_1} \to [\ ]_{h_2} b$, where $a, b \in O$ and $h_1, h_2 \in H$,

(e_0') $[\ a]_{h_1} \to [\ b]_{h_2}[\ c]_{h_3}$, where $a, b, c \in O$ and $h_1, h_2, h_3 \in H$,

(g_0') $[\ a]_{h_1} \to [\ O_1]_{h_2}[\ O_2]_{h_3}$, for $h_1, h_2, h_3 \in H$, $a \in O$.

In the following, we consider the efficiency and universality of active P systems without polarization with separation rules instead of division.

4.1 Efficiency

Theorem 2. *P systems with rules of types $(a_0), (b_0), (c_0), (g_0')$ can solve* `SAT` *in linear time in a uniform and confluent way.*

Proof. Let us consider again a propositional formula in the conjunctive normal form:

$$\begin{aligned} \beta &= C_1 \wedge \cdots \wedge C_m, \\ C_i &= y_{i,1} \vee \cdots \vee y_{i,l_i},\ 1 \le i \le m, \text{ where} \\ y_{i,k} &\in \{x_j, \neg x_j \mid 1 \le j \le n\},\ 1 \le i \le m, 1 \le k \le l_i. \end{aligned}$$

As above, the instance β (to which the size (m, n) is associated) is encoded as a multiset over

$$V(\langle n, m\rangle) = \{x_{i,j}, \bar{x}_{i,j} \mid 1 \le i \le m, 1 \le j \le n\}.$$

The object $x_{i,j}$ represents the variable x_j appearing in the clause C_i without negation, and object $\bar{x}_{i,j}$ represents the variable x_j appearing in the clause C_i with negation. Thus, the input multiset is

$$\begin{aligned} w = &\{x_{i,j} \mid x_j \in \{y_{i,k} \mid 1 \le k \le l_i\}, 1 \le i \le m, 1 \le j \le n\} \\ &\cup \{\bar{x}_{i,j} \mid \neg x_j \in \{y_{i,k} \mid 1 \le k \le l_i\}, 1 \le i \le m, 1 \le j \le n\}. \end{aligned}$$

For given $(n,m) \in \mathbb{N}^2$, we construct a recognizing P system $(\Pi(\langle n,m\rangle), V(\langle n,m\rangle), 2)$ with:

$$\begin{aligned}
\Pi(\langle n,m\rangle) &= (O(\langle n,m\rangle), H, \mu, w_1, w_2, w_7, R),\\
O(\langle n,m\rangle) &= O_1 \cup O_2,\\
O_1 &= \{x_{i,j}, \bar{x}_{i,j} \mid 1 \le i \le m, 0 \le j \le n\} \cup \{d_i \mid 0 \le i \le 2n+2m+6\}\\
&\quad \cup \{c_i \mid 1 \le i \le m\} \cup \{\lambda, e, f_0, f_1, \mathtt{yes}, \mathtt{no}\},\\
O_2 &= \{x'_{i,j}, \bar{x}'_{i,j} \mid 1 \le i \le m, 0 \le j \le n\} \cup \{d'_i \mid 0 \le i \le n-1\}\\
&\quad \cup \{c'_i \mid 1 \le i \le m\},\\
\mu &= [\,[\,]_2[\,]_7]_1,\\
w_1 &= \lambda,\ w_2 = w_7 = d_0,\\
H &= \{0,1,2,3,4,5,6,7\},
\end{aligned}$$

and the following rules (we also give explanations about the use of these rules):

Generation phase

G1 $[\,d_i]_2 \to [\,O_1]_3[\,O_2]_4,\ 0 \le i < n.$
$[\,d_i \to d_i d'_i]_2,\ 0 \le i < n.$

G2 $[\,d_i]_3 \to [\,O_1]_2[\,O_2]_0,\ 0 \le i < n.$
$[\,d_i \to d_{i+1} d'_0]_3,\ 0 \le i < n.$

G3 $[\,d'_i]_4 \to [\,O_1]_2[\,O_2]_0,\ 0 \le i < n.$
$[\,d'_i \to d_{i+1} d'_0]_4,\ 0 \le i < n.$

G4 $[\,d_n]_2 \to [\,O_1]_5[\,O_2]_0.$
$[\,d_n \to d_0 d'_0]_2.$

The objects d_i and d'_i are used to control the generation process. In $2n+1$ steps, 2^n membranes with label 5 are created, corresponding to the truth assignments of the variables $x_1, \cdots, x_n$. During this process, the object d_i inside the membrane with label 3 corresponds to the *true* value of variable x_{i+1}, and the object d'_i inside the membrane with label 4 corresponds to the *false* value of variable x_{i+1}. The created membranes with label 0 are dummy membranes: no rules can be applied to them.

G5 $[\,x_{i,j} \to x_{i,j-1} x'_{i,j-1}]_2,\ 1 \le i \le m,\ 1 \le j \le n.$
$[\,\bar{x}_{i,j} \to \bar{x}_{i,j-1} \bar{x}'_{i,j-1}]_2,\ 1 \le i \le m,\ 1 \le j \le n,$

G6 $[\,x_{i,0} \to c_i]_3,\ 1 \le i \le m.$
$[\,\bar{x}_{i,0} \to \lambda]_3,\ 1 \le i \le m.$

G7 $[\,\bar{x}'_{i,0} \to c_i]_4,\ 1 \le i \le m.$
$[\,x'_{i,0} \to \lambda]_4,\ 1 \le i \le m.$

G8 $[\ x'_{i,j} \to x_{i,j}]_4$, $1 \le i \le m$, $1 \le i \le n-1$.

G9 $[\ c_i \to c_i c'_i]_2$, $1 \le i \le m$.

G10 $[\ c'_i \to c_i]_4$, $1 \le i \le m$.

The labels of the created membranes toggles between 2 at even steps and 3 or 4 at odd steps. Every object $x_{i,j}$ of the input evolves to $x_{i,0}$ or $x'_{i,0}$ in $2j-1$ steps. Then, it evolves to c_i in membranes where *true* value was chosen for x_j (recall that $x_{i,j} = \mathit{true}$ satisfies clause C_i) and is erased in membranes where *false* value was chosen for x_j. Similarly, $\bar{x}_{i,j}$ changes to c_i if $x_j = \mathit{false}$ and is erased if $x_j = \mathit{true}$. After $2n+1$ steps, the membranes with label 5 will represent all possible truth assignments of the variables in β. Every such membrane will contain d_0 and the objects representing the clauses satisfied by the present truth assignment.

Checking phase

C1 $[\ c_1]_5 \to [\ O_1]_6[\ O_2]_0$.
$[\ c_1 \to c_0 d'_0]_5$.
$[\ c_0 \to \lambda]_5$.

C2 $[\ c_i \to c_{i-1}]_6$, $1 \le i \le m$,

C3 $[\ d_i]_6 \to [\ O_1]_5[\ O_2]_0$.
$[\ d_i \to d_{i+1} d'_0]_6$.

C4 $[\ d_m \to e f_0]_5$.

A membrane with label 5 where object c_1 appears will change the label to 6 (recall that no rule is ever applied in membranes with label 0 created by separation). In a membrane with label 6, the subscripts of all objects c_j are decremented by one, and at the same time the subscript of d_i is incremented by one and the label of membrane changes back to 5.

If in the beginning of the checking phase $c_1, \cdots, c_i$ are present $(1 \le i \le m-1)$, but c_{i+1} is absent, then the evolution of the membrane finishes after $2i$ steps with label 5, with d_i and without c_1. If all objects $c_i, 1 \le i \le m$, are present in the beginning of the checking phase, then after $2m$ steps they will all be rewritten into c_0, then removed, and d_0 will evolve into d_m (and into ef_0 in one more step).

C5 $[\ e]_5 \to [\ ]_5 e$.
$[\ f_0 \to f_1]_5$.

C6 $e[\ ]_7 \to [\ e]_7$.
$[\ f_1]_5 \to [\ O_1]_6[\ O_2]_0$.
$[\ f_1 \to d_{2m+2n+4} d'_0]_5$.

C7 $e[\ ]_6 \to [\ e]_6$.

If β has solutions (suppose β has s solutions, $1 \leq s \leq 2^n$), then at step $2n + 2m + 3$, every membrane corresponding to a solution of β ejects e in the skin region, and at the same time f_0 changes to f_1. At step $2n + 2m + 4$, one copy of e enters the membrane with label 7, and s membranes change label from 5 to 6 by the separation rule $[\ f_1]_5 \to [\ O_1]_6[\ O_2]_0$. At step $2n + 2m + 5$, $s - 1$ copies of e enter in $s - 1$ membranes of the $s + 1$ membranes with labels 6 and 7. If β has no solution, then no object e enters the membrane labeled 7.

Output phase

O1 $[\ d_i \to d_{i+1}]_7$, $0 \leq i \leq 2m + 2n + 5$.

O2 $[\ e]_7 \to [\ O_1]_6[\ O_2]_0$.
$[\ e \to \mathtt{yes}d_0']_7$.

O3 $[\ \mathtt{yes}]_6 \to [\ ]_6\mathtt{yes}$.

O4 $[\ \mathtt{yes}]_1 \to [\ ]_1\mathtt{yes}$.

O5 $[\ d_i \to \lambda]_6$, $i \in \{2n + 2m + 5, 2n + 2m + 6\}$.

O6 $[\ d_{2n+2m+6}]_7 \to [\ ]_7\mathtt{no}$.

O7 $[\ \mathtt{no}]_1 \to [\ ]_1\mathtt{no}$.

If β has solutions, then at step $2n + 2m + 4$ the membrane with label 7 receives a copy of e by the rule C6. In this case, the rule O2 will be applied either at step $2n + 2m + 5$ or at step $2n + 2m + 6$ (this can happen if $s > 1$ and the rule C6 is applied at step $2n + 2m + 5$), changing the label of the membrane from 7 to 6. It will take two more steps to eject object `yes` in the skin and then into the environment. If β has no solutions, then after step $2n + 2m + 6$ the membrane with label 7 remains with label 7 and then the rule O6 is applied, ejecting object `no` into the skin and then into the environment.

If β has at least two solutions, then the behavior of this system is not deterministic: at step $2n+2m+5$ either one of the rules C6 and O2 can be applied to the membrane with label 7 (applying C6 at step $2n+2m+5$ results in one extra copy of e in membrane with label 7 and one copy of e missing in some membrane with label 6). However, the system is confluent: in either case mentioned above, after at most three further steps, the system produces the output `yes` and halts in the same configuration (the membrane with label 7 changes its label to 6 and the counter $d_{2n+2m+5}$ or $d_{2n+2m+6}$ is erased). □

From the confluence point of view, using rules of type (c_0') allows to obtain a stronger result:

Theorem 3. *P systems with rules of types* $(a_0), (c'_0), (g_0)$ *can solve* SAT *in linear time in a uniform and deterministic way.*

Proof. We consider a propositional formula β in the conjunctive normal form and in the previous proofs, encoded as a multiset w over $\Sigma(\langle n, m\rangle)$ in the same way as above. For given $(n, m) \in \mathbb{N}^2$, we construct a recognizing P system $(\Pi(\langle n, m\rangle), V(\langle n, m\rangle), 2)$, with

$$
\begin{aligned}
\Pi(\langle n,m\rangle) &= (O(\langle n,m\rangle), H, \mu, w_1, w_2, R),\\
O(\langle n,m\rangle) &= O_1 \cup O_2,\\
O_1 &= \{x_{i,j}, \bar{x}_{i,j} \mid 1 \le i \le m, 1 \le j \le n\} \cup \{d_i \mid 0 \le i \le 4n+2m+2\}\\
&\cup \{e_i, \bar{e}_i \mid 0 \le i \le n-1\} \cup \{c_{i,0}, c_{i,1}, c_{i,2} \mid 0 \le i \le m\}\\
&\cup \{a, u, \bar{u}, v, \bar{v}, t, \lambda, \mathtt{yes}, \mathtt{no}\},\\
O_2 &= \{x'_{i,j}, \bar{x}'_{i,j} \mid 1 \le i \le m, 1 \le j \le n\} \cup \{\bar{e}'_i \mid 0 \le i \le n-1\}\\
&\cup \{f, \bar{u}', \bar{v}'\},\\
\mu &= [\,[\,]_2]_1,\\
w_1 &= w_2 = d_0,\\
H &= \{1,2,3,4,5,6,7\},
\end{aligned}
$$

and the following rules:

Generation phase

G1 $[\,d_i \to e_i a u]_2$, $0 \le i \le n-2$.
$[\,d_{n-1} \to e_{n-1} a v]_2$.

G2 $[\,a]_2 \to [\,O_1]_2[\,O_2]_2$.
$[\,a \to tf]_2$.
$[\,e_i \to \bar{e}_i \bar{e}'_i]_2$, $0 \le i \le n-1$.
$[\,u \to \bar{u}\bar{u}']_2$.
$[\,v \to \bar{v}\bar{v}']_2$.

G3 $[\,t]_2 \to [\,]_3\lambda$,
$[\,f]_2 \to [\,]_4\lambda$,

G4 $[\,\bar{e}_i \to d_{i+1}]_3$, $0 \le i \le n-2$.
$[\,\bar{e}'_i \to d_{i+1}]_4$, $0 \le i \le n-2$.
$[\,\bar{e}_{n-1} \to d_0 u]_3$.
$[\,\bar{e}'_{n-1} \to d_0 u]_4$.

G5 $[\,\bar{u}]_3 \to [\,]_2\lambda$.
$[\,\bar{u}']_4 \to [\,]_2\lambda$.
$[\,\bar{v}]_3 \to [\,]_5\lambda$.
$[\,\bar{v}']_4 \to [\,]_5\lambda$.

In $4n$ steps, 2^n membranes are created, corresponding to the truth assignments of the variables $x_1, \cdots, x_n$. During this process, object d_i inside the membrane with label 3 corresponds to the *true* value of variable x_{i+1}, and object d_i inside the membrane with label 4 corresponds to the *false* value of variable x_{i+1}. Object a is used to choose the truth assignment of variables, and objects u and u' are used to change the membrane label back to 2. At step $4n$, the labels 3 and 4 of internal membranes are changed to 5 by the objects v and v'. The membranes with label 5 represent all possible truth assignments of the variables in β. Every such membrane will contain d_0, u, and the objects representing the clauses satisfied.

G6 $[\, x_{i,j} \to x_{i,j}x'_{i,j}]_2$, $1 \le i \le m$, $1 \le j \le n$.
$[\, \bar{x}_{i,j} \to \bar{x}_{i,j}\bar{x}'_{i,j}]_2$, $1 \le i \le m$, $1 \le j \le n$.

G7 $[\, x_{i,1} \to c_{i,0}]_3$, $1 \le i \le m$.
$[\, \bar{x}_{i,1} \to \lambda]_3$, $1 \le i \le m$.

G8 $[\, \bar{x}'_{i,1} \to c_{i,0}]_4$, $1 \le i \le m$.
$[\, x'_{i,1} \to \lambda]_4$, $1 \le i \le m$.

G9 $[\, x_{i,j} \to x_{i,j-1}]_3$, $1 \le i \le m$, $2 \le i \le n$.
$[\, \bar{x}_{i,j} \to \bar{x}_{i,j-1}]_3$, $1 \le i \le m$, $2 \le i \le n$.
$[\, x'_{i,j} \to x_{i,j-1}]_4$, $1 \le i \le m$, $2 \le i \le n$.
$[\, \bar{x}'_{i,j} \to \bar{x}_{i,j-1}]_4$, $1 \le i \le m$, $2 \le i \le n$.

The label of the created membranes is 2 and then changes to 3 or 4 at steps $4i+3$, $0 \le i < n$. Every object $x_{i,j}$ of the input evolves to $x_{i,1}$ or $x'_{i,1}$ in $4(i-1)$ steps. Then, it evolves to $c_{i,0}$ in membranes where *true* value was chosen for x_j (recall that $x_{i,j} = \textit{true}$ satisfies clause C_i) and is erased in membranes where *false* value was chosen for x_j. Similarly, $\bar{x}_{i,j}$ changes to $c_{i,0}$ if $x_j = \textit{false}$, and is erased if $x_j = \textit{true}$.

G10 $[\, c_{i,0} \to c_{i,1}]_2$, $1 \le i \le m$.

G11 $[\, c_{i,1} \to c_{i,2}c'_{i,2}]_2$, $1 \le i \le m$.

G12 $[\, c_{i,2} \to c_{i,0}]_3$, $1 \le i \le m$.
$[\, c'_{i,2} \to c_{i,0}]_4$, $1 \le i \le m$.

The rules of types (G10), (G11), and (G12) are representing the fact that if the clause C_i is satisfied in an internal membrane, then in the new created membranes from it C_i is also satisfied.

Checking phase

C1 $[\, c_{i,0} \to c_{i-1,0}]_5$, $1 \le i \le m$.

C2 $[\ u]_5 \rightarrow [\ ]_6\lambda$.

C3 $[\ c_{0,0}]_6 \rightarrow [\ ]_5\lambda$.

C4 $[\ d_i \rightarrow d_{i+1}u]_6$, $0 \leq i < m-1$.

C5 $[\ d_{m-1} \rightarrow d_m]_6$.

By expelling object u, membrane changes label from 5 to 6. At the same time the subscripts of all objects $c_{j,0}$ are decremented by one. A membrane with label 6 where object $c_{0,0}$ appears will change the label back to 5. At the same time the subscript of d_i is incremented by one and u is reproduced (except for $i = m-1$).

If in the beginning of the checking phase $c_{1,0}, \cdots, c_{i,0}$ are present $(1 \leq i < m)$, but $c_{i+1,0}$ is absent, then after $2i+1$ steps rule C3 will no longer be applicable and the membrane will have label 6, no object $c_{0,0}$, and will never change the label again. After $m+i+1$ steps from the beginning of the checking phase, the membrane will stop evolving. If all objects $c_{i,0}$, $1 \leq i \leq m$, are present in the beginning of the checking phase, then after $2m$ steps they will all be erased, d_0 will evolve into d_m and the membrane label will be 5.

Output phase

O1 $[\ d_m]_5 \rightarrow [\ ]_5\texttt{yes}$.

O2 $[\ \texttt{yes}]_1 \rightarrow [\ ]_7\texttt{yes}$.

O3 $[\ d_i \rightarrow d_{i+1}]_1$, $0 \leq i \leq 4n+2m+1$.

O4 $[\ d_{4n+2m+2}]_1 \rightarrow [\ ]_1\texttt{no}$.

At step $4n+2m+1$, every membrane corresponding to a solution of β expels `yes` in the skin region, and in the next step one copy of `yes` (if any) is ejected into the environment, changing the label of the skin from 1 to 7. If β has no solutions, then after step $4n+2m+2$ the skin membrane remains with label 1 and then rule O4 is applied, ejecting the object `no` into the environment. □

4.2 Universality

Because the notion of a matrix grammar will be used below, here we introduce it.

A *matrix grammar with appearance checking* is a construct $G = (N, T, S, M, F)$, where N, T are disjoint alphabets, $S \in N$, M is a finite set of sequences of the form $(A_1 \rightarrow x_1, \ldots, A_n \rightarrow x_n)$, $n \geq 1$, of context-free rules over $N \cup T$ (with $A_i \in N, x_i \in (N \cup T)^*$, in all cases), and F is a set of occurrences of rules in M (N is the nonterminal alphabet, T is the terminal alphabet, S is the axiom, while the elements of M are called matrices).

For $w, z \in (N \cup T)^*$ we write $w \Longrightarrow z$ if there is a matrix $(A_1 \rightarrow x_1, \ldots, A_n \rightarrow x_n)$ in M and the strings $w_i \in (N \cup T)^*, 1 \leq i \leq n+1$, such that $w = w_1, z = w_{n+1}$, and, for all $1 \leq i \leq n$, either $w_i = w'_i A_i w''_i, w_{i+1} = w'_i x_i w''_i$, for some $w'_i, w''_i \in (N \cup T)^*$, or $w_i = w_{i+1}$, A_i does not appear in w_i, and the rule $A_i \rightarrow x_i$ appears in F. (The rules of a matrix are applied in order, possibly skipping the rules in F if they cannot be applied – therefore we say that these rules are applied in the *appearance checking* mode.)

The language generated by G is defined by $L(G) = \{w \in T^* \mid S \Longrightarrow^* w\}$. The family of languages of this form is denoted by MAT_{ac}. It is known that $MAT_{ac} = RE$.

A matrix grammar $G = (N, T, S, M, F)$ is said to be in the *binary normal form* if $N = N_1 \cup N_2 \cup \{S, \#\}$, with these three sets mutually disjoint, and the matrices in M are in one of the following forms:

1. $(S \rightarrow XA)$, with $X \in N_1, A \in N_2$,
2. $(X \rightarrow Y, A \rightarrow x)$, with $X, Y \in N_1, A \in N_2, x \in (N_2 \cup T)^*, |x| \leq 2$,
3. $(X \rightarrow Y, A \rightarrow \#)$, with $X, Y \in N_1, A \in N_2$,
4. $(X \rightarrow \lambda, A \rightarrow x)$, with $X \in N_1, A \in N_2$, and $x \in T^*, |x| \leq 2$.

Moreover, there is only one matrix of type 1 (that is why one uses to write it in the form $(S \rightarrow X_{init} A_{init})$, in order to fix the symbols X, A present in it), and F consists exactly of all rules $A \rightarrow \#$ appearing in matrices of type 3; $\#$ is a trap-symbol, because once introduced, it is never removed. A matrix of type 4 is used only once, in the last step of a derivation.

For each matrix grammar there is an equivalent matrix grammar in the binary normal form. Details can be found in [Dassow and Păun 1989].

The following theorem shows that by using membrane separation rules to change the labels of the membranes, the universality can be reached. Here $Ps(\Pi)$ denotes the set of vectors of natural numbers describing the multiplicity of objects expelled into the environment by the various halting computations in system Π; by $PsOP(a_0, c_0, g'_0)$ we denote the family of sets $Ps(\Pi)$ computed by P systems using the types of rules a_0, c_0, and g'_0.

Theorem 4. $PsOP(a_0, c_0, g'_0) = PsRE$.

Proof. Consider a matrix grammar $G = (N, T, S, M, F)$ with appearance checking, in the binary normal form, hence with $N = N_1 \cup N_2 \cup \{S, \#\}$ and with the matrices of the four forms introduced above. Assume that all matrices are injectively labeled with elements of a set B. Replace the rule $X \rightarrow \lambda$ from matrices of type 4 by $X \rightarrow f$, where f is a new symbol.

We construct the P system of degree 2

$$\begin{aligned}
\Pi &= (O, H, [\,[\ \ ]_{X_{init}}]_1, w_1 = \lambda, w_{X_{init}} = c_0 A_{init}, R),\\
O &= O_1 \cup O_2,\\
O_1 &= T \cup N_2 \cup \{A_m \mid A \in N_2, m \in B\} \cup \{c, c_0, c_1, c_2, \#\},\\
O_2 &= \{d\},\\
H &= N_1 \cup \{X_m \mid X \in N_1, m \in B\} \cup \{0, 1, f\},
\end{aligned}$$

and the set R containing the following rules.

The simulation of a matrix $m : (X \to Y, A \to x)$, with $X \in N_1, Y \in N_1 \cup \{f\}$, is done in three steps, using the next rules:

1. $[\,A]_X \to [\,O_1]_{Y_m}[\,O_2]_0.$
   $[\,A \to A_m d]_X.$
2. $[\,A_m \to xc]_{Y_m}.$
3. $[\,c]_{Y_m} \to [\,O_1]_Y[\,O_2]_0.$
   $[\,c \to d]_{Y_m}.$
4. $[\,c_1 \to c_2]_{Y_m}.$
   $[\,c_2 \to c_0]_{Y_m}.$

The first rule of the matrix is simulated by the change of the label of the inner membrane (the "dummy" object d and membrane 0 play no further role). Note that if $X \in N_1$ also appears in a matrix of type 3, then when A activates the separation, c_0 can evolve to $c_1 d$ by the second rule of type (5), then to c_2 and c_0 by the rules of type (4). The correctness of the simulation is obvious, because one cannot simulate one rule of the matrix without simulating the other rule.

The simulation of a matrix $m : (X \to Y, A \to \#)$, with $X, Y \in N_1$ and $A \in N_2$, is done also in three steps, using the next rules:

5. $[\,c_0]_X \to [\,O_1]_{Y_m}[\,O_2]_0.$
   $[\,c_0 \to c_1 d]_X.$
6. $[\,c_1 \to c_2]_{Y_m}.$
   $[\,A \to \#]_{Y_m}.$
7. $[\,c_2]_{Y_m} \to [\,O_1]_Y[\,O_2]_0.$
   $[\,c_2 \to c_0 d]_{Y_m}.$

While the membrane with label X is used by object c_0, no other rule can be used. In the next step, if any copy of A is present, then it introduces the trap-object $\#$ and the computation never stops. If no A is present, then the objects c_j evolve, returning the label of the membrane to Y and introducing the auxiliary object c_0, for iterating the procedure.

We also consider the following rules:

8. $[\ A \rightarrow \#]_f$, for all $A \in N_2$.
9. $[\ \# \rightarrow \#]_h$, for all $h \in H$.
10. $[\ a]_f \rightarrow [\ \ ]_f a$.
11. $[\ a]_1 \rightarrow [\ \ ]_1 a$, for all $a \in T$.

The equality $\Psi_T(L(G)) = Ps(\Pi)$ easily follows from the above explanations. □

Remark. In the above proof, the rules of type (c_0) are only used for sending the result of a computation out of the system. Therefore, rules of types (a_0) and (g_0') are sufficient to reach universality for membrane systems with internal output.

5 Final Remark

In this paper, separation is introduced into active P systems, and the efficiency and universality of active P systems with separation rules instead of division are investigated. The universality result is obtained by a direct proof. It still remains open using P systems with separation rules to simulate P systems with division rules.

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