

## On Determinism of Evolution–Communication P Systems

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**Abstract:** It is commonly believed that a significant part of the computational power of membrane systems comes from their inherent non-determinism. Recently, R. Freund and Gh. Păun have considered deterministic P systems, and formulated the general question whether the computing (generative) capacity of non-deterministic P systems is strictly larger than the (accepting) capacity of their deterministic counterpart.

In this paper, we study the computational power of deterministic P systems in the evolution–communication framework. It is known that, in the generative case, two membranes are enough for universality. For the deterministic systems, we obtain the universality with three membranes, leaving the original problem open.

**Key Words:** Membrane computing, P system, Determinism, Computational completeness

**Category:** F.4.2, F.4.3

### 1 Introduction

We assume the reader to be familiar with membrane computing (see [P Systems Web Page] for the bibliography of the domain). The *evolution–communication* P systems, introduced in [Cavaliere 2002], are the P systems with two types of rules: simple (i.e., without targets) rewriting rules, and communication (i.e., symport/antiport rules).

A *generative* P system starts from a fixed configuration, and (possibly) halts with a resulting number of objects (or multiset, or a sequence) in a specified region. An *accepting* P system starts from a fixed configuration plus the input number (or multiset), and the input is accepted if and only if the computation halts.

The purpose of this paper is to prove universality of deterministic accepting evolution–communication (in short, EC) P systems. In the non-deterministic generative case, EC P systems are known to be universal even when using only two membranes (and symport/antiport rules of a rather small weight). At the price of using one further membrane, we show that the universality holds true also in the deterministic accepting case; the symport/antiport rules used in the proof are still of a small weight. We do not know whether our results can be improved in the number of membranes.

## 2 Definitions

A P system is deterministic if for every reachable non-halting configuration the next configuration is unique.

In what follows, we consider P systems which accept numbers: to accept a number  $N$ , the system starts with the initial configuration, to which  $N$  copies of a specified object  $a$  are added in a specified region. The number is accepted if and only if the computation halts. The set of numbers accepted by a system  $\Pi$  is denoted by  $N(\Pi)$ . The system being deterministic, there is only one computation (either halting, or non-halting) possible for every input.

Let the P system have  $m$  membranes and the set  $O$  of objects. In this paper, the evolution-communication systems are considered, so the rules (applied in the maximally-parallel manner) are of the following forms:

1.  $a \rightarrow x$ ,  
associated with the region  $i$ ,  $1 \leq i \leq m$ , where  $a \in O$ ,  $w \in O^*$ ,
2.  $(x, out), (y, in), (x, out; y, in)$ ,  
associated with the membrane  $i$ , where  $1 \leq i \leq m$ ,  $x, y \in O^+$ .

Thus, the accepting P system can be denoted as

$$\Pi = (O, \mu, w_1, \dots, w_m, R_1, \dots, R_m, R'_1, \dots, R'_m, i_0),$$

where  $\mu$  is the membrane structure,  $w_i$  is the starting multiset of objects in region  $i$ ,  $R_i$  is the set of rules of the first form (evolution),  $R'_i$  is the set of rules of the second form (communication), and  $i_0$  is the input region.

By  $NO P_m(ncoo, sym_p, anti_q)$  we denote the family of sets  $N(\Pi)$  generated by EC P systems with at most  $m$  membranes, using non-cooperative evolution rules, symport rules of weight at most  $p$ , and antiport rules of weight at most  $q$ . When dealing with accepting systems, we add the subscript  $a$  to the front  $N$ , while, moreover, a  $D$  is added in the case of using only deterministic systems. As usual,  $NRE$  is the family of Turing computable sets of numbers.

## 3 The Power

It is known from [Alhazov 2003] and [Krishna Păun 2003] that

$$NO P_2(ncoo, sym_1, anti_1) = NO P_2(ncoo, sym_2) = NRE.$$

We now present the deterministic counterparts of these results, using 3 membranes in the accepting case.

**Theorem 1.**  $DN_a OP_3(ncoo, sym_1, anti_1) = NRE$ .

*Proof.* Given a set  $M \in NRE$ , consider a deterministic register machine  $G = (m, e_{init}, e_{halt}, P)$  with  $m$  registers, initial label  $e_{init}$ , halting label  $e_{halt}$ , instruction set  $P$ , and set  $Lab(P)$  of labels, accepting  $M$ . We construct the following P system (object  $a_i$  represents the value of the  $i$ th register of  $G$ ):

$$\begin{aligned}
\Pi &= (O, \mu = [_1[_2[_3]_3]_2]_1, w_1 = \lambda, w_2 = e_{init}, w_3 = \lambda, \\
&\quad R_1, R_2, R_3, R'_1 = \emptyset, R'_2, R'_3, 2), \\
O &= \{e, e_0, e_1, e_2, e_3, e_4, e_5, e_6 \mid e \in Lab(P)\} \\
&\quad \cup \{a_r \mid 1 \leq r \leq m\} \cup \{s_1, s_2, s_3, q, z\}, \\
R_1 &= \{s_1 \rightarrow s_2\} \cup \{a_r \rightarrow \lambda \mid 1 \leq r \leq m\} \\
&\quad \cup \{e_3 \rightarrow e_4 \mid (e : dec(r), f, g) \in P\}, \\
R_2 &= \{s_3 \rightarrow \lambda\} \cup \{e \rightarrow a_r f \mid (e : inc(r), f) \in P\} \\
&\quad \cup \{e \rightarrow s_1 e_0, e_0 \rightarrow e_1, e_1 \rightarrow e_2, e_2 \rightarrow e_3, e_4 \rightarrow e_5 q, e_5 \rightarrow e_6, e_6 \rightarrow z \\
&\quad \mid (e : dec(r), f, g) \in P\}, \\
R_3 &= \{s_2 \rightarrow s_3, q \rightarrow \lambda\} \cup \{e_3 \rightarrow f \mid (e : dec(r), f, g) \in P\}, \\
R'_2 &= \{(s_1, out), (a_r, out; s_2, in)\} \\
&\quad \cup \{(e_3, out; s_2, in), (e_4, in) \mid (e : dec(r), f, g) \in P\}, \\
R'_3 &= \{(s_2, in), (s_3, out; q, in)\} \cup \{(f, out) \mid (e : dec(r), f, g) \in P\} \\
&\quad \cup \{(s_3, out; e_3, in) \mid (e : dec(r), f, g) \in P\}.
\end{aligned}$$

The P system above accepts a number  $N$  if and only if the computation, starting with  $a_1^N$  (the input register of  $G$  is the first one) placed in region 2, halts. Below are the simulations of individual instructions.

Instruction  $(e : inc(r), f)$  is simulated in the following way:

$$[_1[_2 e w[_3]_3]_2]_1 \Rightarrow [_1[_2 a_r f w[_3]_3]_2]_1.$$

The object  $e$  (corresponding to the instruction label) simply evolves into  $a_r f$ , thus changing instruction label from  $e$  to  $f$  and adding one to the counter  $r$ .

Simulation of instruction  $(e : dec(r), f, g)$  (in case register  $r$  is non-zero):

$$\begin{aligned}
&[_1[_2 e a_r w[_3]_3]_2]_1 \Rightarrow [_1[_2 s_1 e_0 a_r w[_3]_3]_2]_1 \Rightarrow [_1 s_1 [_2 e_1 a_r w[_3]_3]_2]_1 \\
&\Rightarrow [_1 s_2 [_2 e_2 a_r w[_3]_3]_2]_1 \Rightarrow [_1 a_r [_2 s_2 e_3 w[_3]_3]_2]_1 \Rightarrow [_1 [_2 e_3 w[_3 s_2]_3]_2]_1 \\
&\Rightarrow [_1 [_2 e_3 w[_3 s_3]_3]_2]_1 \Rightarrow [_1 [_2 s_3 w[_3 e_3]_3]_2]_1 \Rightarrow [_1 [_2 w[_3 f]_3]_2]_1 \\
&\Rightarrow [_1 [_2 f w[_3]_3]_2]_1.
\end{aligned}$$

The object  $e$  (corresponding to the instruction label) evolves into  $e_0$  (changing in 3 steps into  $e_3$ ) and  $s_1$ , which goes in region 1, then changes into  $s_2$ , and then returns in region 2 in exchange for  $a_r$  (which is then erased). Then,  $s_2$  travels into region 3, changes to  $s_3$  and returns to region 2 (where it is then erased)

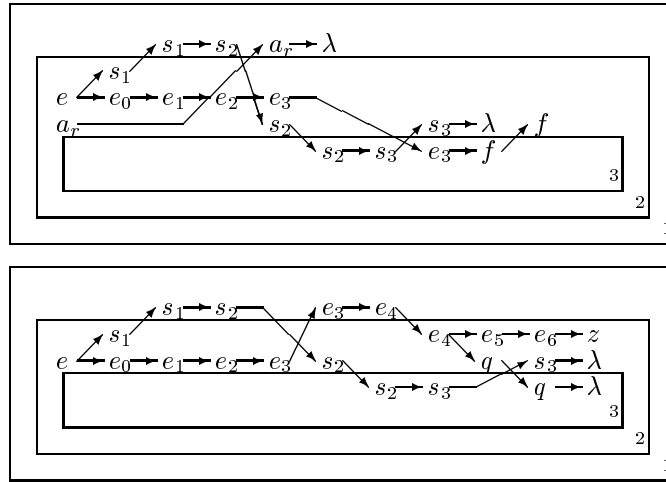
in exchange for  $e_3$ . Finally,  $e_3$ , being in region 3, changes into  $f$  and returns in region 2, finishing the simulation of the instruction.

Instruction  $(e : dec(r), f, g)$  (in case register  $r$  is zero) is simulated as follows:

$$\begin{aligned} & [{}_1[{}_2ew[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2s_1e_0rw[{}_3]_3]_2]_1 \Rightarrow [{}_1s_1[{}_2e_1w[{}_3]_3]_2]_1 \\ & \Rightarrow [{}_1s_2[{}_2e_2w[{}_3]_3]_2]_1 \Rightarrow [{}_1s_2[{}_2e_3w[{}_3]_3]_2]_1 \Rightarrow [{}_1e_3[{}_2s_2w[{}_3]_3]_2]_1 \\ & \Rightarrow [{}_1e_4[{}_2w[{}_3s_2]_3]_2]_1 \Rightarrow [{}_1[{}_2e_4w[{}_3s_3]_3]_2]_1 \Rightarrow [{}_1[{}_2e_5qw[{}_3s_3]_3]_2]_1 \\ & \Rightarrow [{}_1[{}_2e_6ws_3[{}_3q]_3]_2]_1 \Rightarrow [{}_1[{}_2zw[{}_3]_3]_2]_1. \end{aligned}$$

(Note that  $|w|_{a_r} = 0$ .) Like in the previous case, the object  $e$  evolves into  $e_0$  (changing in 3 steps into  $e_3$ ) and  $s_1$ , which goes in region 1, and then changes into  $s_2$ . Now there is no object  $a_r$  in region 2 to bring  $s_2$  to region 2, so  $s_2$  remains in region 3 until the next step, when it is exchanged with  $e_3$ . Then  $s_2$  travels to region 3 and changes into  $s_3$ . Now,  $e_3$ , being in region 1, changes into  $e_4$ , returns to region 2, where it evolves into  $e_5$  (changing it two steps into  $z$ ) and  $q$ , which exchanges with  $s_3$  and then both  $q$  and  $s_3$  are erased.

These two cases can be graphically represented by the following diagrams:



□

In the next theorem, symport of weight two is used instead of antiport of weight one, leading to one more universality result.

**Theorem 2.**  $DN_aOP_3(ncoo, sym_2) = NRE$ .

*Proof.* Given a set  $M \in NRE$ , consider a deterministic register machine  $G = (m, e_{init}, e_{halt}, P)$  as above, accepting  $M$ . We construct the following P system:

$$\Pi = (O, \mu = [{}_1[{}_2[{}_3]_3]_2]_1, w_1 = \lambda, w_2 = e_{init}, w_3 = \lambda,$$

$$\begin{aligned}
& R_1, R_2, R_3, R'_1 = \emptyset, R'_2, R'_3, 2), \\
O &= \{e, e_0, e_1, e_2, e_3, e_4, e_5, e_6 \mid e \in \text{Lab}(P)\} \\
& \cup \{a_r \mid 1 \leq r \leq m\} \cup \{s_1, s_2, s_3, q, z\}, \\
R_1 &= \{q \rightarrow \lambda, s_2 \rightarrow \lambda\} \cup \{e_0 \rightarrow e_1 \mid (e : \text{dec}(r), f, g) \in P\} \\
& \cup \{a_r \rightarrow \lambda \mid 1 \leq r \leq m\}, \\
R_2 &= \{s_1 \rightarrow s_2\} \cup \{e \rightarrow s_1 e_0, e_1 \rightarrow e_2 q, e_2 \rightarrow e_3, e_3 \rightarrow f \\
& \mid (e : \text{dec}(r), f, g) \in P\} \\
& \cup \{e \rightarrow a_r f \mid (e : \text{inc}(r), f) \in P\}, \\
R_3 &= \{s_2 \rightarrow \lambda\} \cup \{e_0 \rightarrow z \mid (e : \text{dec}(r), f, g) \in P\}, \\
R'_2 &= \{(qs_2, \text{out})\} \cup \{(e_0 a_r, \text{out}), (e_1, \text{in}) \mid (e : \text{inc}(r), f) \in P\}, \\
R'_3 &= \{(s_2 e_0, \text{in}), (z, \text{out}) \mid (e : \text{dec}(r), f, g) \in P\}.
\end{aligned}$$

The P system above accepts a number  $N$  if and only if the computation, starting with  $a_1^N$  (the first register is the input one of  $G$ ) placed in region 2, halts. Below is the simulation of the instructions of  $G$ .

Instruction  $(e : \text{inc}(r), f)$  is simulated like in the previous theorem:

$$[{}_1[{}_2ew[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2Rfw[{}_3]_3]_2]_1.$$

Instruction  $(e : \text{dec}(r), f, g)$  is simulated in the following way: The object  $e$  evolves in  $e_0$  (used to subtract) and  $s_1$  (which changes into  $s_2$ , the helper).

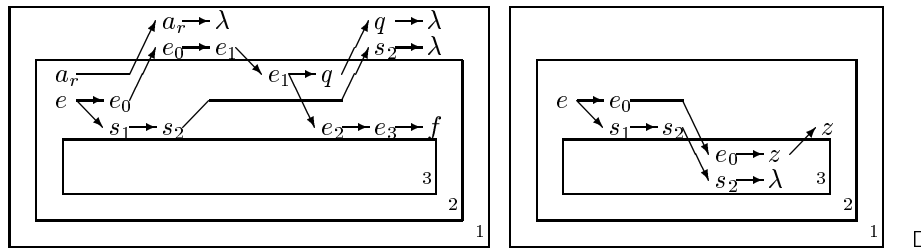
$$\begin{aligned}
& [{}_1[{}_2ea_rw[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2s_1e_0a_rw[{}_3]_3]_2]_1 \Rightarrow [{}_1e_0a_r[{}_2s_2w[{}_3]_3]_2]_1 \\
& \Rightarrow [{}_1e_1[{}_2s_2w[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2e_1s_2w[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2e_2qs_2w[{}_3]_3]_2]_1 \\
& \Rightarrow [{}_1qs_2[{}_2e_3w[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2fw[{}_3]_3]_2]_1.
\end{aligned}$$

If  $a_r$  is present in region 2, then (one copy of)  $a_r$  goes to region 1 (where it is erased) together with  $e_0$ , which changes into  $e_1$ , returns to region 2, and then evolves into  $e_2$  (which changes into  $f$  in two steps) and  $q$ , which exists to region 1 together with  $s_2$ , where both  $q$  and  $s_2$  are erased.

$$\begin{aligned}
& [{}_1[{}_2ew[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2s_1e_0w[{}_3]_3]_2]_1 \Rightarrow [{}_1[{}_2s_2e_0w[{}_3]_3]_2]_1 \\
& \Rightarrow [{}_1[{}_2w[{}_3s_2e_0]_3]_2]_1 \Rightarrow [{}_1[{}_2w[{}_3z]_3]_2]_1 \Rightarrow [{}_1[{}_2zw[{}_3]_3]_2]_1.
\end{aligned}$$

(Note that  $|w|_{a_r} = 0$ .) If  $a_r$  is not present in region 2, then  $e_0$  waits for  $s_2$ , they both come to region 3, where  $s_2$  is erased, while  $e_0$  changes to  $z$  and returns to region 2, finishing the simulation of the instruction.

These two cases can be graphically represented by the following diagrams:



#### 4 Conclusions

This paper gives two three-membrane constructions for the universal deterministic evolution-communication P systems, one using symport of weight at most two, and the other one using symport and antiport of weight one. These results are incomparable with the existing (nondeterministic) universality results with two membranes, as the proofs rely on having three regions where evolution rules take place. It is an *open question* whether the EC P systems with two membranes are universal in the deterministic way with symport of weight at most two, or with symport and antiport of weight one.

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#### References

- [Alhazov 2003] Alhazov A.: “Minimizing Evolution-Communication P Systems and EC P Automata”; “Brainstorming Week on Membrane Computing” (M. Cavaliere, C. Martín-Vide, Gh. Păun, eds.), Rovira i Virgili University, Technical Report 26/03, Tarragona (2003), 23–31, and *New Generation Computing*, accepted for publication
- [Alhazov 2004] Alhazov A.: “On the Power of Deterministic EC P Systems”; “Second Brainstorming Week on Membrane Computing” (Gh. Păun, A. Riscos-Núñez, A. Romero-Jiménez, F. Sancho-Caparrini), TR 01/2004, Research Group on Natural Computing, Sevilla (2004), 11–15
- [Cavaliere 2002] Cavaliere M.: “Evolution-Communication P Systems”; “Membrane Computing. International Workshop, WMC-CdeA 2002”, Curtea de Argeş (Gh. Păun, G. Rozenberg, A. Salomaa, C. Zandron, eds.), Springer, LNCS 2597 (2003), 134–145

- [Freund Păun 2003] Freund R., Păun Gh.: “On Deterministic P Systems”; submitted, 2003.
- [Krishna Păun 2003] Krishna S.N., Păun A.: “Some Universality Results on Evolution-Communication P Systems”; “Brainstorming Week on Membrane Computing” (M. Cavaliere, C. Martín-Vide, Gh. Păun, eds.), Rovira i Virgili University, Technical Report **26/03**, Tarragona (2003), 207–215
- [Păun 2002] Păun Gh.: “Membrane Computing. An Introduction”; Springer, Berlin (2002)
- [P Systems Web Page] <http://psystems.disco.unimib.it>