Journal of Universal Computer Science, vol. 16, no. 17 (2010), 2394-2414
submitted: 15/2/10, accepted: 30/8/10, appeared: 1/6/10 © J.UCS

Developing and Analyzing the MP (Materialization Pattern) Model for Math Educational Standards

Namyoun Choi
(Drexel University, College of Information Science and Technology
Philadelphia, PA 19104, USA
namyoun.choi@drexel.edu)

Il-Yeol Song
(Drexel University, College of Information Science and Technology
Philadelphia, PA 19104, USA
songiy@drexel.edu)

Yuan An
(Drexel University, College of Information Science and Technology
Philadelphia, PA 19104, USA
yuan.an@ischool.drexel.edu)

Abstract: Educational standards alignment, which matches similar or equivalent concepts of educational standards, is a necessary task for educational resource retrieval. In order to automate the alignment task, it is important to model the semantics of educational standards statements that are imperative mood sentences. In this paper, we present the MP (Materialization Pattern) model for representing the semantics of math educational standards for the purpose of aligning math educational standards. This article first classifies math educational standards statements into 16 types and then converts them to the MP model. The MP model is based on the Reed-Kellogg sentence diagrams, and created as MP diagrams using the UML notation. The MP model explicitly represents the semantics of the sentences by capturing math concepts and the cognitive process of the math concepts from math educational standards statements. The MP model is developed for automating the alignment of math educational standards.

Keywords: Unified Modeling Language, MP (Materialization Pattern) model
Categories: H.1

1 Introduction

Edcational standards that describe what knowledge and skills students should acquire in a K-12 setting become a primary focus of educational systems in the United States. Due to the standards-based reform movement [Hamilton et al., 2008], classroom activities or curriculum materials often have been made based on educational standards. Teachers can utilize and retrieve educational resources tagged with other state educational standards statements through educational standards alignment between states. In order to support these educational resource retrieval or educational resource discovery, educational standards have to be aligned. This tests whether or not two educational standards statements are similar or equivalent to each other.

Math educational standards (i.e. math standards) express the mathematical understanding, knowledge, and skills that students should obtain from pre-kindergarten through grade 12. All the math educational standards statements are imperative mood sentences. Two examples of math educational standards statements are as follows: 1) *Add and subtract whole numbers with and without regrouping (Ohio State).* 2) *Add and subtract decimals using money as a model (Nevada State).*

In this paper, we present the detailed level of the MP (Materialization Pattern) model for modeling imperative mood sentences used in math educational standards statements. The purpose of the MP model is to represent the semantics of imperative mood sentences using the UML [Booch et al., 2005] notation so that the semantics of sentences from different math standards can be compared for alignment. Our MP model captures math concepts and the cognitive process of math concepts from math educational standards statements. Hence, the MP model enables us to compare the level of similarity of two statements from different math standards in terms of math concepts and the cognitive process of math concepts. In other words, the MP model enables us to align math educational standards. For examples, "Whole number" and "decimal" in 1) and 2) are math concepts which students should learn. "Add" and "subtract" are the cognitive process of math concepts "whole number" and "decimal," respectively. By comparing two math concepts "whole number" and "decimal" the two statements can be aligned as they have related math concepts. However, in this paper, we focus on the MP model for representing math educational standards.

The MP model is developed at a sentence level for each statement from typical math educational standards. This MP model can explicitly model the semantics of imperative mood sentence structures used in math standards. Our sentence analysis is based on the Reed-Kellogg sentence diagram [Reed & Kellogg, 2006]. We classify math educational standards statements based on the sentence structures into 16 different types for a comprehensive coverage of the possible cases of math educational standards statements. This is an iterative process and new types can be added in the future if necessary. We refer to these 16 different types of math educational standards statements as *MP statements*. Each type of MP statements can have many math educational standards statements. A math educational standards statement belongs to only one type of MP statements. Each type of MP statements has different relationships between concepts (i.e. classes) when it is converted to the MP model. These different relationships are association, aggregation, dependency, prepositional, transitive verb, and realization relationships. We also identify a math concept as an MP class or a noun class, and the cognitive process [Bloom & Krathwohl, 1956] of a math concept as a verb stereotype class. A distinct feature of the MP model is to extend the granularity of modeling with a verb stereotype class, in which a verb is reified as a class, and thus simplifies modeling of sentences by a Materialization Pattern in a domain class diagram.

The rest of the paper is organized as follows: In Section 2, we present terminologies used in this paper. We review related work in Section 3. In Section 4, we present the components of the MP model, heuristics and examples of the MP model, and conversion of 16 types of MP statements to the MP model using UML. We present an analysis of the MP model in Section 5. Section 6 presents conclusions and future work.

2 Terminology

- *Reed-Kellogg system*: It is a graphic representation of a sentence structure. It also represents relationships between the elements of sentences and their modifiers. The horizontal main line is for elements such as the subject, the verb, the direct object, and the complement. Modifiers of the subject, the verb, or the object are placed under elements they modify. A simple English sentence in the Reed-Kellogg System is shown as Figure 2.

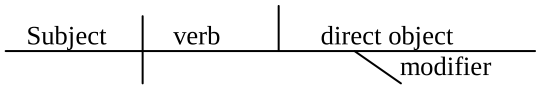

Figure 2: A simple sentence in the Reed-Kellogg System

- *Educational standards alignment*: It matches educational standards that describe similar or equivalent concepts.
- *Imperative mood sentence*: It only has a predicate which consists of verbs with verb modifiers, and nouns with noun modifiers. For example, *"Write fractions with numerals and number words."* is an imperative mood sentence
- *Math educational standards statement:* Math educational standards express the mathematical understanding, knowledge, and skills that students should obtain from pre-kindergarten through grade 12. In the United States, every state has its own math educational standards. All math educational standards statements have only one subject "student," and the subject is omitted because all the statements are imperative mood sentences. *"Write fractions with numerals and number words (Ohio State)."* is a math educational standards statement.
- *MP statement*: We classify math educational standard statements based on the sentence structures into 16 different types for a comprehensive coverage of the possible cases of math educational standard statements. We refer to these 16 different types of math educational standard statements as MP statements. For example, "Write fractions with numerals and number words (Ohio State)." belongs to a Type 3 MP statement. See Figure 3 for the sentence structure of a math standard statement *"Write fractions with numerals and number words (Ohio State)."* in the Reed-Kellogg System. "With numerals and words" modifies fractions. It is placed under "fractions" using a slanted line and a solid line. A dotted line indicates "and" or "or". A vertical line can be used between a verb and a direct object, and between a subject and a verb.

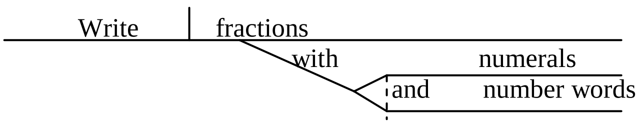

Figure 3: A Type 3 MP statement in the Reed-Kellogg System

- *MP verb*: It is a verb in Figure 3 at the beginning of an MP statement. For example, “write” is an MP verb.
- *MP noun*: It is a noun in Figure 3 in an MP statement. For example, “fractions” is an MP noun.
- *MP verb modifier*: A verb modifier which modifies an MP verb.
- *MP noun modifier*: A noun modifier which modifies an MP noun. For example, “with numerals and words” is an MP noun modifier. If more than one nouns in a noun modifier they are connected using “and” or “or”.
- *MP nouns with MP noun modifiers* are *math concepts* and their *properties.* They imply *what students are learning*.
- *Cognitive process:* It means an operation which affects mental contents. The cognitive process has been referred as the verbs in the educational standards [Williamson and Williams, 2010].
- *Cognitive process of a math concept*: It describes *how students are learning regarding a math concept.*
- *MP diagram:* It is a diagram of an MP statement which is modeled using UML notation.
- *Materialization Pattern (MP)*: It represents an *MP class* and *its verb materialization hierarchy that realizes the behaviors of the MP class*. An *MP class* represents a *concept* represented by a noun. A *materialization hierarchy* is a verb *hierarchy that models the behaviors of the MP class*. The *relationship* between the MP class and the materialization hierarchy is represented as *a realization relationship of UML*. See Figure 4 as an MP diagram for a sentence “Recognize, compare, and classify whole numbers.”. A connective “and” in a sentence is simply used for enumeration of classes “Recognize”, “Compare”, and “Classify”. From that sentence we extract an MP class “Whole number” as a math concept, and three verb stereotype classes “Recognize”, “Compare, and “Classify” as the cognitive process of the MP class “Whole number”. These three verb stereotype classes are subclasses of a class “Realize” which is an abstract class with no instance. A verb materialization hierarchy has verb stereotype classes “Realize”, “Recognize”, “Compare”, and “Classify”. A realization relationship exists between the classes “Whole number” and these verb stereotype classes.

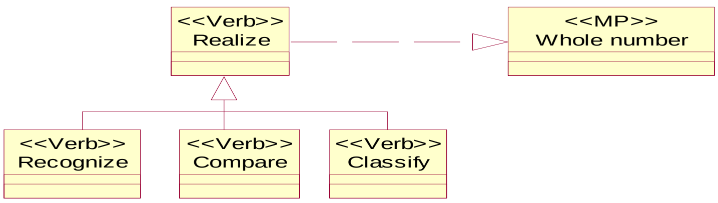

Figure 4: An MP diagram for a math standard statement: “Recognize, compare, and classify whole numbers.” A class “Whole number*” is an MP class.* “Realize*”, “Recognize”, “Compare”, and* “Classify*” are verb stereotype classes.*

3 Related work

This related work is divided into two sections. The first area is educational standards alignment, and the second is representing natural languages with UML.

3.1 Educational standards alignment

Alignment is a term used in a variety of contexts within the standards-based reform movement, which currently dominates decisions and actions in schools. The terminology "alignment" [Nasstrom and Henrikssson, 2008] is summarized as *when two or all three components in certain education system are* ***in agreement*** [Bhola et al., 2003; Webb, 1997], *and* ***matched*** [La Marca, 2001; Olson, 2003]. In alignment systems for educational standards, Yilmazel et al. (2007) mentions standards alignment as "*standards describing similar concepts are correlated*". Sutton and Golder (2008) state it as "*one statement is more-or-less equivalent to another statement.*"

The need of searching for resources by educational standards has been recently increased due to the increasing availability of on-line K-12 curriculum and the standards-based reform movement. Educational resources assigned with one state's standards can be searched or retrieved by teachers in other states through educational standards alignment. See Fig. 1 for educational standards alignment.

The MP model represents the semantics of math educational standards so that the semantics of statements from different math standards can be easily compared for alignment.

Further detail of how to utilize the MP model for math educational standards alignment will be included as our future work.

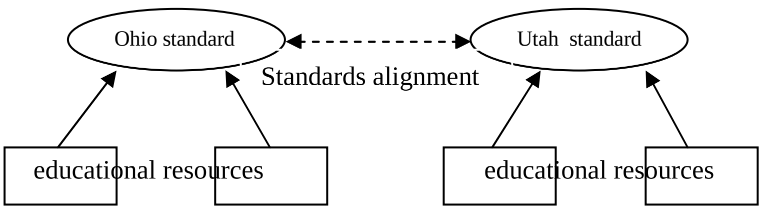

Figure 1: Educational standards alignment: Educational standards between Ohio and Utah are aligned. A teacher in Ohio can retrieve educational resources tagged with a statement in Utah which is equivalent or similar to a statement in Ohio.

3.2 Representing natural language with UML

Representing natural languages with UML (United Modeling Language), for instance English, has been an important research issue for various reasons – including the transition of natural language software requirements into modeling [Ilieva and Ormandjieva, 2005; Bryant et al., 2003; Takahashi et al., 2008], natural language query sentence processing [Tseng and Chen, 2008], and representation of knowledge [Ilieva and Boley, 2008] which is extracted from the text by an automatic tool.

Illieva (2007) and Illieva and Boley (2008) divide English sentences into three basic groups such as the subject, the predicate, and the object in a tabular presentation of sentences. They then build a graphical natural language for UML diagram generation. If the sentences lack a subject, the position of the subject is kept empty in a table and it will be filled by the analyst in an interactive mode. Math standard statements have only one subject "student", and the subject is omitted because all the statements are imperative mood sentences. Tseng and Chen (2008) briefly mention how to model an imperative mood sentence of English sentences in UML. They aim for transforming natural language queries into relational algebra through the UML class diagram notation. Their approach for modeling an imperative mood sentence of English sentences is summarized as follows: 1) Find out hidden associations between classes. or 2) If the verb does not transfer an action, the English sentence is modeled using only classes without including a verb as an association or a class. In math standards, it is not easy to find out hidden associations on a sentence level, and MP verbs are reified as classes in the MP model. Bryant et al. (2003) describe the method of translating requirements in natural language into UML models and/or executable models of software components. Their method depends on whole requirements in natural language rather than a sentence level. The requirements are refined and processed for creating a knowledge base using natural language processing techniques. And then the Knowledge base is converted into TLG (Two-Level Grammar) [Bryant and Lee, 2002] which is used as an intermediate representation between the informal knowledge base and the formal specification language representation. TLG can be converted into UML at the final step.

Our survey shows that most research in this area has not been focused on an imperative mood sentence. Our MP model is focused on capturing the semantics of imperative mood sentences for modeling math educational standards.

4 Developing the MP model

In this section, we discuss terminologies used in the MP model, the components of the MP model, conversion of 16 types of MP statements to MP diagrams in detail using UML, and an analysis and validation of the MP model.

4.1 The Component of the MP model

The MP model is developed at the sentence level. A math educational standard statement is an imperative mood sentence. The predicate and the object are two basic types which form this sentence. In this section, the predicate means a verb group which includes an MP verb(s) and an MP verb modifier(s). The object is a noun group which includes an MP noun(s) and an MP noun modifier(s). The basic building blocks of the MP model are classes and relationships between classes. These classes consist of an MP class, a noun class, and a verb stereo type class. In the MP model, predicative relationships, prepositional relationships, transitive verb relationships, and realization relationships, generalization relationships, and dependency relationships are used. All the detail of classes and relationships are mentioned in the Section 4.2.1.

See Table 1 for a summary of classes and relationship in the MP model.

			semantics	notation
class	MP class		Abstraction class	<<MP>>
	Verb stereotype class		A verb is reified as a class.	<<Verb>>
	Noun class		Regular UML class	Regular UML notation
relationship	Predicative relationship	Association	It connects two concepts using a verb [Ilieva and Boley, 2008].	
		Aggregation	It is more specific than association and represents a whole-part relationship.	
	Dependency relationship		One class depends on another.	
	Prepositional relationship		It connects two concepts using a preposition [Ilieva and Boley, 2008].	<<preposition>>
	Transitive verb relationship		It relates a noun to another noun or a verb.	<<preposition>>
	Realization relationship		The verb stereotype class realizes the behavior that the MP class specifies.	
	Generalization relationship		" isa " relationship	

Table 1: A summary of classes and relationships in the MP model

4.2 Heuristics and examples of the MP Model

By analyzing almost 700 math educational standards statements from 3 different states, we classify math educational standard statements based on the sentence structures into 16 different types of MP statements. Each type of MP statements has different relationships between concepts (i.e. classes). These relationships are association, aggregation, dependency, prepositional, transitive, realization relationships when each type of MP statements is converted to the MP model. We now present heuristics for converting each different type of MP statements to an MP diagram.

4.2.1 Heuristics in the MP Modeling. We present heuristics to determine classes and relationships.

1. Heuristic to determine classes
   1) All *MP verbs* are converted to *verb stereotype classes*, which represent the *cognitive process of math concepts.*
   2) All *MP nouns* except *Type 2 MP Statement* are converted into *MP classes*. These *MP classes* are *math concepts.* A superclass is created as an MP class if a superclass exists *when* more than one MP class exist.
   3) All nouns in an MP noun modifier or an MP verb modifier are converted to noun classes. A superclass is created if a superclass exists *when* more than one noun class in an MP modifier or an MP verb modifier exist.
2. Heuristic to determine relationsips

   We identify relationships as follows:
   - between an MP class and noun classes in an MP noun modifier,
   - between noun classes in an MP noun modifier or an MP verb modifier,
   - between an MP verb class and noun classes in an MP verb modifier.

   1) There is always a realization relationship between an MP class and a verb stereo type class.
   2) There is a predicative relationship (associtation or aggregation) when two concepts are connected using a verb.
   3) There is a prepositional relationship when two concepts are connected using a preposition.
   4) There is a transitive verb relationship when a transitive verb relates a concept to another concept [Hartman and Link, 2007] or a verb stereo type class.
   5) There is a dependency relationship when an MP verb modifier or an MP noun modifier starts with "using."
   6) There is a generalization relationship if a superclass exists.

4.2.2 Examples of the MP Model. In order to develop an MP diagram, we take four different steps for each type of MP statements as follows:

1. Step 1: Write an MP statement which is a math educational standard statement.
2. Step 2: Create the table format of the MP statement.
3. Step 3: Draw a diagram of the MP statement based on the Reed-Kellogg system.
4. Step 4: Develop an MP diagram using the UML notation from the Reed-Kellogg diagram of the MP statement.

Due to the lack of space we only present all steps of Type 1 and Type 2.

1. Type 1 MP Statement

1) Step 1: Recognize, classify, and compare whole numbers.
2) Step 2: Create the table format of the Type 1 MP statement.

MP verb	MP noun
Recognize, classify, compare	whole numbers

Table 2: A Type 1 MP Statement

3) Step 3: Draw the Reed-Kellogg diagram of a Type 1 MP Statement "Recognize, classify, and compare whole numbers.

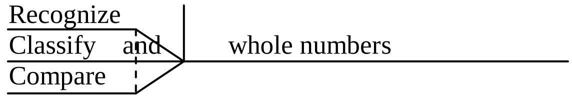

Figure 5: A Type 1 MP Statement in the Reed-Kellogg System

4) Step 4: Verbs (for example, recognize, classify, and compare) and nouns (for example whole number) in the horizontal line of the Reed-Kellogg System (except Types 2) are converted to classes in the MP model. A dotted line implies "and". A connective "and" in a sentence is simply used for enumeration of classes in the MP model.

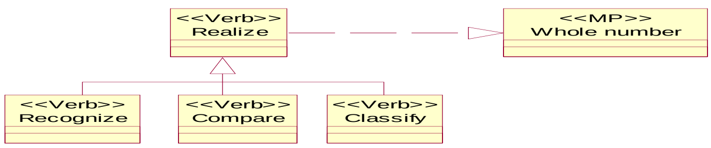

Figure 6: An MP diagram of a Type 1MP statement: A class "Whole number" is an MP class which is a math concept. Verb stereo type classes "Recognize", "Classify", and "Compare" are the cognitive process of the class "Whole number".

2. Type 2 MP Statement:

1) Step 1: A math standard statement: Estimate the value of irrational numbers.
2) Step 2: Create the table format of a Type 2 MP statement.

MP verb	MP noun	MP noun modifier
Estimate	value	of irrational numbers

Table 3: A Type 2 MP Statement

3) Step 3: Draw the Reed-Kellogg diagram of a Type 2 MP statement “Estimate the value of irrational numbers.“.

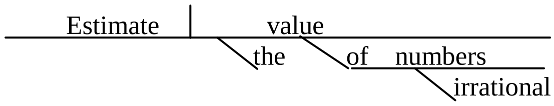

Figure 7: A Type 2 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a complement in the form of a prepositional phrase such as “of irrational numbers”.
- Model a noun (for example, irrational number) in an MP noun modifier as an MP class, which is a *math concept* and an MP noun(s) (for example, value) as an *attribute* of the MP class. See Figure 8. A class “Irrational number” is an MP class which is a math concept. “value” is an attribute of the class “Irrational number”. “Estimate“ is a verb stereotype class which represents the cognitive process of the MP class “Irrational number“.

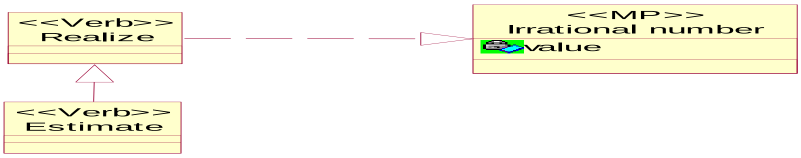

Figure 8: An MP diagram of a Type 2 MP Statement: A class “Irrational number” is an MP class. A realization relationship exists between the class “Irrational number” and a verb stereotype class “Estimate”.

3. Type 3 MP Statement:

A math educational standards statement: Write fractions with numerals number words**.**

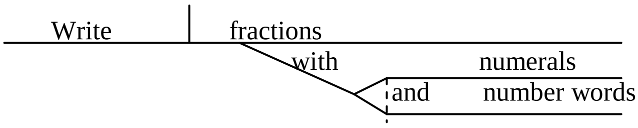

Figure 9: A Type 3 MP Statement in the Reed-Kellogg System

- A verb (for example, write) and nouns (for example, fraction, numeral, and number word) which are in the horizontal lines of the Reed-Kellogg System are converted to classes in the MP model.
- An MP noun modifier is a prepostinal phrase such as “with numerals and number words“. A slanted line implies an existance of a modifier in the Reed-Kellogg System. A relationship always exists between an MP noun

(an MP class) and nouns (noun classes) in an MP noun modifier (an except Type 2) when an MP statement is converted to the MP model.

- There is a prepostinal relationship (for example, <<with>>) between an MP noun class (for example, Fraction) and noun classes (for example, Numeral and Number word) in an MP noun modifier.

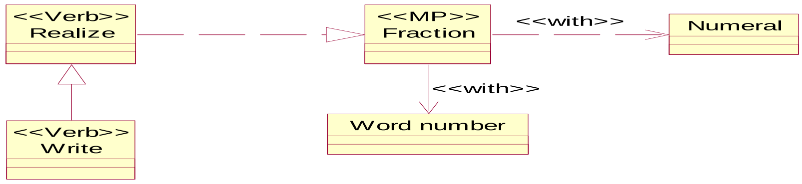

Figure 10: An MP diagram of Type 3 MP Statement

4. Type 4 MP Statement:

- A math standards statement: Estimate and use measuring devices with standard units and non-standard units to measure length.
- An MP noun modifier is a prepositional phrase such as "with standard unit and non-standard units to measure length ".
- There is a prepostinal relationship (for example, <<with>>) between an MP noun class (for example, Measuring device) and noun classes (for example, Standard unit and Non-standard unit) in an MP noun modifier.
- There is an association relationship (for example, measure) between noun classes (for example, Standard unit and Non-standard unit, and length). A super class "Unit of measre" of classes "Standard unit" and "Non-standard unit" has been created.

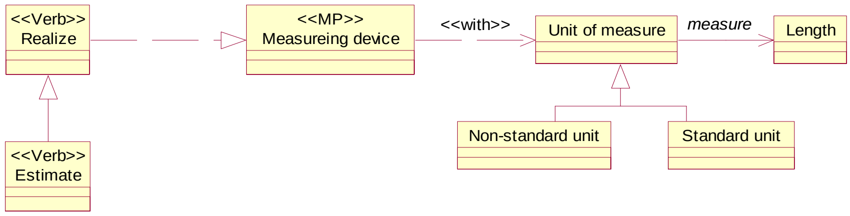

Figure 11: An MP diagram of Type 4 MP Statement

5. Type 4A MP Statement:

- A math educational standards statement: Identify functions with a graph that has a rotation symmetry and a reflection symmetry about x-axis and y-axis.
- An MP noun modifier is a prepostional phrase such as "with a graph that has a rotation symmetry and a relfection symmetry about x-axis and y-axix".
- There is a prepostinal relationship (for example, <<with>>) between an MP class (for example, Function) and noun classes (for example, Graph) in an MP noun modifier.

- There is a prepositional relationship (for example, <<about>>) between noun classes (for example, Rotation symmetry, Reflection symmetry and X-axis, Y-axis). There is an association relationship (for example, has) between noun classes (Graph, and Rotation symmetry and Reflection symmetry).

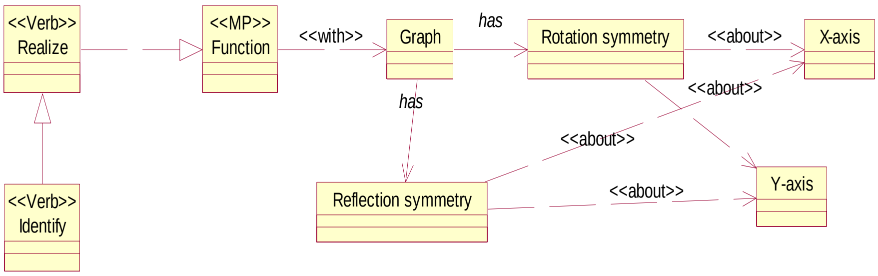

Figure 12: An MP diagram of Type 4A MP Statement

6. Type 5 MP Statement:

- A math standard statement: Develop formulas for determining measurements.
- An MP noun modifier is an infinitive phrase, a prepositional phrase with gerund (for example, for determining measurements), or a pronoun clause.
- An association relationship (for example, determine) exists between an MP class (for example, Formula and Procedure) and a noun class (for example, Measurement) in the MP noun modifier. This noun class is also a *math concept*.

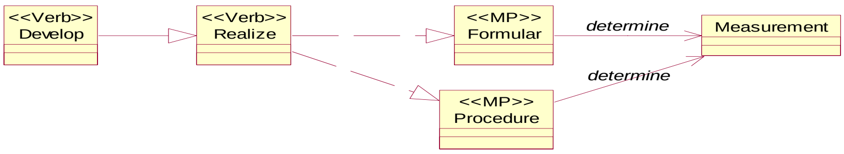

Figure 13: An MP diagram of Type 5 MP Statement

7. Type 5A MP Statement:

- A math standard statement: Create a plan that collects data for a purpose.
- An MP noun modifier is a pronoun clause (for example, that collects data for a purpose).
- An association relationship (for example, collect) exists between an MP class (for example, Plan) and a noun class (for example, Data) in the MP noun modifier. A prepositional relationship (for example, <<for>>) exists between noun classes (for example, Data and Purpose).

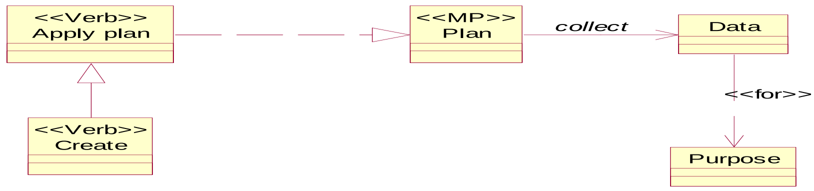

Figure 14: An MP diagram of Type 5A MP Statement

8. Type 6 MP Statement:

- A math educational standard statement: Create a two-dimensional design that contains line of symmetry. An MP noun modifier is a relative noun clause such as “that contins line of symmetry“.
- There is an aggregation relationship between an MP class “Two-dimensional design“ and a noun class “Line of symmetry“.

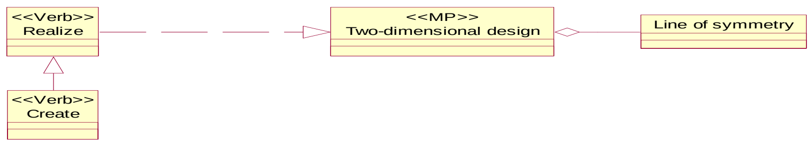

Figure 15: An MP diagram of Type 6 MP Statement

9. Type 7 MP Statement:

- A math educational standards statement: Demonstrate skills using fractions to confirm computation, to verify conjecture, and to explore complex problem-solving situation
- An MP noun modifier is “using fractions to confirm computation, to verify conjecture, and to explore complex problem-solving situation
- There is an independency relationship between an MP class “Skill“ and a noun class “Fraction“. There is also assocition relationships between a noun class “Fraction“ and noun classes “Computation“, “Problem-solving situation“, and “Conjecture“.

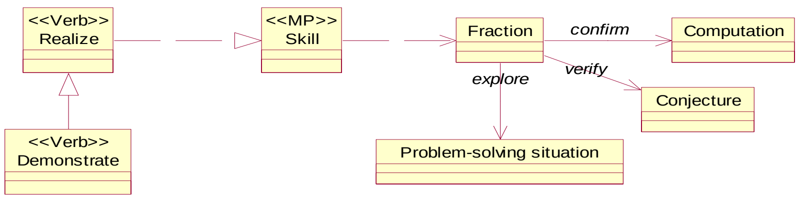

Figure 16: An MP diagram of Type 7 MP Statement

10. Type 8 MP Statement:

- A class "Number theory" is an MP class and a math concept.
- An MP verb modifier is " to rename number quantity". It also has an MP verb (for example, rename) and an MP noun(for example, number quantity). This MP noun is also an MP class and a math concept in the MP model.
- There is an transitive verb relationship (<<to>>) between MP verbs "Apply" and "Rename".

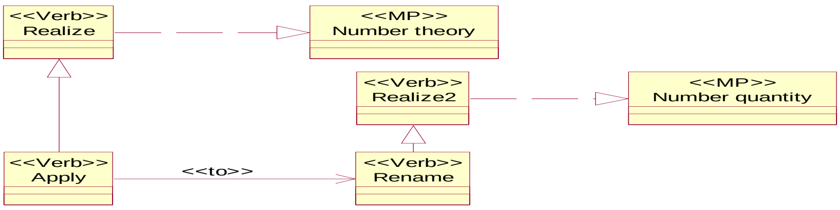

Figure 17: An MP diagram of Type 8 MP Statement

11. Type 8A MP Statement:

- A math educational standards statement: Apply factorials and exponents including factorial exponents to solve problems.
- An MP verb modifier is " to solve problems". It has an MP verb(for example, Solve) and an MP noun (for example, Problem).
- There is a transitive verb relationship between MP verbs "Apply" and "Solve". There is an aggregation relationship between MP classes (Factorial and Exponent) and a noun class (Factorial exponent).

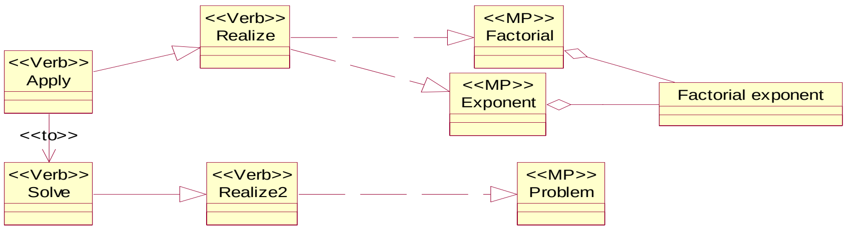

Figure 18: An MP diagram of Type 8A MP Statement

12. Type 8B MP Statement:

- Relate addition and subtraction as an inverse operation.
- An MP verb modifier is " as an inverse operation"
- There is an transitive verb relationship between an MP verbs "Apply" and a noun class "Inverse operation".

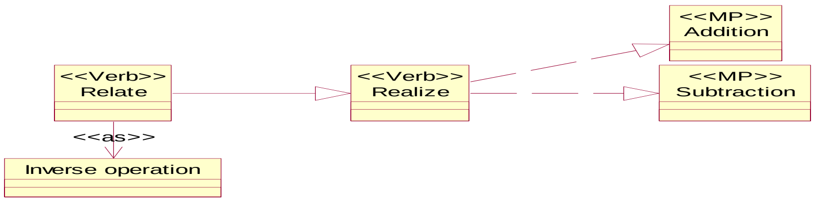

Figure 19: An MP diagram of Type 8B MP Statement

13. Type 8C MP Statement:

- Math standards statement: Translate contextual situation involving area, surface area, volume, and density to Mathematical symbols.
- There is an aggregation relatonship between an MP class " Contextual situation" , and noun classes "Area", "Surface area", "Volume", "Density". There is a transitive verb relationship between an MP verb "Translate" and a noun class "Mathematical symbol" in an MP verb modifier.

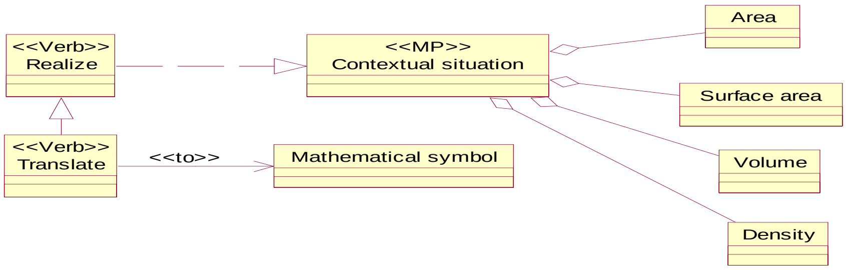

Figure 20: An MP diagram of Type 8C MP Statement

14. Type 8D MP Statement

- A math educational standards statement: Apply combination as a method which creates coefficient of binomial theorem.
- There is an association relationship (create) between noun classes "Mathematical method", and "Bionomial theorem". There is a transitive verb relationship (<<as>>) between MP verb "Apply" and a noun class "Method" in an MP verb modifier.

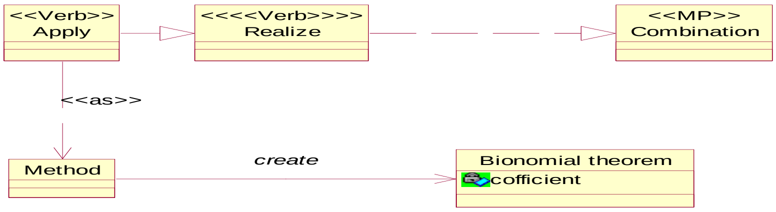

Figure 21: An MP diagram of Type 8 D MP Statement

15. Type 9 MP Statement:

- A math educational standards statement: Create and solve word problems involving addition, subtraction, multiplication, and division of a whole number.
- An MP noun modifier is "involving addition, subtraction, multiplication, and division of a whole number". It has an MP noun "Whole number" , and MP verbs "Add", "Subtract", "Multiply", and "Divide".
- There is an aggregation relationship between two MP classes "Word problem" and "Whole number".

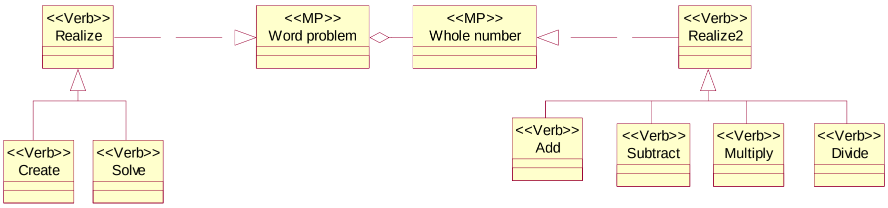

Figure 22: An MP diagram of Type 9MP Statement

16. Type 10 MP Statement:

It can be any combination of a Type 1 MP Statement thru a Type 9 MP statement.

5 An analysis of the MP model

5.1 Why UML for the MP model

We propose the MP model for representing the semantics of mathd educational standards for the purpose of aligning math educational standards. Educational standard alignment can be referred as matching educational standards which describe similar or equivalent concepts. Our alignment method for math educational standards gives the different level of similarity instead of "yes or no" Boolean decision of current existing alignment methods. The different level of similarity is measured in terms of math concepts and the cognitive process of math concepts for our alignment method. Therefore, capturing math concepts and the cognitive process of math concepts from math educational standards statements is a necessarily important task. In the MP model, an MP class represents a math concept represented by a noun and a verb materialization hierarchy realizes the behaviour of an MP class. A verb materialization hierarchy represents the cognitive process of math concepts. UML has been chosen for the MP model for capturing math concepts and the cognitive process of math concepts for the following reasons: 1) UML has a realization relationship which semantically well represents a relationship between an MP class and a verb materialization hierarchy. 2) UML has a stereotype which extends current feature, and a stereotype has been used for defining an MP class for math concepts and a verb stereotype class for the cognitive process of math concepts. However, aligning math educational standards is beyond the scope of this paper.

We use a math ontology which has math concept hierarchies for related math concepts and WordNet for word similarity in our math educational standards alignment method. However, ontology has not been used for representing the semantics of math educational standards because the concept of the MP (Materialization Pattern) model which includes a realization relationship, an MP class, and a verb stereotype class cannot be expressed well with OWL as an ontology. In general, a verb describes a relation between resources as a property in RDF or RDFS, and a relationship between entities as an entity relationship in ER model. A verb also relates an object to other object as an object property in OWL. Math educational standards statements have only one subject "student," and the subject is omitted because they are imperative mood sentences. For example, "*Write fractions with numerals and number words (Ohio State).*" is a math educational standards statement. If we model a verb as a relationship between a "student" and math concepts using OWL, a concept "student" will be repeatedly shown as a domain in an object property because math educational standard statement has only one subject "student". The concept "student" in math educational standards statement is insignificant because math educational standards states what kinds of math concepts should be learned and how they can be learned. Instead, a verb is significant as the cognitive process of a math concept and is modeled as a verb stereotype class. Therefore, we think the MP model using the UML notation is a right choice for capturing math concepts and the cognitive process of math concepts for aligning math educational standards.

5.2 An analysis of the conversion of different types of MP statements to the MP model

The MP model explicitly represents the semantics of sentences by capturing math concepts and the cognitive process of the math concepts from math educational standards statements. The MP model is developed at a sentence level for each sentence from typical math educational standards. Math educational standards consist of well-defined sentences which have math concepts and cognitive process verbs. In the MP model, stereotypes in UML have been used for defining an MP class, a verb stereotype class, a prepositional relationship, and a transitive verb relationship. The MP model will be created by a semi-automatic tool "MPVitz" using UML from 16 different types of MP statements. Each type of MP statements has been classified based on the sentence structure of math educational standards. In these sentences, concepts are represented by nouns or verbs. These concepts are connected by prepositions or verbs. These concepts and connections determine classes and relationships, respectively in the MP model. Each type of MP statements has different relationships between concepts (i.e. classes) when it is converted to the MP model. These different relationships are association, aggregation, dependency, prepositional, transitive verb, and realization relationships. In this section, we analyze how different types of MP statements corresponded to the MP model in terms of classes and relationships between classes. In Table 4, we analyze the conversion of 9 different types of MP statements to the MP model.

6 Conclusions and future work

This article has presented the MP (Materialization Pattern) model for representing the semantics of imperative mood sentences used in math educational standards. Based on the analysis of 700 math educational standards statements from 3 different states, we first classified math educational standard statements into 16 types for a comprehensive coverage of the possible cases of math educational standards statements. We then provided heuristics for determining classes and relationships in the MP model. We illustrated our method using an example of MP statements for each type. We also stated an anlaysis of the MP model. The MP model has captured the semantics of English sentence diagrams based on the Reed-Kellogg system by identifying math concepts, the cognitive process of math concepts, and relationships bewteen concepts from math educational standards. Our MP model is useful for aligning math educational standard statements. With minor or no modification, our approach can be utilized for modeling other educational standards which have impertative mood sentence structures.

We are currently developing a semi-automatic tool “MPVitz” for creating the MP model and an alignment tool which uses MPVitz. The MP model has been proposed for representing the semantics of math educational standards for the purpose of aligning math educational standards. Our alignment tool will be used as a formal validation tool for the MP model because model validation is based on the purpose of a model and its intended use. Validation guarantees that the model satisfies its intended use in terms of the methods employed and the results produced [Macal, 2005]. The validation of the MP model will be clearly shown when we will evaluate our alignment tool for the intended use of the MP model. As a validation (evaluation) method we set up a domain expert judgment as the gold standard. We want to prove whether or not our results from the alignment tool are comparable to a domain expert judgment. We will compare a result from the alignment tool to a domain expert result using the Cohen’s Kappa test.

MP Statement (**MP model**)	MP verb (**Verb stereo type class**)	MP noun (**MP class**: **Math concept** except Type 2)	Relationship between an MP noun and nouns in an MP noun modifier (Relationship between an **MP class** and **noun classes**)	Nouns in an MP noun modifier (**Noun classes** except Type 2, Type 9, Type 9A)	Relationship between nouns in an MP noun modifier (Between **noun classes**)
TYPE 1	Recognize, classify, and order whole numbers.				
	Recognize, Classify, Order	**Whole number**			
TYPE 2	Demonstrate the value of irrational numbers.				
	Demonstrate	**value** (attribute)	MP noun: attribute of an MP class.	**Irrational number** (MP class, Math concept)	
TYPE 3	Write fractions with numeral and number word.				
	Write	**Fraction**	**Prepositional** (with)	**Numeral, Number word**	
TYPE 4	Estimate, use measuring devices with standard and non-standard unit to measure quantity.				
	Estimate, Use	**Measuring device**	**Prepositional** (with)	**Standard unit, Non standard unit, quantity**	**Association** (measure)
TYPE 5	Develop formulas and procedures that determine measurement.				
	Develop	**Formula, Procedure**	**Association** (determine)	**Measurement**	
TYPE 6	Create a two-dimensional design that contains line of symmetry.				
	Create	**Two-dimensional design**	**Aggregation** (contain)	**Line of symmetry**	
TYPE 7	Demonstrate skills using fractions to confirm computation, to verify conjectures, and to explore complex problem-solving situation.				
	Demonstrate	**Skill**	**Dependency** (use)	**Fraction, Computation, Conjecture, Problem-solving situation**	**Association** (confirm, verify, explore)
TYPE 8:	Apply number theory to rename number quantity. A transitive verb relationship exists between "Apply" and "Rename". Verb stereo type classes: Apply, Rename MP nouns (MP classes): Number theory, Number quantity				
TYPE 9	Create and solve word problems involving addition, subtraction, multiplication, and division of a whole number.				
	Create, Solve, Add, Subtract, Multiply, Divide	**Word problem (MP class)**	**Aggregation** (involving)	**Whole number (MP class)**	

Table 4: An analysis of the conversion of MP statements to the MP model

References

[Bhola et al., 2003] Bhola, D.S., Impara, J.C. and Buckendahl, C.W.: Aligning tests with states' content standards: Methods and issues, Educational Measurement: Issues and Practice, 22(3), 21-29, 2003.

[Bloom and Krathwohl, 1956] Bloom, B., Krathwohl, D.: Taxonomy of educational objectives: the classification of educational goals by a committee of college and university examiners Handbook 1: Cognitive domain, New York, Longmans, 1956.

[Booch et al., 2005] Booch, G., Rumbaugh, J., Jacobson, I.: The Unified Modeling Language User Guide, Addison-Wesley, Upper Saddle River, NJ, 2005.

[Bryant and Lee, 2002] Bryan, B., Lee, B.: Two-level Grammar as an Object-Oriented Requirements Specification. In: 35 Hawaii Int. Conf. System Sciences, 2002.

[Bryant et al., 2003] Bryant, B., Lee, B., Cao, F., Zhao, W., Gray, J., Burt, C.: From Natural Language Requirements to Executable Models of Software Components, In: Monterey Workshop on Software Engineering for Embedded Systems, pp. 51—58, 2003.

[Hamilton et al, 2008] Hamilton, L. S., Stecher, B. M., & Yuan, K.: Standard-based reform in the United States; History, research, and future directions, Paper commissioned by the Center on Education Policy, Washington, DC, December 2008.

[Hartman and Link, 2007] Hartman, S., Link S.: English Sentence Structures and EER Modeling, In: the fourth Asia-Pacific conference on conceptual modeling, pp. 27—35, 2007.

[Ilieva and Ormandjieva, 2005] Ilieva, M., Ormandjieva, O.: Automatic Transition of Natural Language Software Requirements Specification into Formal Presentation, In: NLDB 2005, pp. 392—397.

[Ilieva, 2007] Ilieva, M.: Graphical notation for natural language and knowledge representation, In: SEKE 2007, pp. 361—367, 2007.

[Ilieva and Boley, 2008] Ilieva, M., Boley, H.: Representing textual requirements as graphical natural language for UML diagram generation, In: SEKE 2008, pp. 478—483, 2008.

[La Marca, 2001] La Marca, P.M.: Alignment of standards and assessments as an accountability criterion,. Practical Assessment, Research & Evaluation, 7(21), 2001.

[Macal, 2005] Macal C.M.: Model verification and Validation, In Workshop on "Threat Anticipation: Social Science Methods and Models", The University of Chicago and Argonne National Laboratory, April 7-9, 2005.

[Nasstrom and Henriksson, 2008] Nasstrom, G., Henriksson, W. Alignment of Standards and Assessemnt: A theoretical and empirical study of methods for alignment. Electronic Journal of Research in Educational Psychology, No. 16, Vol 6 (3) 2008, 667-690, 2008.

[Olson, 2003] Olson, L.: Standards and test: Keeping them aligned, Research Points, 1(1), 1-4, 2003.

[Reed and Kellogg, 2006] Reed, A., Kellogg, B.: Graded Lessons in English, ISBN 1-4142-8639-2, 2006.

[Sutton and Golder, 2008] Sutton, S., Golder, D.: Achievement Standards Network (ASN): An Application Profile for Mapping K-12 Educational Resoruces to Achievement Standards, Proceeding of International Conf. on Dublin Core and Metadata Applications, 2008.

[Takahashi et al., 2008] Takahashi, M., Takahashi, S., Fujita, Y.: A development method of UML documents from requirement specifications using NLP. International Journal of Computer Applications in Technology, 33, 2-3, pp. 164-175, 2008.

[Tseng and Chen, 2008] Tseng, F., Chen, C.: Enriching the class diagram concepts to capture natural language semantics for database access, Data & Knowledge Engineering 67, pp. 1—29, 2008.

[Yilmazel et al., 2007] Yilmazel, O., Balasubramanian, N., Harwell, S., Bailey, J., Diekema, A., and Liddy, E. Text Categorization for Aligning Educational Standards, Proceedings of the 40th Hawaii International conference on System Sciences 2007.

[Webb, 1999] Webb, N. L.: Alignment of science and mathematics standards and assessments in for states (Research monograph, No. 18), Madison: National Institute for Science Education, 1999.

[Williamson and Williamson, 2010] Williamson, C., Williamson, G.: ACRE: Accountability and Curriculum Report: Essential Standard, http://www.ncpublicschools.org/acre/standards/, access to 02-13-2010.