The Method of Logistic Optimization in E-commerce

Robert Bucki
(The College of Informatics and Management in Bielsko-Biała
Bielsko-Biała, Poland
rbucki@wsi.edu.pl)

Petr Suchánek
(Silesian University in Opava, School of Business Administration in Karviná
Karviná, Czech Republic
suchanek@opf.slu.cz)

Abstract: Rapidly changing business environment requires new approaches and methods for supporting management systems in all types of companies. Modern companies doing business use e-commerce systems by default. One of the key areas of e-commerce systems is logistics and the supply chain. The optimal way to ensure the success of logistics and supply chains is to use the methods of modeling and simulation based on appropriate models and especially its mathematical representation. In this paper, authors highlight the customer-oriented model of the e-commerce system and deal with logistic optimization and simulations. As an example, a sample logistic structure which requires the adequate control approach is presented. This is realized by means of heuristic algorithms which are responsible for meeting the set criterion. Moreover, the criteria to either maximize the production output or minimize the lost flow capacity of the logistic system or minimize the tool replacement criterion are introduced. Equations of state are given in order to represent the flow of material through the logistic system.

Keywords: e-commerce system, logistics, simulation, logistic process, production maximization criterion.

Categories: C.4, H.1.1, H.4.2, I.6.8

1 Introduction

E-commerce development has acquired great importance in the contemporary business. It has helped business organizations, businessmen and end users to overcome the barriers of time and distance to sell, buy and carry out other business transactions across the globe. The development of e-commerce makes it necessary to search for new management methods and techniques designed to optimize the entire e-commerce system. The emergence of e-commerce has changed the relationship between customers and retailers and created a new need to restructure the entire supply chain and logistic system. The main objective remains the maximization of the whole system effectiveness which is to be achieved at the lowest possible costs. E-commerce systems are by their nature considered as systems with low operating costs. This must be provided by means of the adequate optimization approach. The area, which could significantly affect the costs associated with running an e-commerce system, is either logistics or the supply chain (SC). The initial point for the
Implementation of all types of systems is a suitable model with its own architecture and mathematical description. Different types of models in conjunction with a suitable mathematical representation allow us to perform the simulation process whose outputs can help managers make suitable decisions. The major support of simulation is provided by advanced computer systems. In this context, we often talk about the so-called computational logistics. Computational logistics involves planning and implementation of large and complex tasks using computations and higher mathematics. Computational logistics is implemented in many areas, including the flow and storage of goods, services, and related information from their point of origin to the point of consumption. In computational logistics, optimization models and algorithms must be developed and verified for the plan and execution of complex logistics and supply chain systems. The main objective of this paper which emerges from the above discussion is to present the importance of logistic optimization in customer-oriented e-commerce systems as well as a sample logistic structure which requires the adequately specified control approach. These days it is evident that production activities are treated as sub-processes of trade processes. Should it be thought that an e-commerce system is directly oriented on the order of the specified product e.g. furniture, it is necessary to properly emphasize the problem of order realization as well as supply of a ready product. Moreover, it is also indispensable to optimize logistics within realizing the determined products. The main goal of the paper is to present how the order manufacturing process is controlled by means of heuristic algorithms while implementing the required criterion.

2 E-commerce system

The structural model describes an e-commerce system as a set of functionally connected components (Figure 1). The main basic components of e-commerce systems are:

- customers and generally business environment,
- the Internet,
- the web server,
- LAN (Local Area Network),
- CRM (Customer Relationship Management) characterized among others in [Pradeep, 10],
- ERP (Enterprise Resource Planning) [Bredford, 09],
- payment system [White, 08],
- delivery of goods [Chopra, 09],
- after-delivery (after-sales) services,
- information systems (CRM/ERP) of cooperating suppliers.

Important and integral parts of the whole system are hardware, software, people, co-operative suppliers, legislation and generally information and communication technologies (ICT). All shown parts of e-commerce systems are supported and controlled by the management at all control levels (tactical, operational, strategic). Most important for control and support functions are SCM - Supply Chain Management, FRM - Financial Resource Management, HRM - Human Resource Management...
Management, IBP – Integrated Business Planning and Information system/Information technology administration (IS/ICT management).

3 Customer-oriented e-commerce system

The e-commerce system should be seen as the part of a complete business environment. Here, the main decisive part of the environment is understood as the customer whose orientation is one of the main prerequisites of an effective management system.

Based on its own research, Simulation Systems Ltd. [Simulation Systems, 11] presented the fundamental definition and description of customer-oriented systems. They say that each customer-oriented system results from the individual work with the client taking into account specific details of his business processes. The customer-oriented system is made according to the principle emphasizing the well-known truth which reads that the system must be created for the user, avoiding the need to adjust the user for the system.

The development of customer-oriented systems includes the full cycle of tasks, such as: collection and analysis of data needed for automation, definition of the requirements, designing, implementation, introducing, testing and maintenance of the system. The specific features of the approach to designing of customer-oriented IS are formulated as follow:

Figure 1: Basic structure of e-commerce system Source: after [Suchánek, 10]

SCM – Supply Chain Management
FRM – Financial Resource Management
HRM – Human Resource Management
IBP – Integrated Business Planning
Administrator IS/IT, and so on
fundamental analysis of the customer’s business as a base for working out the solutions corresponding to the actual goals and problems of the customer;

detail elaboration and coordination with the customer of all stages of the project development, control points and required resources;

support of the convenient mode of maintenance, modification and extension of the system;

provision of openeness, mobility and scalability of the system.

Customer-oriented functionalities of e-commerce comprise business-to-customer (B2C) applications such as remote shopping, banking and infotainment-on-demand. To study the possibilities of e-commerce functionality improvement, the structure model presented in Figure 1 does not seem to be sufficient. Any company strives to accomplish its business strategy. Following the business strategy, long-term and short-term objectives and targets are set.

After measuring the results achieved in a certain period of time, the differences from the target values are evaluated and corrective actions for the next term are specified by the company management. Thus, in a very general way, any business system can be looked upon as a specific form of a control loop (Figure 2). In our opinion, this holds true also for e-commerce systems.

Let us set an example in which a customer faces a billboard showing a car. Then the customer decides to buy this car immediately. There is a credit card in his pocket with the necessary financial coverage and the telephone number of the seller on the billboard. The customer contacts the dealer and tells him he wants to buy a car immediately specifying required parameters and the geographical localization (the customer does not have to call, but he can use the Internet via his mobile phone or another mobile device). The dealer should be able to determine immediately where the desired car is available and how long it will take them to deliver the car to the customer. Let us suppose that using the information system the dealer determines that the car can be delivered to the customer’s destination in e.g. half an hour and subsequently makes a quote. If the customer agrees, he can sit on a bench next to the billboard and wait. Within this period of time, a responsible person will bring him the required car. The customer makes a payment by means of a mobile credit card reader and takes over the car. This example presents another possible direction of e-commerce development. The scheme in Figure 3 fully complies with the need of direct orientation to the customer. However, it requires to solve a wide range of technical, personnel and legal issues. The discussed model seems to be realistic. In order to make this model fully operational in the real environment, the following conditions have to be met:

- ensuring unambiguous identification of the customer (the customer who has made the order);
- defining the legal framework for this type of business in particular;
- defining principles of the legal protection of the dealers (for example it is necessary to ensure that the customer could not deny the ordered goods – click-wrap and browse-wrap contracts);
- ensuring technical support of the mobile credit card readers;
- defining and implementing the logistic system (especially the supply chain).
The controlled subsystem consists of the actual e-Commerce system itself. The inputs constitute the management decisions on the e-Commerce parameters like prices, investments, marketing decisions, safety rules etc. The outputs include sales value, profits and margins, return on investment (ROI), stability, security and other key performance factors. The outputs are measured in the measuring element and compared with the targets in the differential component.

There is a general rule on the basis of which the customer wants to find products quickly and easily and get products in the shortest time as well as to pay for goods in the selected way and to have the longest possible warranty. One way of meeting this rule is to accept the basic philosophy of orientation on the customer and use all available technologies and business and management practices in company operations. The general business pattern following the stated conditions can be presented as transformation of Figure 1 into Figure 3. Using the pattern shown in Figure 3, it is possible to advance much further in the considerations.
Logistic and supply chain and its optimization in e-commerce

For the discussed purpose, namely the area of electronic commerce, logistics is defined as a business planning framework for the management of material, goods, services, information and capital flows. It includes the increasingly complex information, communication and control systems required in today's business environment [Logistics, 96], [Langevin, 10]. Especially, in business, the logistics has the meaning of the way of the supply of goods by which the customers' requirements can be solved. It determines the path of transporting goods and a lot of important information exchanged between the consumer, production or source points. It has the special term production logistics which means that the supply of the high quality products in time along with the security must be assured (for tax related products: like iphone, iPod, etc. which are for sale). Logistics and the entire system must be controlled. In this context, we are talking about the management of logistics and supply chain management.

Logistics management is that part of the Supply Chain Management process which is responsible for planning, implementing and controlling the efficient, effective forwarding and reversing the flow and storage of goods, services, and related information between the point of origin and the point of consumption in order to meet customers' requirements. Supply Chain Management (SCM) encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all Logistics Management activities. Importantly, it also includes coordination and collaboration with channel partners which can be suppliers, intermediaries, third-party service providers and customers. In essence, Supply Chain Management integrates supply and demand management within and across companies [Cantrell, 04].

One of the key areas of e-commerce system is to optimize the whole e-commerce system. It is clear that to be optimized, the entire e-commerce system and all of its subsystems must be optimized. The rules defined by [Ratliff, 03] as the general principles of logistic (as a subsystem of the e-commerce system) optimization can be
adopted:
- Objectives - must be quantified and measurable;
- Models - must faithfully represent required logistic processes;
- Variability - must be explicitly considered;
- Data - must be accurate, timely, and comprehensive;
- Integration - must support fully automated data transfer;
- Delivery - must provide results in a form that facilitates execution, management and control;
- Algorithms - must intelligently exploit individual problem structure;
- People - must have the domain and technology expertise required to support the models, data, and optimization engines;
- Process - must support optimization and have the ability to continuously improve;
- Return on Investment (ROI) - must be provable considering the total cost of technology, people and operations.

Optimization is a highly timely topic in the field of manufacturing and supply chain management. Optimal control of a substitutable inventory system, structured assemble-to-order systems and the impact of advance demand information on various production-inventory control mechanisms are the key factors which must be taken into account while planning order realization procedures [Shanthikumar, 03].

A deterministic system does not involve any randomness in the development of subsequent states of the logistic system. Therefore, such a model will always produce the same output from a given initial state. Stochastic ordering is a fundamental guide for decision making under uncertainty. It is also an essential tool in the study of structural properties of complex stochastic systems [Shaked, 07].

Supply chain optimization is the application of processes and tools to ensure the optimal operation of a manufacturing and distribution supply chain. This includes the optimal placement of inventory within the supply chain, minimizing operating costs (including manufacturing costs, transportation costs, and distribution costs). This often involves the application of mathematical modelling techniques using computer software.

The general objective is to find optimal logistic network design. In the logistic network design problem (LNDP), decisions must be made regarding the selection of suppliers, the location of plants and warehouses, the assignment of activities to these facilities, and the flows of raw materials and finished products in the network [Cordeau, 08]. Optimal network design/redesign minimizes inventory carrying, warehousing, and transportation costs while satisfying customer response-time requirements. Specifics include the network’s distribution levels and centers, location and mission of each facility, assignment of supplier and customer locations to each center, and inventory deployment [Frazelle, 06]. To create an optimal network design/redesign, Frazelle recommends a 10-step of logistics network design process shown in the Table 1 (analogous approach can be found for example in [Groothedde, 11]).
10-step of logistic network design process

1. Assess/evaluate the current network
2. Design and populate the network optimization database.
3. Create network design alternatives such as more or fewer hierarchies, multi-commodity flows, pooling opportunities, merge-in-transit, direct shipping, cross docking, and supply-flow optimization concepts.
4. Develop a network optimization model.
5. Choose a network optimization tool.
6. Implement the network model in the chosen tool.
7. Evaluate alternative network designs.
8. “Practicalize” the recommended network structure.
9. Compute the reconfiguration cost.
10. Make a go/no-go decision.

Table 1: The 10-step of logistic network design process

Simulations can be effectively used in order to support optimization procedures in the state of uncertainty. Simulations are based on modelling and in particular mathematical models. Computer simulations are used extensively as models of real systems to evaluate output responses. Applications of simulation are widely found in many areas including supply chain management, finance, manufacturing, engineering design and medical treatment [Fu, 95], [Kim, 05], [Semini, 06]. Supply chain simulations [Chang, 03]:

- help to understand the overall supply chain processes and characteristics by graphics and/or animation;
- are able to capture system dynamics: using probability distribution, user can model unexpected events in certain areas and understand the impact of these events on the supply chain.
- could dramatically minimize the risk of changes in planning process: By what-if simulation, user can test various alternatives before changing plan.

Traditionally, the formal modelling of systems has been carried out by means of a mathematical model which attempts to find analytical solutions to problems which enable the prediction of the system behaviour on the basis of the set of parameters and initial conditions. There are many methods for the purposes of simulation and especially simulation-based optimization techniques. Kleijnen claims that supply chain simulations can be carried out by means of the use of spreadsheet simulation methods, system dynamics, discrete event simulation and/or business games [Kleijnen, 03]. E-commerce systems can be classified into the category of intelligent systems. Evolutionary optimization can be effectively used for the optimization of intelligent systems. Evolutionary optimization is becoming an omnipresent technique in almost every process of intelligent system design. Just to name few, engineering, control, economics and forecasting are some of the scientific fields that take advantage of an evolutionary computational process that supports engineering systems with intelligent behavior [Nedjah, 08]. Detailed breakdown of the simulation-
Simulation-based Optimization Techniques

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Table 2: Simulation-based Optimization Techniques

For the purpose of presenting a sample logistic manufacturing system, deterministic search techniques and especially heuristic search techniques are implemented. Professor Ashram further states that the heuristic search technique is probably most commonly used in optimizing response surfaces. It is also the least sophisticated scheme from the mathematical point of view and it can be thought of as an intuitive and experimental approach. The analyst determines the starting point and the stopping rule based on the previous experience with the system. Generally, heuristic search methods generate and test algorithms in order to meet the set manufacturing criteria.

In e-commerce, logistic optimization must be based upon business objectives to find the schedule producing the product with the least cost or the shortest production lead time. It all means that the planner ought to be presented with the best algorithm enabling him to manufacture the best sequence of products. Traditionally simulation models have been used effectively on a project-by-project basis to analyze changes to a factory physical equipment. Simulation models contribute more effectively to the design of new methods for production scheduling and new supply chain configurations. By the nature of the control the simulation model is to assist with the day-to-day scheduling of the company doing business.

Knowing that there is an order to be realized, we can easily find out in advance what kind of charge will be necessary and what amount of it will be needed. In this context we are talking about supply chain master planning presented, for example, in
Running the simulation process for just a few hours, the results show a completely different but more accurate story. This is why simulation is so important. Simulation, and only simulation, takes into account the combined effect of variability, uncertainty, and complex interdependencies between processes. Simulation is helping companies improve their businesses and become more competitive. Increasingly competitive markets bring new challenges and customers’ demands are constantly changing. As a result, manufacturers need to become more responsive, more quickly, more efficiently, yet often within tighter budgets and timescales. Simulators are helping them to visualize, analyze and optimize their processes to achieve these business performance improvements [Bucki, 09].

Following the above remarks, in the next section we present the sample logistic structure which requires the adequate control approach.

5 General assumptions of the sample logistic system

The problem itself consists in determining the sequence of elements of the order vector which are to be realized subsequently. The proposed heuristic algorithms choose the required element on which certain operations are carried out. The state of orders decreases after each production decision which influences the state of the whole logistic system at every stage. The given criteria are used on condition that each of them is associated by adequate bounds. Assuming that the results of calculations which are made for one chosen heuristic algorithm do not deliver a satisfactory solution, there arises a need to test other algorithms.

Let us assume that the logistic system consists of \( I \) active logistic blocks arranged in a series. Each logistic block is meant to carry out a production operation. There are no buffer stores placed between logistic blocks in the discussed case. Each logistic block consists of a certain number of production stands. If a production stand in the logistic block does not have enough production capacity to accept all production flow, another parallel stand carrying out the same production operation is activated in this exact logistic block. If a production stand is not needed any more, it is deactivated in the discussed logistic block [Bucki, 10].

Let us introduce the vector of charges:

\[
W = [w_l], \quad l = 1, ..., L,
\]

where: \( w_l \) - the \( l \)-th charge material.

Let us introduce the vector of orders:

\[
Z = [z_n], \quad n = 1, ..., N,
\]

where: \( z_n \) - the \( n \)-th production order (given in units).

Having defined the vector of charges \( W \) as well as the vector of orders \( Z \), the assignment matrix of products to charges is proposed as follows:

\[
\Omega = [\omega_{n,l}], \quad n = 1, ..., N, \quad l = 1, ..., L,
\]

where: \( \omega_{n,l} \) - the assignment of the \( n \)-th product to the \( l \)-th charge material.

Elements of the assignment matrix take the following values:
\[ \omega_{n,l} = \begin{cases} 
1 & \text{if the } n\text{-th product is realized from the } l\text{-th charge,} \\
0 & \text{otherwise.} 
\end{cases} \]

The assignment matrix \( \Omega \) of products to charges is presented as follows:

\[
\Omega = \begin{bmatrix}
\omega_{1,1} & \cdots & \omega_{1,j} & \cdots & \omega_{1,L} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\omega_{N,1} & \cdots & \omega_{N,j} & \cdots & \omega_{N,L}
\end{bmatrix}
\]

We also assume that used charge vector elements are immediately supplemented, which means that we treat them as the constant source of charge material. However, for simplicity reasons, it is assumed that each \( n \)-th product is made from the universal charge which enables realization of the given \( n \)-th product from any \( l \)-th charge material element, \( l = 1, \ldots, L \). The determined charge material enters the production system itself. Machines in each logistic block \( M_i \), \( i = 1, \ldots, I \) in the production system carry out autonomous operations on the specified material. The operations are realized subsequently. If the logistic block \( M_i \) is able to accept the material from which the \( n \)-th product is made, it is passed to this block and after carrying out the adequate operation is passed to the block \( M_{i+1} \). After leaving the \( I \)-th block, it fills the elements of the order vector in the serial logistic system.

Let us introduce the vector of production stands corresponding with the logistic blocks:

\[ M_i = [m_{i,j}] \quad j = 1, \ldots, J \]

where: \( m_{i,j} \) - the \( j \)-th production stand in the \( i \)-th logistic block.

The stage \( k, \ k = 1, \ldots, K \) is the moment at which the manufacturing process at any \( j \)-th production stand in the \( i \)-th logistic block begins. We need to consider that decisions are made at the stage \( k = 1, \ k = 1, \ldots, K \).

It is assumed that operations in the stands in the system are realized subsequently. The structure scheme of the serial logistic system is presented as follows:
where: $i = 1, \ldots, I$, $j = 1, \ldots, J$, $k = 1, \ldots, K$, $n = 1, \ldots, N$, $l = 1, \ldots, L$.

Elements representing production stands take the following values:

$$e_{i,j}^k = \begin{cases} 
1 & \text{if the } n\text{-th product is realized by the } j\text{-th production stand in the } i\text{-th logistic block at the } k\text{-th stage,} \\
0 & \text{otherwise.} 
\end{cases}$$

Moreover, it is assumed that $\forall e_{i,j}^k = 1$.

Having assumed the above, we can introduce the life vector of the logistic system for a new brand set of tools:

$$G = \begin{bmatrix} g_{n,i} \end{bmatrix}, \ n = 1, \ldots, N, \ i = 1, \ldots, I,$$

where: $g_{n,i}$ - the number of the $n$-th product units which can be realized in any $j$-th production stand of the $i$-th logistic block before the tool in this stand is completely worn out and requires an immediate replacement.

The elements of the matrix $G$ take the following values:

$$g_{n,i} = \begin{cases} 
\psi & \text{if the } n\text{-th product is realized in the } j\text{-th production stand of the } i\text{-th logistic block, } \psi = 1, \ldots, \Psi, \\
0 & \text{otherwise.} 
\end{cases}$$

Moreover, we can assume that $\forall g_{n,i} = \psi$, $\psi = 1, \ldots, \Psi$.

If the number $\psi$ is reached for the given $n$-th element of the vector $Z$ in the $j$-th production stand of the $i$-th logistic block, the tool in this exact stand has to be replaced by a new one.

Let $S_{n}^{k-1} = \begin{bmatrix} s(n)_{i,j}^{k-1} \end{bmatrix}$ be the matrix of state of the logistic system for the $n$-th product realization at the stage $k - 1$ where $s(n)_{i,j}^{k-1}$ is the number of units of the $n$-th product already realized in the $j$-th stand of the $i$-th logistic block with the use of the installed tool. The matrix of state can be shown in the following extended form:
Let $P_n^{k-1} = [p(n)_{i,j}^{k-1}]$ be the matrix of the flow capacity of the logistic system for the $n$-th product realization at the stage $k-1$ where $p(n)_{i,j}^{k-1}$ is the number of units of the $n$-th product which still can be realized in the $j$-th stand of the $i$-th logistic block.

The matrix of state can be shown in the following extended form:

$$
P_n^{k-1} = \begin{bmatrix}
p(n)_{i,j}^{k-1} & p(n)_{i,j}^{k-1} & \ldots & p(n)_{i,j}^{k-1} \\
p(n)_{i,j}^{k-1} & p(n)_{i,j}^{k-1} & \ldots & p(n)_{i,j}^{k-1} \\
\vdots & \vdots & \ddots & \vdots \\
p(n)_{i,j}^{k-1} & p(n)_{i,j}^{k-1} & \ldots & p(n)_{i,j}^{k-1}
\end{bmatrix}
$$

On the basis of the above assumptions we can determine the flow capacity of the $j$-th production stand in the $i$-th logistic block for the $n$-th element of the order vector $Z$ at the stage $k-1$:

$$p(n)_{i,j}^{k-1} = g_{n,j} - s(n)_{i,j}^{k-1}$$

The manufacturing procedure consists in realizing orders in sequence (manufacturing of the order may begin when the previously realized one leaves the logistic system). Its disadvantage consists in the need of waiting for completing the manufacturing process of a certain product before resuming it again for the next one. This results in not using the available flow capacity of the whole production system. Moreover, during the production course tools must be replaced. When another element of the order vector $Z$ enters the production system, the state of the system has to be recalculated.

Let us define the production times for the $n$-th product, $n = 1, \ldots, N$ in the $j$-th production stand of the $i$-th logistic block in the matrix form:

$$T^{pr} = \begin{bmatrix}
t_{1,j}^{pr} & t_{2,j}^{pr} & \ldots & t_{N,j}^{pr} \\
t_{1,j}^{pr} & t_{2,j}^{pr} & \ldots & t_{N,j}^{pr} \\
\vdots & \vdots & \ddots & \vdots \\
t_{1,j}^{pr} & t_{2,j}^{pr} & \ldots & t_{N,j}^{pr}
\end{bmatrix}
$$

If the $n$-th product is not realized in the $j$-th production stand of the $i$-th logistic block, then $t_{n,j}^{pr} = 0$ (in this case the implemented technology excludes production operations in certain logistic blocks).

Let us define the vector of replacement times for the tools in the logistic system:

$$T^{repl} = \begin{bmatrix}
t_1^{repl} & t_2^{repl} & \ldots & t_j^{repl} \\
t_1^{repl} & t_2^{repl} & \ldots & t_j^{repl} \\
\vdots & \vdots & \ddots & \vdots \\
t_1^{repl} & t_2^{repl} & \ldots & t_j^{repl}
\end{bmatrix},$$
where: $\tau_{ij}^{repl}$ - the replacement time of the tool in the $j$-th production stand of the $i$-th logistic block.

If $\tau_{ij}^{repl} \leq \tau_{ij+1}^{repl}$, then the $j$-th production stand of the $i$-th logistic block becomes blocked while manufacturing the $n$-th product so there is the need to activate the production stand $m_{i+1,j+1}$ which is based on the assumption that the number of active production stands in the $i$-th logistic block is increased by 1 (not to block the production process in the logistic block $M_{i,j}$) on condition that the number $J_i$ is not exceeded, where $J_i$ is the number of the $j$-th production stand in the $i$-th logistic block. It is justified by the fact that the production of the subsequent part of the $n$-th order from the vector $Z$ can be started instantly without having to await for completing the current production operation in the logistic block $M_i$. The structure of such a system guarantees continuing the production activity in any stand $j+1$ in the $i$-th logistic block if the stand $j_i$ requires tool replacement.

Let us introduce the production rate vector $V = [v_{n,i}]$. Its element $v_{n,i}$ is the number of units of the $n$-th product made in the time unit.

Let us calculate the total manufacturing time of all elements from vector $Z$:

$$T = \sum_{n=1}^{N} \sum_{i=1}^{I} \tau_{ij}^{repl} + \sum_{j=0}^{J} \sum_{k=1}^{K} y_{ij}^{k} \tau_{ij}^{repl}$$

The coefficient $y_{ij}^{k}$ takes the following values:

$$y_{ij}^{k} = \begin{cases} 1 & \text{ if the replacement procedure of the tool in the } i\text{-th stand is carried out,} \\ 0 & \text{ otherwise.} \end{cases}$$

6 Production criteria

The criteria presented hereby are to either maximize the production output or minimize the lost flow capacity of the production stands or minimize the tool replacement time. Let us propose production criteria for the logistic system along with the necessary bounds:

6.1 The production maximization criterion

Let us introduce the production maximization criterion:

$$Q_i = \sum_{k=1}^{K} q_i^{k} = \sum_{k=1}^{K} \sum_{n=1}^{N} x_{n}^{k} \rightarrow \max,$$

where: $x_{n}^{k}$ - the number of units of the $n$-th element realized at the $k$-th stage.

The tool replacement bound: $\sum_{j=1}^{J} \sum_{i=1}^{I} y_{ij}^{k} \tau_{ij}^{repl} \leq c$,

where: $c$ - the maximal allowable tool replacement time,
$t_{i}^{repl}$ - the replacement time of the used tool in the $j$-th stand of the $i$-th logistic block.

The flow capacity bound:

$$y_{i}^{k} \sum_{j=1}^{l} p_{n,i}^{k} \leq g_{n,i},$$

where: $p_{n,i}^{k}$ - the lost flow capacity of the $j$-th stand of the $i$-th logistic block at the $k$-th stage.

The production-maximizing criterion is reduced to the replacement time of tools and flow capacity bounds.

6.2 The lost flow capacity criterion

Let us introduce the flow capacity criterion:

$$Q_2 = \sum_{k=1}^{K} q_{k}^{2} = \sum_{k=1}^{K} \sum_{i=1}^{l} y_{i}^{k} \sum_{j=1}^{i} p_{n,i}^{k} \rightarrow \min$$

The tool replacement bound:

$$\sum_{i=1}^{l} y_{i}^{k} t_{i}^{repl} \leq c$$

The order bound:

$$\sum_{n=1}^{N} x_{n} \leq z_{n}$$

The lost flow capacity criterion is reduced to the replacement time of tools and order bounds.

6.3 The minimal tool replacement time criterion

Let us introduce the flow capacity criterion:

$$Q_3 = \sum_{k=1}^{K} \sum_{i=1}^{l} y_{i}^{k} t_{i}^{repl} \rightarrow \min$$

The flow capacity bound:

$$y_{i}^{k} \sum_{j=1}^{l} p_{n,i}^{k} \leq g_{n,i}$$

The order bound:

$$\sum_{n=1}^{N} x_{n} \leq z_{n}$$

The minimal tool replacement time criterion is reduced to the flow capacity bound and the order bound.

7 Equations of state

The state of the discussed serial logistic system for the $n$-th element changes in the production course as follows:

$$S_{n}^{0} \rightarrow S_{n}^{1} \rightarrow \ldots \rightarrow S_{n}^{k} \rightarrow \ldots \rightarrow S_{n}^{K}$$

The state of the $j$-th production stand of the $i$-th logistic block in case of the $n$-th product manufacturing changes consequently:
\[ s(n)_{i,j}^0 \rightarrow s(n)_{i,j}^1 \rightarrow \ldots \rightarrow s(n)_{i,j}^k \rightarrow \ldots \rightarrow s(n)_{i,j}^K \]

which can be written in the following form:

\[
s(n)_{i,j}^k = \begin{cases} 
  s(n)_{i,j}^{k-1} & \text{if no } n\text{-th product is realized in the } j\text{-th stand of the } i\text{-th logistic block at the } k\text{-th stage,} \\
  s(n)_{i,j}^{k-1} + x_n^k & \text{otherwise.} 
\end{cases}
\]

Let \( \rho_i \) be the tool to be replaced with a new one, \( 1 \leq \rho_i \leq I \). The state of the \( j \)-th production stand of the \( i \)-th logistic block in case of replacement of tools changes in the way shown below:

\[
s(n)_{n,j}^k = \begin{cases} 
  s(n)_{n,i}^{k-1} & \text{if } i \neq \rho_i \text{ at the stage } k-1, \\
  0 & \text{if } i = \rho_i \text{ at the stage } k-1. 
\end{cases}
\]

If tools in all stands are totally worn out, then \( S = G \) and they need an immediate replacement procedure to resume the production process.

The order vector \( Z \) changes after every production decision:

\[ Z^0 \rightarrow Z^1 \rightarrow \ldots \rightarrow Z^k \rightarrow \ldots \rightarrow Z^K \]

The order vector is modified after every decision about production:

\[
z_n^k = \begin{cases} 
  z_n^{k-1} - x_n^k & \text{if the number of units } x_n^k \text{ of the } n\text{-th order is realized at the } k\text{-th stage,} \\
  z_n^{k-1} & \text{otherwise.} 
\end{cases}
\]

### 8 Heuristics

In order to control the logistic process we need to implement heuristics for determining elements from the vector \( Z \) for the production process. The following control algorithms are put forward:

#### 8.1 The algorithm of the maximal order

This algorithm chooses the biggest order vector element characterized by the biggest coefficient \( \gamma_n^{k-1} \) in the state \( S^{k-1} \).

To produce the element \( a \), the following condition must be met:

\[(q^k = a) \Leftrightarrow \left[ \gamma_n^{k-1} = \max_{1 \leq n \leq N} \gamma_n^{k-1} \right],\]

where: \( \gamma_n^{k-1} = z_n^{k-1} \).
The above approach is justified by avoiding excessive bringing the production line to a standstill in order to change an element to be manufactured. If in state $S^{k-1}$ only minimal orders were chosen, in consequence the number of orders might be reduced. Such control is favorable because the serial production line is blocked and must be stopped only in order to replace the tools in certain stands (on condition that the replacement process disturbs the flow of the material).

8.2 The algorithm of the minimal order

This algorithm chooses the smallest order vector element characterized by the smallest coefficient $\gamma_n^{k-1}$ in the state $S^{k-1}$.

To produce the element $a$, the following condition must be met:

$$(q^k = a) \Leftrightarrow \left[ \gamma_a^{k-1} = \min_{1 \leq n \leq N} \gamma_n^{k-1} \right],$$

where: $\gamma_n^{k-1} = z_n^{k-1}$.

The above approach is justified by the need to eliminate the elements of the order vector $Z$ which could be sent to the customer just after the $n$-th product leaves the production line on condition that the customer sets such a requirement.

8.3 The algorithm of the relative order

This algorithm chooses the order element characterized by the maximal relative order coefficient $\gamma_n^{k-1}$ in the state $S^{k-1}$.

To produce the element $a$, the following condition must be met:

$$(q^k = a) \Leftrightarrow \left[ \gamma_a^{k-1} = \max_{1 \leq n \leq N} \gamma_n^{k-1} \right],$$

where: $\gamma_n^{k-1} = \frac{z_n^{k-1}}{z_{\alpha}^{k-1}}$.

It is assumed that the orders are realized one after another that is to say each order element $z_n$ in the state $S^{k-1}$ is reduced partly. Such control is advantageous when some parts of the order are needed earlier and the rest can be manufactured later.

9 The block diagram of the logistic process

In order to create the block diagram of the logistic manufacturing system (Figure 4) criteria $Q_{\beta}$, $\beta = 1,...,B$ must be implemented where:

- the production maximization criterion $\rightarrow Q_1$
- the lost flow capacity criterion $\rightarrow Q_2$
- the minimal tool replacement time criterion $\rightarrow Q_3$
Moreover, a heuristic algorithm $h_2, \lambda = 1, \ldots, \Lambda$ is responsible for choosing the $n$-th element of the order vector where:

- the algorithm of the maximal order $\rightarrow h_1$
- the algorithm of the minimal order $\rightarrow h_2$
- the algorithm of the relative order $\rightarrow h_3$

10 Conclusions

The problem presented in the paper discusses the issue of the serial production system with no buffer stores. The system delivers ready products corresponding with the elements of the order vector. The main goal is to fulfil the task by the set criteria. There is also a possibility to implement a two- or three-criterion model. Such models may lead to delivering a solution which would satisfy criteria included in the discussed model only partly as there should be bounds added. Heuristic algorithms
proposed in the paper enable the operator to choose the satisfactory production sequence on the basis of which a certain element of the order vector is determined to be realized. The use of one specified algorithm does not mean that we will achieve the result satisfying the given criterion. It is advisable to implement other algorithms and decide which one minimizes the order realization time, the loss of residual capacity, and the total replacement time or satisfies a hypothetical customer’s demand not specified hereby. Another idea already used in available works is to simulate the combination of heuristic algorithms. By means of this method, we are able to combine two or more algorithms. It also seems reasonable to draw products for manufacturing. To achieve a satisfactory result a big number of simulations must be carried out. In conclusion, it must be admitted that a simulator imitating real environment should be built to continue this work. Simulation experiments carried out in the synthetic environment may deliver an answer which heuristic approach is the expected one for a certain criterion. The idea of time scaling by means of the simulation method on condition a satisfactory number of simulation experiments is carried out seems to able to deliver the results minimizing the total order realization time.

In conclusion, we can say that the interactive software combines real-life images, with a step-by-step immersion process, which successfully replicates the experience of controlling a real system. By providing the user with both the flexibility to operate the whole system and the support of the data files, which can be easily downloaded, it is possible to fit the simulator into any given system. Although the approach is flexible, it is far from being an easy option. The design of the simulator ensures its reliability at every stage of the technological process. Even in situations where there can immerse a dose of uncertainty, the user will be able to count on the intuitive support of the program. By using a simulator there will be an impact on further production performance. The advantages far outweigh the costs of building a simulator in a very short period of time proving its market value, which will increase with every production unit. Operators will gain self-confidence as well as improve their communication skills and learn how to contribute more effectively in a plant activity, which is required of them in return.

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